

أدرب وأحل المسائل

التكامل بالكسور الجزئية

أجد كلاً من التكاملات الآتية:

$$\int (x-10x(x+5))dx \quad (1)$$

$$x-10x(x+5)=Ax+Bx+5 \Rightarrow x-10=A(x+5)+Bx \Rightarrow A=-2 \quad B=3$$

$$\int \frac{1}{x+5} + C|x| + 3 \ln|x-10x(x+5)| dx = \int (-2x+3x+5) dx = -2 \ln$$

$$\int x^2 dx \quad (2)$$

$$x^2=2(1-x)(1+x)=A(1-x)+B(1+x) \Rightarrow 2=A(1-x)+B(1+x) \Rightarrow A=1 \quad B=1$$

$$\int \frac{1}{|1+x|} + C = \ln|1-x| + \ln|1+x| = -1 \Rightarrow B=1$$

$$\int 2(1-x)^2 dx = \int (1-2x+1+x^2) dx = -\ln|x-1-x| + C$$

$$\int (x-2)(x-4) dx \quad (3)$$

$$(x-2)(x-4)=Ax-2+Bx-4 \Rightarrow 4=A(x-4)+B(x-2) \Rightarrow A=-2 \quad B=4$$

$$\int \frac{1}{|x-4|} + C = 2 \ln|x-2| + 2 \ln 2$$

$$\int 4(x-2)(x-4) dx = \int (-2x-2+2x-4) dx = -2 \ln|x-4x-2| + C$$

$$\int (3x+4x^2+x) dx \quad (4)$$

$$3x+4x^2+x=3x+4x(x+1)=Ax+Bx+1 \Rightarrow 3x+4=A(x+1)+Bx \Rightarrow A=4 \quad B=-1$$

$$\int \frac{1}{|x+1|} + C|x| - \ln|x-1| = -1 \Rightarrow B=-1$$

$$\int 3x+4x^2+x dx = \int (4x+-1x+1) dx = 4 \ln$$

$$\int x^2(x^2-4) dx \quad (5)$$

$$x^2(x^2-4) dx = \int (1+4x^2-4) dx$$

$$4x^2-4=4(x-2)(x+2)=Ax-2+Bx+2 \Rightarrow 4=A(x+2)+B(x-2) \Rightarrow A=1 \quad B=-1$$

$$\int x^2(x^2-4) dx = \int (1+1x-2+-1x|x-2x+2| + C|x+2| + C = x + \ln|x-2| - \ln|x+2|) dx = x + \ln$$

$$\int (3x-6x^2+x-2) dx \quad (6)$$

$$3x-6x^2+x-2=3x-6(x+2)(x-1)=Ax+2+Bx-1 \Rightarrow 3x-6=A(x-1)+B(x+2) \Rightarrow A=4 \quad B=-1$$

$$\int 3x-6x^2+x-2 dx = \int (4x+2+-1x-1) dx = 4$$

$$|x-1|+C|x+2|-\ln|x-1|$$

$$(4x+104x^2-4x-3)dx \quad (7f)$$

$$4x+104x^2-4x-3=4x+10(2x-3)(2x+1)=A2x-3+B2x+1 \Rightarrow 4x+10=A(2x+1)+B(2x-3)$$

$$x=3 \Rightarrow A=4x=-12 \Rightarrow B=-2 \int 4x+104x^2-4x-3 dx = \int (2x+1)+C|2x-3|-\ln|2x-3|+2x+1 dx = 2\ln$$

$$(2x^2+9x-11x^3+2x^2-5x-6)dx \quad (8f)$$

$$2x^2+9x-11x^3+2x^2-5x-6=2x^2+9x-11(x-2)(x+1)(x+3)=Ax-2+Bx+1+Cx+3$$

$$\Rightarrow 2x^2+9x-11=A(x+1)(x+3)+B(x-2)(x+3)+C(x-2)(x+1)$$

$$x=2 \Rightarrow A=1x=-1 \Rightarrow B=3x=-3 \Rightarrow C=-2 \int 2x^2+9x-11x^3+2x^2-5x-6 dx = \int |x+3|+C|x+1|-2\ln|x-2|+3\ln(1x-2+3x+1+-2x+3) dx = \ln$$

