

## أُتدرب وأحل المسائل

### التكامل بالكسور الجزئية

أجد كلاً من التكاملات الآتية:

$$(x-10x(x+5))dx \quad (1) \int$$

$$x-10x(x+5)=Ax+Bx+5 \Rightarrow x-10=A(x+5)+Bx \quad x=0 \Rightarrow A=-2 \quad x=-5 \Rightarrow B=3 \int$$

$$|x+5|+C \int x-10x(x+5)dx = \int (-2x+3x+5)dx = -2 \ln$$

$$(x^2)dx \quad (2) \int$$

$$x^2=2(1-x)(1+x)=A(1-x)+B(1+x) \Rightarrow 2=A(1-x)+B(1+x) \quad x=1 \Rightarrow A=1 \quad x=-1 \Rightarrow B=1 \int$$

$$|1-x|+C = \ln|1-x| + \ln|1+x| + C = -1 \Rightarrow B=1 \int 2(1-x^2)dx = \int (1-x+1+x)dx = -\ln$$

$$+x-1-x|+C$$

$$(x-2)(x-4)dx \quad (3) \int$$

$$x-2)(x-4)=Ax-2+Bx-4 \Rightarrow 4=A(x-4)+B(x-2) \quad x=2 \Rightarrow A=-2 \quad x=4 \Rightarrow B=4 \int$$

$$|x-4|+C = 2 \ln|x-2| + 2 \ln|x-4| \int 4(x-2)(x-4)dx = \int (-2x-2+2x-4)dx = -2 \ln$$

$$|x-4x-2|+C$$

$$(3x+4x^2+x)dx \quad (4) \int$$

$$3x+4x^2+x=3x+4x(x+1)=Ax+Bx+1 \Rightarrow 3x+4=A(x+1)+Bx \quad x=0 \Rightarrow A=4 \quad x=-1 \Rightarrow B=-1 \int$$

$$|x+1|+C \int x-1 \ln = -1 \Rightarrow B=-1 \int 3x+4x^2+x dx = \int (4x+-1x+1)dx = 4 \ln$$

$$(x^2x^2-4)dx \quad (5) \int$$

$$x^2x^2-4dx = \int (1+4x^2-4)dx \quad 4x^2-4=4(x-2)(x+2)=Ax-2+Bx+2 \Rightarrow 4 = \int$$

$$A(x+2)+B(x-2) \quad x=2 \Rightarrow A=1 \quad x=-2 \Rightarrow B=-1 \int x^2x^2-4dx = \int (1+1x-2+-1x$$

$$|x-2x+2|+C |x+2|+C = x + \ln|x-2| - \ln|x+2| dx = x + \ln$$

$$(3x-6x^2+x-2)dx \quad (6) \int$$

$$3x-6x^2+x-2=3x-6(x+2)(x-1)=Ax+2+Bx-1 \Rightarrow 3x-6=A(x-1)+B(x$$

$$+2) \quad x=-2 \Rightarrow A=4 \quad x=1 \Rightarrow B=-1 \int 3x-6x^2+x-2 dx = \int (4x+2+-1x-1)dx = 4$$

$$|x-1|+C|x+2|-\ln|x-1|$$

$$(4x+104x^2-4x-3)dx \quad (7f)$$

$$4x+104x^2-4x-3=4x+10(2x-3)(2x+1)=A2x-3+B2x+1 \Rightarrow 4x+10=A(2x+1)+B(2x-3)$$

$$x=3 \Rightarrow A=4x=-12 \Rightarrow B=-2 \int 4x+104x^2-4x-3 dx = \int (|2x+1|+C|2x-3|-\ln|2x-3|+2x+1) dx = 2 \ln$$

$$(2x^2+9x-11x^3+2x^2-5x-6)dx \quad (8f)$$

$$2x^2+9x-11x^3+2x^2-5x-6=2x^2+9x-11(x-2)(x+1)(x+3)=Ax-2+Bx+1+Cx+3$$

$$\Rightarrow 2x^2+9x-11=A(x+1)(x+3)+B(x-2)(x+3)+C(x-2)(x+1)$$

$$x=2 \Rightarrow A=1x=-1 \Rightarrow B=3x=-3 \Rightarrow C=-2 \int 2x^2+9x-11x^3+2x^2-5x-6 dx = \int (|x+3|+C|x+1|-2 \ln|x-2|+3 \ln(1x-2+3x+1+-2x+3)) dx = \ln$$

