أتدرب وأحل المسائل

التكامل بالكسور الجزئية

أجد كلاً من التكاملات الآتية:

(x−10x(x+5)dx (1∫

x−10x(x+5)=Ax+Bx+5⇒x−10=A(x+5)+Bxx=0⇒A=−2x=−5⇒B=3∫|x+5|+C|x|+3lnx−10x(x+5)dx=∫(−2x+3x+5)dx=−2ln

(x2dx (2−21∫

x2=2(1−x)(1+x)=A1−x+B1+x⇒2=A(1+x)+B(1−x)x=1⇒A=1x−21|1|1+x|+C=ln|1−x|+ln=−1⇒B=1∫21−x2dx=∫(11−x+11+x)dx=−ln+x1−x|+C

(x−2)(x−4)dx (3)4∫

x−2)(x−4)=Ax−2+Bx−4⇒4=A(x−4)+B(x−2)x=2⇒A=−2x=4⇒B=)4|x−4|+C=2ln|x−2|+2ln2∫4(x−2)(x−4)dx=∫(−2x−2+2x−4)dx=−2ln|x−4x−2|+C

(3x+4x2+xdx (4∫

3x+4x2+x=3x+4x(x+1)=Ax+Bx+1⇒3x+4=A(x+1)+Bxx=0⇒A=4x|x+1|+C|x|−ln=−1⇒B=−13x+4x2+xdx=∫(4x+−1x+1)dx=4ln

(x2x2−4dx (5∫

x2x2−4dx=∫(1+4x2−4)dx4x2−4=4(x−2)(x+2)=Ax−2+Bx+2⇒4=∫A(x+2)+B(x−2)x=2⇒A=1x=−2⇒B=−1∫x2x2−4dx=∫(1+1x−2+−1x|x−2x+2|+C|x+2|+C=x+ln|x−2|−ln+2)dx=x+ln

(3x−6x2+x−2dx (6∫

3x−6x2+x−2=3x−6(x+2)(x−1)=Ax+2+Bx−1⇒3x−6=A(x−1)+B(x+2)x=−2⇒A=4x=1⇒B=−1x3x−6x2+x−2dx=∫(4x+2+−1x−1)dx=4

|x−1|+C|x+2|−lnln

(4x+104x2−4x−3dx (7∫

4x+104x2−4x−3=4x+10(2x−3)(2x+1)=A2x−3+B2x+1⇒4x+10=A(2x+1)+B(2x−3)x=32⇒A=4x=−12⇒B=−2∫4x+104x2−4x−3dx=∫(|2x+1|+C|2x−3|−ln42x−3+−22x+1)dx=2ln

(2x2+9x−11x3+2x2−5x−6dx (8∫

2x2+9x−11x3+2x2−5x−6=2x2+9x−11(x−2)(x+1)(x+3)=Ax−2+Bx+1+Cx+3⇒2x2+9x−11=A(x+1)(x+3)+B(x−2)(x+3)+C(x−2)(x+1)x=2⇒A=1x=−1⇒B=3x=−3⇒C=−2∫2x2+9x−11x3+2x2−5x−6dx=∫|x+3|+C|x+1|−2ln|x−2|+3ln(1x−2+3x+1+−2x+3)dx=ln

(4xx2−2x−3dx (9∫

4xx2−2x−3=4x(x−3)(x+1)=Ax−3+Bx+1⇒4x=A(x+1)+B(x−3)x=3|x|x−3|+ln⇒A=3x=−1⇒B=1∫4xx2−2x−3dx=∫(3x−3+1x+1)dx=3ln+1|+C

(8x2−19x+1(2x+1)(x−2)2dx (10∫

8x2−19x+1(2x+1)(x−2)2=A2x+1+Bx−2+C(x−2)2⇒8x2−19x+1=A(x−2)2+B(2x+1)(x−2)+C(2x+1)x=−12⇒A=2x=2⇒C=−1x=0⇒1=4A−2B+C⇒B=3∫8x2−19x+1(2x+1)(x−2)2dx=∫(22x+1+3x−2+−1(x|x−2|+1x−2+C|2x+1|+3ln−2)2)dx=ln

(9x2−3x+29x2−4dx (11∫

9x2−3x+29x2−4dx=∫(1+6−3x9x2−4)dx6−3x9x2−4=6−3x(3x−2∫)(3x+2)=A3x−2+B3x+2⇒6−3x=A(3x+2)+B(3x−2)x=23⇒A=1x=−|323⇒B=−2∫9x2−3x+29x2−4dx=∫(1+13x−2+−23x+2)dx=x+13ln|3x+2|+Cx−2|−23ln