$$(4xx^2-2x-3)dx \quad (9f)$$

$$4xx^2-2x-3=4x(x-3)(x+1)=Ax-3+Bx+1 \Rightarrow 4x=A(x+1)+B(x-3)$$

$$x=3 \Rightarrow A=3x=-1 \Rightarrow B=1 \int 4xx^2-2x-3 dx = \int (3x-3+1x+1) dx = 3\ln|x-3|+\ln|x+1|+C$$

$$(8x^2-19x+1(2x+1)(x-2)^2)dx \quad (10f)$$

$$8x^2-19x+1(2x+1)(x-2)^2=A2x+1+Bx-2+C(x-2)^2 \Rightarrow 8x^2-19x+1=A(x-2)^2+B(2x+1)(x-2)+C(2x+1)$$

$$x=-12 \Rightarrow A=2x=2 \Rightarrow C=-1x=0 \Rightarrow 1=4A-2B+C \Rightarrow B=3 \int 8x^2-19x+1(2x+1)(x-2)^2 dx = \int (22x+1+3x-2+-1(x-2)^2) dx = \ln$$

$$(9x^2-3x+29x^2-4)dx \quad (11f)$$

$$9x^2-3x+29x^2-4 dx = \int (1+6-3x)(9x^2-4) dx$$

$$6-3x(9x^2-4)=6-3x(3x-2)(3x+2)=A3x-2+B3x+2 \Rightarrow 6-3x=A(3x+2)+B(3x-2)$$

$$x=23 \Rightarrow A=1x=-3 \Rightarrow B=-2 \int 9x^2-3x+29x^2-4 dx = \int (1+13x-2+-23x+2) dx = x+13\ln|3x+2|+Cx-2|-23\ln$$

$$(x^3+2x^2+2x^2+xdx) \quad (12f)$$

$$x^3 + 2x^2 + 2x^2 + x dx = \int (x+1+2-x)x^2 + x dx = 2 - x(x+1) = Ax^2 - x(x+1) = Ax^2 + Bx + 1 \Rightarrow 2 - x = A(x+1) + Bx \Rightarrow 2 - x = Ax + A + Bx \Rightarrow 2 - x = (A+B)x + A$$

$$x=0 \Rightarrow 2 = A \Rightarrow A=2 \quad x=1 \Rightarrow 1 = 3A + B \Rightarrow B = -3$$

$$\int x^3 + 2x^2 + 2x^2 + x dx = \int (x+1+2x-3x+1) dx = \frac{1}{2}x^2 + x + 2 \ln|x+1| + C$$

$$(x^2 + x + 23 - 2x - x^2) dx \quad (13)$$

$$x^2 + x + 23 - 2x - x^2 dx = \int (-1 + 5 - x - x^2 - 2x + 3) dx = 5 - x - x^2 - 2x + 3 = x - 5$$

$$5(x+3)(x-1) = Ax + 3 + Bx - 1 \Rightarrow x - 5 = A(x-1) + B(x+3) \Rightarrow x - 5 = Ax + 3 + Bx + 3B$$

$$x = -3 \Rightarrow A = 2, x = 1 \Rightarrow B = -1$$

$$\int x^2 + x + 23 - 2x - x^2 dx = \int (-1 + 2x + 3 - 1x - 1) dx = -x + 2 \ln|x-1| + C - \ln$$

$$(2x - 4(x^2 + 4)(x + 2)) dx \quad (14)$$

$$2x - 4(x^2 + 4)(x + 2) = Ax + 2 + Bx + Cx^2 + 4 \Rightarrow 2x - 4 = A(x^2 + 4) + (Bx + C)(x + 2)$$

$$x = -2 \Rightarrow A = -1, x = 0 \Rightarrow -4 = 4A + 2C \Rightarrow C = 0, x = 1 \Rightarrow -2 = 5A + 3B + 3C \Rightarrow B = 1$$

$$\int x^2 + 4 + |x + 2| + 12 \ln 2x - 4(x^2 + 4)(x + 2) dx = \int (-1x + 2 + xx^2 + 4) dx = -\ln C$$

$$(x^3 - 4x^2 - 2x^3 + x^2) dx \quad (15)$$

$$x^3 - 4x^2 - 2x^3 + x^2 dx = \int (1 + -5x^2 - 2x^3 + x^2) dx = -5x^2 - 2x^3 + x^2 = -5x^2 - 2x^2(x+1) = Ax + Bx^2 + Cx + 1 \Rightarrow -5x^2 - 2 = Ax(x+1) + B(x+1) + Cx^2$$

