

إجابات كتاب التمارين

تكامل اقترانات خاصة

أجد كلاً من التكاملات الآتية:

(1) $\int 4e^{-5x} dx$

$$\int 4e^{-5x} dx = -45e^{-5x} + C$$

(2) $\int (\sin 2x - \cos 2x) dx$

$$\int (\sin 2x - \cos 2x) dx = -12\cos 2x - 12\sin 2x + C$$

(3) $\int \cos^2 2x dx$

$$\int \cos^2 2x dx = 12 \int (1 + \cos 4x) dx = 12x + 18\sin 4x + C$$

(4) $\int e^x + 4e^{2x} dx$

$$\int e^x + 4e^{2x} dx = \int (e^{-x} + 4e^{-2x}) dx = -e^{-x} - 2e^{-2x} + C$$

(5) $\int (\cos x \sin^2 x - 2e^x) dx$

$$\int (\cot x \csc x - 2e^x) dx = -\csc x - 2e^x + C$$

(6) $\int (3\cos 3x - \tan^2 x) dx$

$$\int (3\cos 3x - \tan^2 x) dx = \int (3\cos 3x - (\sec^2 x - 1)) dx = \sin 3x - \tan x + x + C$$

(7) $\int \cos x(1 + \csc^2 x) dx$

$$\int \cos 3x(1 + \csc^2 x) dx = \int \cos x(1 + 1\sin^2 x) dx = \int \cos x + \cot x \csc x dx = \sin x - \csc x + C$$

(8) $\int x^2 + x - 4x + 2 dx$

$$\int x^2 + x - 4x + 2 dx = \int (x - 1 - 2x + 2) dx = 12x^2 - x - 2\ln |x+2| + C$$

(9) $\int 1e^x dx$

$$\int 1e^x dx = \int e^{-12x} dx = -\frac{1}{12}e^{-12x} + C$$

(10) $\int (1 \cos^2 x + 1/x^2) dx$

$$\int (1 \cos^2 x + 1/x^2) dx = \int (\sec^2 x + x^{-2}) dx = \tan x - \frac{1}{x} + C$$

(11) $\int x^2 - 2xx^3 - 3x^2 dx$

$$\int x^2 - 2xx^3 - 3x^2 dx = \int (x^2 - 6x^3 - 3x^2) dx = \frac{1}{3}x^3 - 6 \cdot \frac{1}{4}x^4 - \frac{3}{2}x^2 + C = \frac{1}{3}x^3 - \frac{3}{2}x^4 - \frac{3}{2}x^2 + C$$

(12) $\int \ln \sec x dx$

$$\int \ln \sec x dx = \int \cos x dx = \sin x + C$$

(13) $\int \sin^2 x dx$

$$\int \sin^2 x dx = \frac{1}{2} \int (1 - \cos 2x) dx = \frac{1}{2}(x - \frac{1}{2}\sin 2x) + C = \frac{1}{2}x - \frac{1}{4}\sin 2x + C$$

(14) $\int 3^{2x-1} dx$

$$\int 3^{2x-1} dx = \frac{1}{2} \int 3^{2x-1} dx = \frac{1}{2} \ln |2x-1| + C$$

(15) $\int 3 - 2 \cos 12x \sin^2 12x dx$

$$\int 3 - 2 \cos 12x \sin^2 12x dx = \int (3 \csc^2 12x - 2 \cot 12x \csc 12x) dx = -\frac{1}{6} \cot 12x + \frac{1}{4} \csc 12x + C$$