(x3+2x2+2x2+xdx (12∫

x3+2x2+2x2+xdx=∫(x+1+2−xx2+x)dx2−xx2+x=2−xx(x+1)=Ax∫
+Bx+1⇒2−x=A(x+1)+Bxx=0⇒A=2x=−1⇒B=−3∫x3+2x2+2x2+xd
|x+1|+C|x|−3lnx=∫(x+1+2x+−3x+1)dx=12x2+x+2ln

(x2+x+23−2x−x2dx (13∫

x2+x+23−2x−x2dx=∫(−1+5−x−x2−2x+3)dx5−x−x2−2x+3=x−∫
5(x+3)(x−1)=Ax+3+Bx−1⇒x−5=A(x−1)+B(x+3)x=−3⇒A=2x=1⇒
|x+3|B=−1∫x2+x+23−2x−x2dx=∫(−1+2x+3+−1x−1)dx=−x+2ln
|x−1|+C−ln

(2x−4(x2+4)(x+2)dx (14∫

2x−4(x2+4)(x+2)=Ax+2+Bx+Cx2+4⇒2x−4=A(x2+4)+(Bx+C)(x+
2)x=−2⇒A=−1x=0⇒−4=4A+2C⇒C=0x=1⇒−2=5A+3B+3C⇒B=1∫
|x2+4|+|x+2|+12ln2x−4(x2+4)(x+2)dx=∫(−1x+2+xx2+4)dx=−ln
C

(x3−4x2−2x3+x2dx (15∫

x3−4x2−2x3+x2dx=∫(1+−5x2−2x3+x2)dx−5x2−2x3+x2=−5x2∫
−2x2(x+1)=Ax+Bx2+Cx+1⇒−5x2−2=Ax(x+1)+B(x+1)+Cx2x=0⇒
B=−2x=−1⇒C=−7x=1⇒−7=2A+2B+C⇒A=2∫x3−4x2−2x3+x2dx=
|x+1|+C|x|+2x−7ln∫(1+2x+−2x2+−7x+1)dx=x+2ln

(x2−5x−12x2dx (16−3∫

x2−5x−12x2=x−312x2+5x−2=x−3(4x−1)(3x+2)=A4x−1+B3x−3
+2⇒x−3=A(3x+2)+B(4x−1)x=14⇒A=−1x=−23⇒B=1∫3−x2−5x−1
|3x+2|+C|4x−1|+13ln2x2dx=∫(−14x−1+13x+2)dx=−14ln

(3x3−x2+12x−6x4+6x2dx (17∫

3x3−x2+12x−6x4+6x2=3x3−x2+12x−6x2(x2+6)=Ax+Bx2+Cx+D
x2+6⇒3x3−x2+12x−6=Ax(x2+6)+B(x2+6)+(Cx+D)(x2)x=0⇒B=−
1x=1⇒8=7A+7B+C+D.....(1)x=−1⇒−22=−7A+7B−C+D.....

((2)x=2⇒38=20A+10B+8C+4D.....(3

بجمع (1)، (2) ينتج أن: 14B+2D=−14، وبتعويض B=−1، نجد أن D=0

وبطرح (2) من (1) ينتج أن 14A+2C=30 أي أن C=15−7A

بالتعويض في (3) ينتج أن:

20A−10+8(15−7A)=38−36A=−72⇒A=2C=15−7(2)=1∫3x3−x2+12|x2+6|+C|x|+1x+12lnx−6x4+6x2dx=∫(2x+−1x2+xx2+6)dx=2ln

(5x−2(x−2)2dx (18∫

5x−2(x−2)2=Ax−2+B(x−2)2⇒5x−2=A(x−2)+Bx=2⇒B=8x=0⇒−2|x−2|−8x=−2A+B⇒A=5∫5x−2(x−2)2dx=∫(5x−2+8(x−2)2)dx=5ln−2+C

ملاحظة: يمكن حل هذا التكامل بالتعويض u=x−2

كما يمكن حله بالأجزاء حيث: u=5x−2,dv=(x−2)−2

أجد قيمة كل من التكاملات الآتية:

(246+3x−x2x3+2x2dx (19∫

6+3x−x2x3+2x2=6+3x−x2x2(x+2)=Ax+Bx2+Cx+2⇒6+3x−x2=Ax(x+2)+B(x+2)+C(x2)x=0⇒B=3x=−2⇒C=−1x=1⇒8=3A+3B+C⇒A|x+2|)|24=0∫246+3x−x2x3+2x2dx=∫24(3x2+−1x+2)dx=(−3x−ln234=34+ln6+32+ln=−34−ln