$$(4xx^2-2x-3)dx \quad (9f)$$

$$4xx^2-2x-3=4x(x-3)(x+1)=Ax-3+Bx+1 \Rightarrow 4x=A(x+1)+B(x-3)$$

$$x=3 \Rightarrow A=3x=-1 \Rightarrow B=1 \int 4xx^2-2x-3 dx = \int (3x-3+1x+1) dx = 3 \ln|x-3|+C$$

$$(8x^2-19x+1(2x+1)(x-2)^2)dx \quad (10f)$$

$$8x^2-19x+1(2x+1)(x-2)^2=A2x+1+Bx-2+C(x-2)^2 \Rightarrow 8x^2-19x+1=A(x-2)^2+B(2x+1)(x-2)+C(2x+1)$$

$$x=-12 \Rightarrow A=2x=2 \Rightarrow C=-1x=0 \Rightarrow 1=4A-2B+C \Rightarrow B=3 \int 8x^2-19x+1(2x+1)(x-2)^2 dx = \int (22x+1+3x-2+-1(x-2)^2) dx = \ln$$

$$(9x^2-3x+29x^2-4)dx \quad (11f)$$

$$9x^2-3x+29x^2-4 dx = \int (1+6-3x9x^2-4) dx$$

$$6-3x9x^2-4=6-3x(3x-2) \int (3x+2)=A3x-2+B3x+2 \Rightarrow 6-3x=A(3x+2)+B(3x-2)$$

$$x=23 \Rightarrow A=1x=-3 \Rightarrow B=-2 \int 9x^2-3x+29x^2-4 dx = \int (1+13x-2+-23x+2) dx = x+13 \ln|3x+2|+Cx-2|-23 \ln$$

$$(x^3+2x^2+2x^2+xdx) \quad (12f)$$

$$x^3 + 2x^2 + 2x^2 + x dx = \int (x+1+2-x)x^2 + x dx = 2 - x(x+1) = Ax^2 - x(x+1) = Ax^2 + Bx + 1 \Rightarrow 2 - x = A(x+1) + Bx \Rightarrow 2 - x = Ax + A + Bx \Rightarrow 2 - x = (A+B)x + A$$

$$x=0 \Rightarrow 2 = A \Rightarrow A=2 \quad x=1 \Rightarrow 1 = 3A + B \Rightarrow B = -3$$

$$\int x^3 + 2x^2 + 2x^2 + x dx = \int (x+1+2x-3x+1) dx = \frac{1}{2}x^2 + x + 2 \ln|x+1| + C$$

$$(x^2 + x + 23 - 2x - x^2) dx \quad (13)$$

$$x^2 + x + 23 - 2x - x^2 dx = \int (-1 + 5 - x - x^2 - 2x + 3) dx = 5 - x - x^2 - 2x + 3 = x - 5$$

$$5(x+3)(x-1) = Ax + 3 + Bx - 1 \Rightarrow x - 5 = A(x-1) + B(x+3)$$

$$x = -3 \Rightarrow A = 2 \quad x = 1 \Rightarrow B = -1$$

$$\int x^2 + x + 23 - 2x - x^2 dx = \int (-1 + 2x + 3 - 1x - 1) dx = -x + 2 \ln|x-1| + C - \ln$$

$$(2x - 4(x^2 + 4)(x + 2)) dx \quad (14)$$

$$2x - 4(x^2 + 4)(x + 2) = Ax + 2 + Bx + Cx^2 + 4 \Rightarrow 2x - 4 = A(x^2 + 4) + (Bx + C)(x + 2)$$

$$x = -2 \Rightarrow A = -1 \quad x = 0 \Rightarrow -4 = 4A + 2C \Rightarrow C = 0 \quad x = 1 \Rightarrow -2 = 5A + 3B + 3C \Rightarrow B = 1$$

$$\int 2x - 4(x^2 + 4)(x + 2) dx = \int (-1x + 2 + xx^2 + 4) dx = -\ln C$$

$$(x^3 - 4x^2 - 2x^3 + x^2) dx \quad (15)$$

$$x^3 - 4x^2 - 2x^3 + x^2 dx = \int (1 + -5x^2 - 2x^3 + x^2) dx = -5x^2 - 2x^3 + x^2 = -5x^2 - 2x^2(x+1) = Ax + Bx^2 + Cx + 1 \Rightarrow -5x^2 - 2 = Ax(x+1) + B(x+1) + Cx^2$$