$$x = 0 \Rightarrow B = -2, x = -1 \Rightarrow C = -7, x = 1 \Rightarrow -7 = 2A + 2B + C \Rightarrow A = 2$$

$$\int x^3 - 4x^2 - 2x^3 + x^2 dx = \int (1 + 2x + -2x^2 + -7x + 1) dx = x + 2 \ln|x+1| + C|x| + 2x - 7 \ln$$

$$(x^2 - 5x - 12x^2) dx \quad (16)$$

$$x^2 - 5x - 12x^2 = x - 3(4x - 1)(3x + 2) = A(4x - 1) + B(3x + 2) + C$$

$$x = 14 \Rightarrow A = -1, x = -23 \Rightarrow B = 1$$

$$\int 3 - x^2 - 5x - 1 |3x + 2| + C|4x - 1| + 13 \ln 2x^2 dx = \int (-14x - 1 + 13x + 2) dx = -14 \ln$$

$$(3x^3 - x^2 + 12x - 6x^4 + 6x^2) dx \quad (17)$$

$$3x^3 - x^2 + 12x - 6x^4 + 6x^2 = 3x^3 - x^2 + 12x - 6x^2(x^2 + 6) = Ax + Bx^2 + Cx + D$$

$$x^2 + 6 \Rightarrow 3x^3 - x^2 + 12x - 6 = Ax(x^2 + 6) + B(x^2 + 6) + (Cx + D)(x^2)$$

$$x = 0 \Rightarrow B = -1, x = 1 \Rightarrow 8 = 7A + 7B + C + D \dots \dots \dots (1) \quad x = -1 \Rightarrow -22 = -7A + 7B - C + D \dots \dots$$

$$((2)x=2 \Rightarrow 38=20A+10B+8C+4D \dots (3)$$

بجمع (1)، (2) ينتج أن: $14B+2D=-14$ ، وبتعويض $B=-1$ ، نجد أن $D=0$

وبطرح (2) من (1) ينتج أن $14A+2C=30$ أي أن $C=15-7A$

بالتعويض في (3) ينتج أن:

$$20A-10+8(15-7A)=38-36A=-72 \Rightarrow A=2 \quad C=15-7(2)=1$$

$$\int (5x-2)(x-2)^2 dx \quad (18)$$

$$5x-2(x-2)^2 = Ax-2+B(x-2)^2 \Rightarrow 5x-2 = A(x-2)+B(x-2)^2 \Rightarrow B=8 \quad x=0 \Rightarrow -2$$

$$|x-2| - 8x = -2A+B \Rightarrow A=5 \quad \int 5x-2(x-2)^2 dx = \int (5x-2+8(x-2)^2) dx = 5 \ln$$

$$-2+C$$

ملاحظة: يمكن حل هذا التكامل بالتعويض $u=x-2$

كما يمكن حله بالأجزاء حيث: $u=5x-2, dv=(x-2)^{-2}$

أجد قيمة كل من التكاملات الآتية:

$$\int (246+3x-x^2)x^3+2x^2 dx \quad (19)$$

$$6+3x-x^2x^3+2x^2 = 6+3x-x^2x^2(x+2) = Ax+Bx^2+Cx+2 \Rightarrow 6+3x-x^2 = A$$

$$x(x+2)+B(x+2)+C(x^2)x=0 \Rightarrow B=3 \quad x=-2 \Rightarrow C=-1 \quad x=1 \Rightarrow 8=3A+3B+C \Rightarrow A$$

$$|x+2|) \int 246+3x-x^2x^3+2x^2 dx = \int 24(3x^2+-1x+2) dx = (-3x-\ln$$

$$234=34+\ln 6+32+\ln = -34-\ln$$

$$\int (1/31/39x^2+49x^2-4 dx \quad (20)$$

$$9x^2+49x^2-4 = 1+89x^2-489x^2-4 = 8(3x-2)(3x+2) = A3x-2+B3x+2$$

$$\Rightarrow 8 = A(3x+2)+B(3x-2) \quad x=23 \Rightarrow A=2 \quad x=-23 \Rightarrow B=-2 \quad \int -13139x^2+49x^2$$