(1/31/39x2+49x2−4dx (20−∫

9x2+49x2−4=1+89x2−489x2−4=8(3x−2)(3x+2)=A3x−2+B3x+2⇒8=A(3x+2)+B(3x−2)x=23⇒A=2x=−23⇒B=−2∫−13139x2+49x2|3x+2|)|3x−2|−23ln−4dx=∫−1313(1+23x−2+−23x+2)dx=(x+23ln3=23−13+13−23ln|3x−23x+2|)|−1313=13+23ln|−1313=(x+23ln343ln

(0117−5x(2x+3)(2−x)2dx (21∫

17−5x(2x+3)(2−x)2=A2x+3+B2−x+C(2−x)2⇒17−5x=A(2−x)2+B(2−x)(2x+3)+C(2x+3)x=−32⇒A=2x=2⇒C=1x=0⇒17=4A+6B+3C⇒B=1∫0117−5x(2x+3)(2−x)2dx=∫01(22x+3+12−x+1(2−x)2)dx=(ln1032−12=12+ln3+ln5+1−ln|2−x|+12−x)|01=ln|2x+3|−ln

(14416x2+8x−3dx (22∫

416x2+8x−3=4(4x−1)(4x+3)=A4x−1+B4x+3⇒4=A(4x+3)+B(4x−1)x=14⇒A=1x=−34⇒B=−1∫14416x2+8x−3dx=∫14(14x−1+−14x1|4x−14x+3|)|14=14(ln|4x+3|)|14=(14ln|4x−1|−14ln+3)dx=(14ln351937)=14ln519−ln

(345x+5x2+x−6dx (23∫

5x+5x2+x−6=5x+5(x−2)(x+3)=Ax−2+Bx+3⇒5x+5=A(x+3)+B(x−2)x=2⇒A=3x=−3⇒B=2∫345x+5x2+x−6dx=∫34(3x−2+2x+3)dx9896=ln7−2ln2+2ln|x+3|)|34=3ln|x−2|+2ln=(3ln

(344x3−4x2+4xdx (24∫

bbb4x3−4x2+4x=4x(x−2)2=Ax+Bx−2+C(x−2)2⇒4=A(x−2)2+Bx(x−2)+Cxx=0⇒A=1x=2⇒C=2x=1⇒4=A−B+C⇒B=−1A=∫344x3−4x|x−2|−2x−2)|34=(l|x|−ln2+4xdx=∫34(1x+−1x−2+2(x−2)2)dx=(ln233+2=1+ln2−1−ln|xx−2|−2x−2)|34=lnn

أجد مساحة المنطقة المظللة في كل من التمثيلين البيانيين الآتيين:

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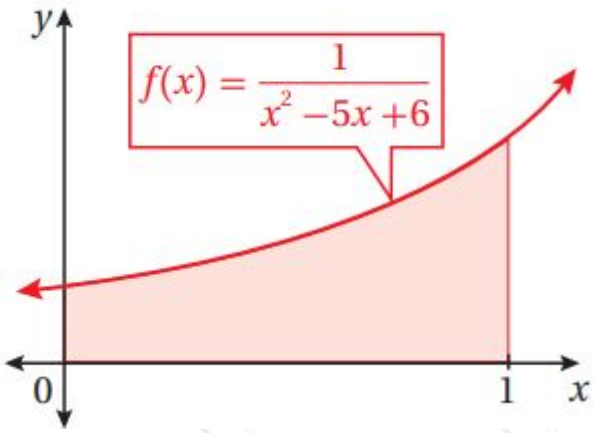

A=∫011x2−5x+6dx1x2−5x+6=1(x−3)(x−2)=Ax−3+Bx−2⇒1=A(x−2)+B(x−3)x=3⇒A=1x=2⇒B=−1A=∫011x2−5x+6dx=∫01(1x−3+4332=ln2−ln|x−3x−2||01=ln|x−2|)|01=ln|x−3|−ln−1x−2)dx=(ln