$$x = 0 \Rightarrow B = -2 \quad x = -1 \Rightarrow C = -7 \quad x = 1 \Rightarrow -7 = 2A + 2B + C \Rightarrow A = 2$$

$$\int x^3 - 4x^2 - 2x^3 + x^2 dx = \int (1 + 2x - 2x^2 - 7x + 1) dx = x + 2 \ln|x+1| + C|x| + 2x - 7 \ln$$

$$(x^2 - 5x - 12x^2) dx \quad (16)$$

$$x^2 - 5x - 12x^2 = x - 3(4x - 1)(3x + 2) = A(4x - 1) + B(3x + 2)$$

$$x = 14 \Rightarrow A = -1 \quad x = -23 \Rightarrow B = 1$$

$$\int 3 - x^2 - 5x - 1 |3x + 2| + C|4x - 1| + 13 \ln 2x^2 dx = \int (-14x - 1 + 13x + 2) dx = -14 \ln$$

$$(3x^3 - x^2 + 12x - 6x^4 + 6x^2) dx \quad (17)$$

$$3x^3 - x^2 + 12x - 6x^4 + 6x^2 = 3x^3 - x^2 + 12x - 6x^2(x^2 + 6) = Ax + Bx^2 + Cx + D$$

$$x^2 + 6 \Rightarrow 3x^3 - x^2 + 12x - 6 = Ax(x^2 + 6) + B(x^2 + 6) + (Cx + D)(x^2)$$

$$x = 0 \Rightarrow B = -1 \quad x = 1 \Rightarrow 8 = 7A + 7B + C + D \dots \dots \dots (1) \quad x = -1 \Rightarrow -22 = -7A + 7B - C + D \dots \dots$$

$$((2)x=2 \Rightarrow 38=20A+10B+8C+4D \dots (3)$$

بجمع (1)، (2) ينتج أن:  $14B+2D=-14$ ، وبتعويض  $B=-1$ ، نجد أن  $D=0$

وبطرح (2) من (1) ينتج أن  $14A+2C=30$  أي أن  $C=15-7A$

بالتعويض في (3) ينتج أن:

$$20A-10+8(15-7A)=38-36A=-72 \Rightarrow A=2 \quad C=15-7(2)=1$$

$$\int (5x-2)(x-2)^2 dx \quad (18)$$

$$5x-2(x-2)^2 = Ax-2+B(x-2)^2 \Rightarrow 5x-2 = A(x-2)+B(x-2)^2 \Rightarrow B=8 \quad x=0 \Rightarrow -2$$

$$|x-2| - 8x = -2A+B \Rightarrow A=5 \quad \int 5x-2(x-2)^2 dx = \int (5x-2+8(x-2)^2) dx = 5 \ln$$

$$-2+C$$

ملاحظة: يمكن حل هذا التكامل بالتعويض  $u=x-2$

كما يمكن حله بالأجزاء حيث:  $u=5x-2, dv=(x-2)^{-2}$

أجد قيمة كل من التكاملات الآتية:

$$\int (246+3x-x^2)x^3+2x^2 dx \quad (19)$$

$$6+3x-x^2x^3+2x^2 = 6+3x-x^2x^2(x+2) = Ax+Bx^2+Cx+2 \Rightarrow 6+3x-x^2 = A$$

$$x(x+2)+B(x+2)+C(x^2)x=0 \Rightarrow B=3 \quad x=-2 \Rightarrow C=-1 \quad x=1 \Rightarrow 8=3A+3B+C \Rightarrow A$$

$$|x+2| |24=0 \int 246+3x-x^2x^3+2x^2 dx = \int 24(3x^2+-1x+2) dx = (-3x-\ln$$

$$234=34+\ln 6+32+\ln = -34-\ln$$

$$\int (1/31/39x^2+49x^2-4 dx \quad (20)$$

$$9x^2+49x^2-4 = 1+89x^2-489x^2-4 = 8(3x-2)(3x+2) = A3x-2+B3x+2$$

$$\Rightarrow 8 = A(3x+2)+B(3x-2) \quad x=23 \Rightarrow A=2 \quad x=-23 \Rightarrow B=-2 \quad \int -13139x^2+49x^2$$