$$|3x+2|) |3x-2| - 23 \ln - 4 dx = \int -1313(1+23x-2+-23x+2) dx = (x+23 \ln$$

$$3=23-13+13-23 \ln |3x-23x+2|) - 1313 = 13+23 \ln - 1313 = (x+23 \ln$$

$$343 \ln$$

$$\int (0117-5x(2x+3)(2-x)^2)dx \quad (21f)$$

$$17-5x(2x+3)(2-x)^2=A(2x+3)+B(2-x)+C(2-x)^2 \Rightarrow 17-5x=A(2-x)^2+B(2-x)(2x+3)+C(2x+3)x=-32 \Rightarrow A=2x=2 \Rightarrow C=1x=0 \Rightarrow 17=4A+6B+3C \Rightarrow B=1$$

$$\int 0117-5x(2x+3)(2-x)^2 dx = \int 01(22x+3+12-x+1(2-x)^2) dx = (\ln 1032-12=12+\ln 3+\ln 5+1-\ln|2-x|+12-x)|01 = \ln|2x+3|-\ln$$

$$\int (14416x^2+8x-3)dx \quad (22f)$$

$$416x^2+8x-3=4(4x-1)(4x+3)=A(4x-1)+B(4x+3) \Rightarrow 4=A(4x+3)+B(4x-1)x=14 \Rightarrow A=1x=-34 \Rightarrow B=-1$$

$$\int 14416x^2+8x-3 dx = \int 14(14x-1+-14x+1|4x-14x+3|)|14=14(\ln|4x+3|)|14=(14\ln|4x-1|-14\ln+3)dx=(14\ln 351937)=14\ln 519-\ln$$

$$\int (345x+5x^2+x-6)dx \quad (23f)$$

$$5x+5x^2+x-6=5x+5(x-2)(x+3)=A(x-2)+B(x+3) \Rightarrow 5x+5=A(x+3)+B(x-2)x=2 \Rightarrow A=3x=-3 \Rightarrow B=2$$

$$\int 345x+5x^2+x-6 dx = \int 34(3x-2+2x+3) dx$$

$$9896=\ln 7-2\ln 2+2\ln|x+3|)|34=3\ln|x-2|+2\ln=(3\ln$$

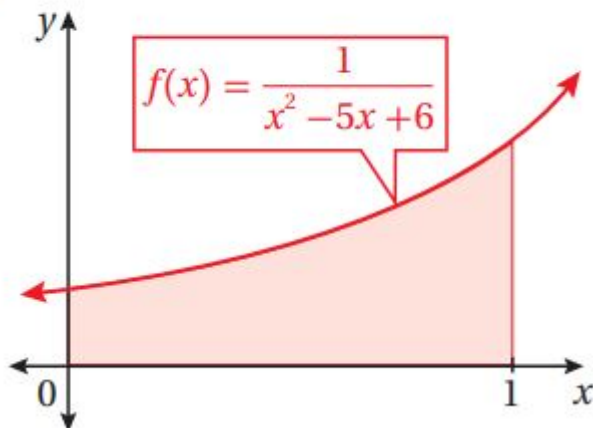
$$\int (344x^3-4x^2+4x)dx \quad (24f)$$

$$4x^3-4x^2+4x=4x(x-2)^2=A(x-2)+B(x-2)^2+Cx \Rightarrow 4=A(x-2)^2+Bx(x-2)+Cx=0 \Rightarrow A=1x=2 \Rightarrow C=2x=1 \Rightarrow 4=A-B+C \Rightarrow B=-1$$

$$A=\int 344x^3-4x^2+4x dx = (\ln|x-2|-2x-2)|34=(\ln|x-2|-2x-2)|34=\ln$$

أجد مساحة المنطقة المظللة في كل من التمثيلين البيانيين الآتيين:

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$$A = \int_0^1 \frac{1}{x^2 - 5x + 6} dx = \int_0^1 \frac{1}{(x-3)(x-2)} dx = \int_0^1 \left(\frac{A}{x-3} + \frac{B}{x-2} \right) dx$$