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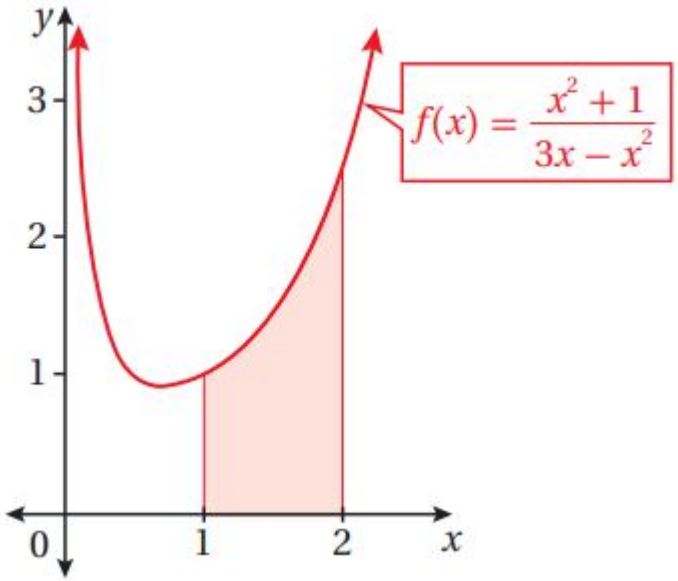
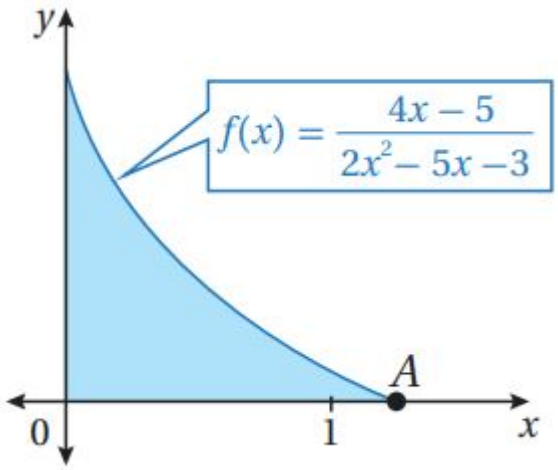


A=∫12x2+13x−x2dxx2+13x−x2=−1+3x+13x−x23x+13x−x2=3x2+1+1x(3−x)=Ax+B3−x⇒3x+1=A(3−x)+Bxx=0⇒A=13=−2+13ln2∫12x2+13x−x2dx=∫12(−1+103+13−x)d2x=3⇒B=−113ln+103lnx

يبين الشكل المجاور جزءاً من منحنى الاقتران: f(x)=4x−52x2−5x−3



(27) أجد إحداثيي النقطة A.

(f(x)=0⇒4x−5=0⇒x=54⇒A(54,0

(28) أجد مساحة المنطقة المظللة.

49243=ln498−ln|2x2−5x−3|054=lnA=∫0544x−52x2−5x−3dx=ln

ملاحظة: البسط هو مشتقة المقام، فلا داعي لتجزئة الكسر.

أجد كلاً من التكاملات الآتية:

(xdx (29x+cos2xcossin∫

xu+u2xdx=∫sinx+cos2xcosx∫sinx⇒dx=du−sinx⇒dudx=−sinu=cos x=∫−1u+u2du−1u+u2=−1u(1+u)=Au+B1+u⇒−1=A(1+u)×du−sin xcos+Buu=0⇒A=−1u=−1⇒B=1∫−1u+u2du=∫(−1u+11+u)du⇒∫sin x|+C=lxcos|1+u|+C1+cosx|+C=ln|1+cosx|+ln|cosxdx=−lnx+cos2 x|+C|1+secn

(1x2+xxdx (30∫

u=x⇒u2=x⇒dx=2udu∫1x2+xxdx=∫1u4+u32udu=∫2u3+u2du2u3+ u2=2u2(u+1)=Au+Bu2+Cu+1⇒2=Au(u+1)+B(u+1)+Cu2u=0⇒B=2 u=−1⇒C=2u=1⇒2=2A+2B+C⇒A=−2∫2u3+u2du=∫(−2u+2u2+2u |u+1u|−2u+C|u+1|+C⇒∫1x2+xxdx=2ln|u|−2u+2ln+1)du=−2ln

(e2xe2x+3ex+2dx (31∫

u=ex⇒dudx=ex=u⟹dx=duu∫e2xe2x+3ex+2dx=∫u2u2+3u+2×duu=∫uu2+3u+2duuu2+3u+2=u(u+1)(u+2)=Au+1+Bu+2⇒u=A(u+2)+B(u+1)u=−1⇒A=−1u=−2⇒B=2∫uu2+3u+2du=∫(−1u+1+2u+2)(ex(ex+1)+2ln|u+2|+C⇒∫e2xe2x+3ex+2dx=−ln|u+1|+2lndu=−ln+2)+C

(x−4)dx (32x(sin2xsincos∫

xu(u2−x−4)dx=∫cosx(sin2xsinx∫cosx⇒dx=ducosx⇒dudx=cosu=sinx=∫1u(u2−4)du1u(u2−4)=1u(u−2)(u+2)=Au+Bu−2+Cu+4)×ducos2⇒1=A(u−2)(u+2)+Bu(u+2)+Cu(u−2)u=0⇒A=−14u=2⇒B=18u=−||u|+18ln2⇒C=18∫1u(u2−4)du=∫(−14u+18u−2+18u+2)du=−14lnx−|sinx|+18ln|sinx−4)dx=−14lnx(sin2xsin|u+2|+C⇒∫cosu−2|+18lnx+2|+C|sin2|+18ln