$$|3x+2| |3x-2| - 23 \ln -4 dx = \int -1313(1+23x-2+-23x+2) dx = (x+23 \ln$$

$$3=23-13+13-23 \ln |3x-23x+2| | -1313 = 13+23 \ln | -1313 = (x+23 \ln$$

$$343 \ln$$

$$\int (0117-5x(2x+3)(2-x)^2)dx \quad (21f)$$

$$17-5x(2x+3)(2-x)^2=A(2x+3)+B(2-x)+C(2-x)^2 \Rightarrow 17-5x=A(2-x)^2+B(2-x)(2x+3)+C(2x+3)x=-32 \Rightarrow A=2x=2 \Rightarrow C=1x=0 \Rightarrow 17=4A+6B+3C \Rightarrow B=1$$

$$\int 0117-5x(2x+3)(2-x)^2 dx = \int 01(22x+3+12-x+1(2-x)^2) dx = (\ln 1032-12=12+\ln 3+\ln 5+1-\ln|2-x|+12-x)|01 = \ln|2x+3|-\ln$$

$$\int (14416x^2+8x-3)dx \quad (22f)$$

$$416x^2+8x-3=4(4x-1)(4x+3)=A(4x-1)+B(4x+3) \Rightarrow 4=A(4x+3)+B(4x-1)x=14 \Rightarrow A=1x=-34 \Rightarrow B=-1$$

$$\int 14416x^2+8x-3 dx = \int 14(14x-1+-14x+3) dx = (14 \ln|4x-1|-14 \ln|4x+3|)|14 = (14 \ln|4x-1|-14 \ln|4x+3|) dx = (14 \ln 351937) = 14 \ln 519 - \ln$$

$$\int (345x+5x^2+x-6)dx \quad (23f)$$

$$5x+5x^2+x-6=5x+5(x-2)(x+3)=A(x-2)+B(x+3) \Rightarrow 5x+5=A(x+3)+B(x-2)x=2 \Rightarrow A=3x=-3 \Rightarrow B=2$$

$$\int 345x+5x^2+x-6 dx = \int 34(3x-2+2x+3) dx = 9896 = \ln 7 - 2 \ln 2 + 2 \ln|x+3| |34 = 3 \ln|x-2| + 2 \ln|x+3| = (3 \ln$$

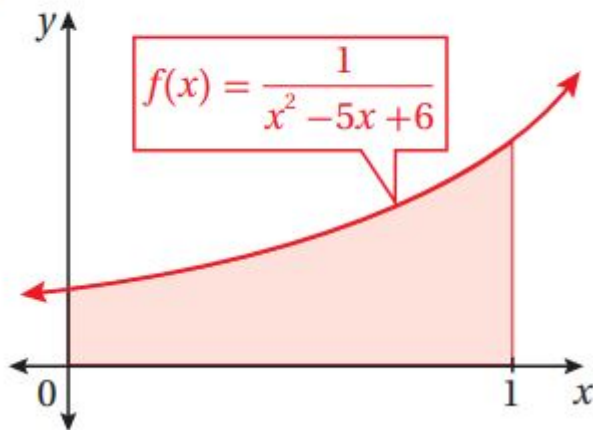
$$\int (344x^3-4x^2+4x)dx \quad (24f)$$

$$4x^3-4x^2+4x=4x(x-2)^2=A(x-2)+B(x-2)^2+Cx \Rightarrow 4=A(x-2)^2+Bx(x-2)+Cx=0 \Rightarrow A=1x=2 \Rightarrow C=2x=1 \Rightarrow 4=A-B+C \Rightarrow B=-1$$

$$A = \int 344x^3-4x^2+4x dx = (\ln|x-2|-2x-2)|34 = (\ln|x-2|-2x-2) dx = \int 34(1x+-1x-2+2(x-2)^2) dx = (\ln 233+2=1+\ln 2-1-\ln|x-2|-2x-2)|34 = \ln$$

أجد مساحة المنطقة المظللة في كل من التمثيلين البيانيين الآتيين:

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$$A = \int_0^1 \frac{1}{x^2 - 5x + 6} dx = \int_0^1 \frac{1}{(x-3)(x-2)} dx = \int_0^1 \left( \frac{A}{x-3} + \frac{B}{x-2} \right) dx$$