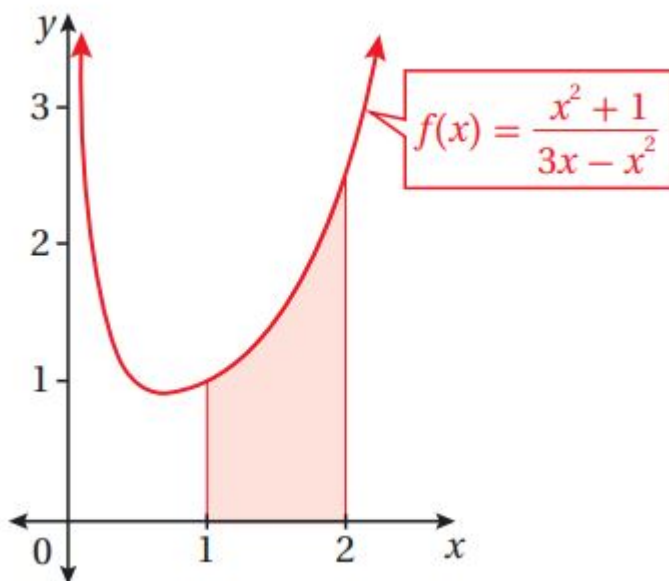
$$1 = A(x-2) + B(x-3) \Rightarrow 1 = Ax - 2A + Bx - 3B$$

$$1 = (A+B)x - 2A - 3B$$

$$\begin{cases} A+B=0 \\ -2A-3B=1 \end{cases} \Rightarrow \begin{cases} A=-B \\ -2(-B)-3B=1 \end{cases} \Rightarrow \begin{cases} A=-B \\ 2B-3B=1 \end{cases} \Rightarrow \begin{cases} A=-B \\ -B=1 \end{cases} \Rightarrow \begin{cases} A=1 \\ B=-1 \end{cases}$$

$$A = \int_0^1 \frac{1}{x^2 - 5x + 6} dx = \int_0^1 \left(\frac{1}{x-3} - \frac{1}{x-2} \right) dx = \left(\ln|x-3| - \ln|x-2| \right) \Big|_0^1 = \ln|1-3| - \ln|1-2| = \ln 2 - \ln 1 = \ln 2$$

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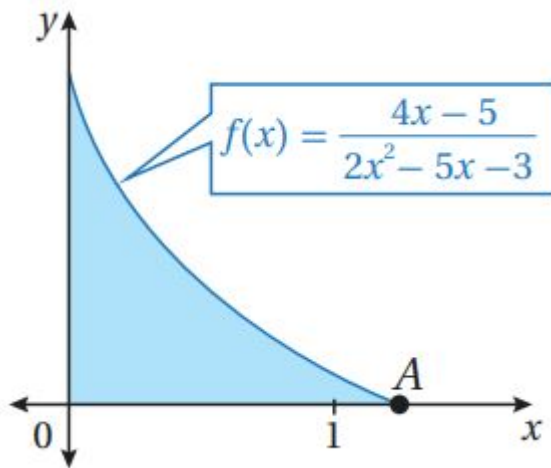


$$A = \int_1^2 \frac{x^2 + 1}{3x - x^2} dx = \int_1^2 \frac{x^2 + 1}{x(3-x)} dx = \int_1^2 \left(\frac{A}{x} + \frac{B}{3-x} \right) dx$$

$$x^2 + 1 = A(3-x) + Bx = 3A - Ax + Bx = (B-A)x + 3A$$

$$\begin{cases} B-A=1 \\ 3A=1 \end{cases} \Rightarrow \begin{cases} B=1+A \\ 3A=1 \end{cases} \Rightarrow \begin{cases} B=1+\frac{1}{3} \\ A=\frac{1}{3} \end{cases}$$

$$A = \int_1^2 \frac{x^2 + 1}{3x - x^2} dx = \int_1^2 \left(\frac{1/3}{x} + \frac{4/3}{3-x} \right) dx = \frac{1}{3} \ln|x| - \frac{4}{3} \ln|3-x| \Big|_1^2 = \frac{1}{3} \ln 2 - \frac{4}{3} \ln 1 - \left(\frac{1}{3} \ln 1 - \frac{4}{3} \ln 2 \right) = \frac{1}{3} \ln 2 - \frac{4}{3} \ln 1 - \frac{1}{3} \ln 1 + \frac{4}{3} \ln 2 = \frac{5}{3} \ln 2$$



يبين الشكل المجاور جزءاً من منحنى
الاقتران: $f(x) = \frac{4x-5}{2x^2-5x-3}$

(27) أجد إحداثيي النقطة A.

$$f(x)=0 \Rightarrow 4x-5=0 \Rightarrow x=5/4 \Rightarrow A(5/4, 0)$$

(28) أجد مساحة المنطقة المظللة.

$$\ln 49/243 = \ln 49/8 - \ln |2x^2 - 5x - 3|_{0 \leq x \leq 5/4} = \ln A = \int_{0 \leq x \leq 5/4} \frac{4x-5}{2x^2-5x-3} dx = \ln$$

ملاحظة: البسط هو مشتقة المقام، فلا داعي لتجزئة الكسر.