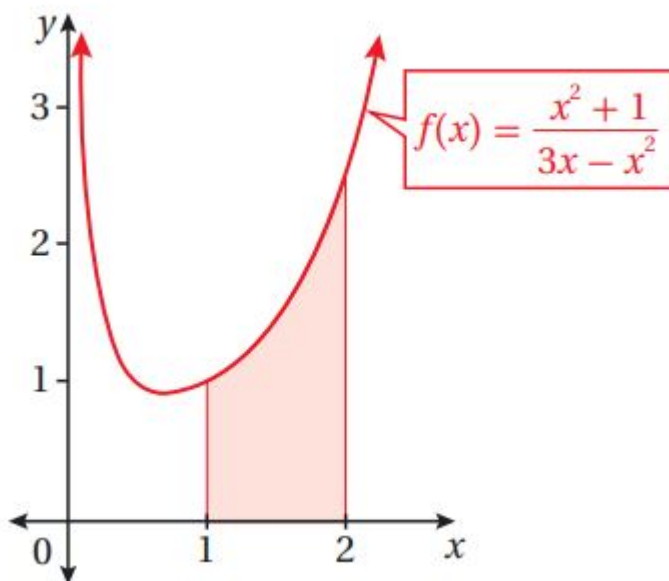
$$1 = A(x-2) + B(x-3) \Rightarrow 1 = A(x-2) + B(x-3)$$

$$1 = Ax - 2A + Bx - 3B = (A+B)x - 2A - 3B$$

$$\begin{cases} A+B=0 \\ -2A-3B=1 \end{cases} \Rightarrow \begin{cases} A=-B \\ -2(-B)-3B=1 \end{cases} \Rightarrow \begin{cases} A=-B \\ 2B-3B=1 \end{cases} \Rightarrow \begin{cases} A=-B \\ -B=1 \end{cases} \Rightarrow \begin{cases} A=1 \\ B=-1 \end{cases}$$

$$A = \int_0^1 \frac{1}{x^2 - 5x + 6} dx = \int_0^1 \left( \frac{1}{x-3} - \frac{1}{x-2} \right) dx = \left( \ln|x-3| - \ln|x-2| \right) \Big|_0^1 = \ln|1-3| - \ln|1-2| = \ln 2 - \ln 1 = \ln 2$$

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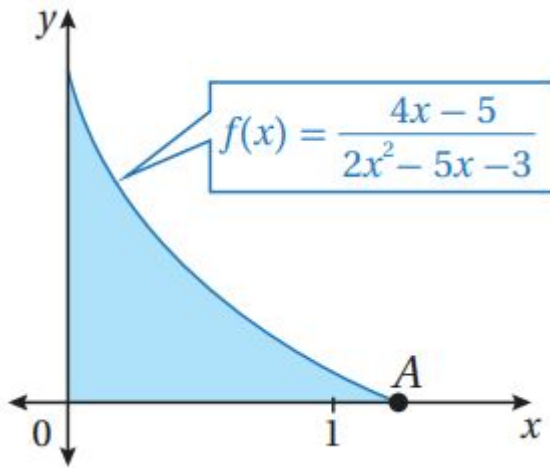


$$A = \int_1^2 \frac{x^2 + 1}{3x - x^2} dx = \int_1^2 \frac{x^2 + 1}{x(3-x)} dx = \int_1^2 \left( \frac{A}{x} + \frac{B}{3-x} \right) dx$$

$$x^2 + 1 = A(3-x) + Bx = 3A - Ax + Bx = (B-A)x + 3A$$

$$\begin{cases} B-A=1 \\ 3A=1 \end{cases} \Rightarrow \begin{cases} B=1+A \\ 3A=1 \end{cases} \Rightarrow \begin{cases} B=1+\frac{1}{3} \\ A=\frac{1}{3} \end{cases}$$

$$A = \int_1^2 \frac{x^2 + 1}{3x - x^2} dx = \int_1^2 \left( \frac{1/3}{x} + \frac{4/3}{3-x} \right) dx = \frac{1}{3} \ln|x| - \frac{4}{3} \ln|3-x| \Big|_1^2 = \frac{1}{3} \ln 2 - \frac{4}{3} \ln 1 - \left( \frac{1}{3} \ln 1 - \frac{4}{3} \ln 2 \right) = \frac{1}{3} \ln 2 - \frac{4}{3} \ln 1 - \frac{1}{3} \ln 1 + \frac{4}{3} \ln 2 = \frac{5}{3} \ln 2$$



يبين الشكل المجاور جزءاً من منحنى  
الاقتران:  $f(x) = \frac{4x-5}{2x^2-5x-3}$

(27) أجد إحداثيي النقطة A.

$$f(x)=0 \Rightarrow 4x-5=0 \Rightarrow x=54 \Rightarrow A(54,0)$$

(28) أجد مساحة المنطقة المظللة.

$$49243 = \ln 498 - \ln |2x^2 - 5x - 3|_{054} = \ln A = \int_{054} 4x - 52x^2 - 5x - 3 dx = \ln$$

ملاحظة: البسط هو مشتقة المقام، فلا داعي لتجزئة الكسر.

أجد كلاً من التكاملات الآتية:

$$\int (x dx + 29x + \cos 2x \cos \sin x)$$

$$\begin{aligned} x u + u^2 x dx &= \int \sin x + \cos 2x \cos x \int \sin x \Rightarrow dx = du - \sin x \Rightarrow du dx = -\sin u = \cos \\ x &= \int -1u + u^2 du - 1u + u^2 = -1u(1+u) = Au + B1 + u \Rightarrow -1 = A(1+u) \times du - \sin \\ x \cos + B u u &= 0 \Rightarrow A = -1u = -1 \Rightarrow B = 1 \int -1u + u^2 du = \int (-1u + 11 + u) du \Rightarrow \int \sin \\ x &+ C = \ln |x \cos| + \ln |1+u| + C = \ln |1 + \cos x| + \ln |\cos x dx = -\ln x + \cos 2 \\ x &+ C | + \sec n \end{aligned}$$

$$\int (1x^2 + xx dx) \quad (30)$$

$$\begin{aligned} u = x \Rightarrow u^2 = x \Rightarrow dx &= 2u du \int 1x^2 + xx dx = \int 1u^4 + u^3 2u du = \int 2u^3 + u^2 du \\ 2u^3 + u^2 &= 2u^2(u+1) = Au + Bu^2 + Cu + 1 \Rightarrow 2 = Au(u+1) + B(u+1) + Cu \\ 2u = 0 &\Rightarrow B = 2 \\ u = -1 \Rightarrow C &= 2u = 1 \Rightarrow 2 = 2A + 2B + C \Rightarrow A = -2 \int 2u^3 + u^2 du = \int (-2u + 2u^2 + 2u \\ |u+1| - 2u + C &|u+1| + C \Rightarrow \int 1x^2 + xx dx = 2 \ln |u| - 2u + 2 \ln |1+u| du = -2 \ln \end{aligned}$$

$$\int (e^{2x} e^{2x} + 3e^x + 2 dx) \quad (31)$$

$$u = ex \Rightarrow du dx = ex = u \Rightarrow dx = du u \int e^{2x} e^{2x} + 3e^x + 2 dx = \int u^2 u^2 + 3u + 2 \times du$$

$$u = \int u u^2 + 3u + 2 du u u^2 + 3u + 2 = u(u+1)(u+2) = Au+1 + Bu+2 \Rightarrow u = A(u+2) + B(u+1)$$

$$u = -1 \Rightarrow A = -1 u = -2 \Rightarrow B = 2 \int u u^2 + 3u + 2 du = \int (-1u+1+2u+2)$$

$$(ex(ex+1) + 2 \ln|u+2| + C \Rightarrow \int e^{2x} e^{2x} + 3e^x + 2 dx = -\ln|u+1| + 2 \ln u = -\ln|u+2| + C$$

$$(x-4) dx \int 2x(\sin^2 x \sin x \cos x)$$

$$x u(u^2 - x - 4) dx = \int \cos x (\sin^2 x \sin x) \int \cos x \Rightarrow dx = du \cos x \Rightarrow du dx = \cos u = \sin x$$

$$x = \int 1 u(u^2 - 4) du 1 u(u^2 - 4) = 1 u(u-2)(u+2) = Au + Bu - 2 + Cu + 4 \times du \cos^2 \Rightarrow 1 = A(u-2)(u+2) + Bu(u+2) + Cu(u-2)$$

$$u = 0 \Rightarrow A = -14 u = 2 \Rightarrow B = 18 u = -|u| + 18 \ln 2 \Rightarrow C = 18 \int 1 u(u^2 - 4) du = \int (-14u + 18u - 2 + 18u + 2) du = -14 \ln|x - |\sin x| + 18 \ln|\sin x - 4| dx = -14 \ln x (\sin^2 x \sin|u+2| + C \Rightarrow \int \cos u - 2| + 18 \ln|x+2| + C|\sin^2| + 18 \ln$$