أجد كلاً من التكاملات الآتية:

$$\int (x dx + \cos 2x \cos \sin x)$$

$$\begin{aligned} \int x u + u^2 x dx &= \int \sin x + \cos 2x \cos x \int \sin x \Rightarrow dx = du - \sin x \Rightarrow du dx = -\sin u = \cos x \\ x &= \int -1/u + u^2 du - 1/u + u^2 = -1/u(1+u) = Au + B/1+u \Rightarrow -1 = A(1+u) \times du - \sin x \\ x \cos + B u u &= 0 \Rightarrow A = -1/u = -1 \Rightarrow B = 1 \int -1/u + 1/1+u du \Rightarrow \int \sin x \\ x &+ C = \ln |\cos x| + \ln |1+u| + C = \ln |1+\cos x| + \ln |\cos x dx = -\ln |x + \cos 2 \\ x &+ C| + \sec x \end{aligned}$$

$$\int (1/x^2 + x) dx \quad (30)$$

$$\begin{aligned} u = x \Rightarrow u^2 = x \Rightarrow dx &= 2u du \int 1/x^2 + x dx = \int 1/u^4 + u^3/2 u du = \int 2u^3 + u^2 du \\ 2u^3 + u^2 &= 2u^2(u+1) = Au + Bu^2 + Cu + 1 \Rightarrow 2 = Au(u+1) + B(u+1) + Cu \\ 2u = 0 &\Rightarrow B = 2 \\ u = -1 &\Rightarrow C = 2 \\ u = 1 &\Rightarrow 2 = 2A + 2B + C \Rightarrow A = -2 \int 2u^3 + u^2 du = \int (-2u + 2u^2 + 2u \\ |u+1| - 2u + C &|u+1| + C \Rightarrow \int 1/x^2 + x dx = 2 \ln |u| - 2u + 2 \ln |1+u| du = -2 \ln \end{aligned}$$

$$\int (e^{2x} e^{2x} + 3e^x + 2) dx \quad (31)$$

$$u = ex \Rightarrow du dx = ex = u \Rightarrow dx = du u \int e^{2x} e^{2x} + 3e^x + 2 dx = \int u^2 u^2 + 3u + 2 \times du$$

$$u = \int u u^2 + 3u + 2 du u u^2 + 3u + 2 = u(u+1)(u+2) = Au+1 + Bu+2 \Rightarrow u = A(u+2) + B(u+1)$$

$$u = -1 \Rightarrow A = -1 u = -2 \Rightarrow B = 2 \int u u^2 + 3u + 2 du = \int (-1u+1+2u+2)$$

$$(ex(ex+1) + 2 \ln|u+2| + C \Rightarrow \int e^{2x} e^{2x} + 3e^x + 2 dx = -\ln|u+1| + 2 \ln u = -\ln|u+2| + C$$

$$(x-4) dx \quad (32x(\sin^2 x \sin x \cos x) \int$$

$$x u(u^2 - x - 4) dx = \int \cos x (\sin^2 x \sin x) \int \cos x \Rightarrow dx = du \cos x \Rightarrow du dx = \cos u = \sin x$$

$$x = \int 1 u(u^2 - 4) du 1 u(u^2 - 4) = 1 u(u-2)(u+2) = Au + Bu - 2 + Cu + 4 \times du \cos^2 \Rightarrow 1 = A(u-2)(u+2) + Bu(u+2) + Cu(u-2)$$

$$u=0 \Rightarrow A = -14 u=2 \Rightarrow B = 18 u = -|u| + 18 \ln 2 \Rightarrow C = 18 \int 1 u(u^2 - 4) du = \int (-14u + 18u - 2 + 18u + 2) du = -14 \ln|x - |\sin x| + 18 \ln|\sin x - 4| dx = -14 \ln x (\sin^2 x \sin|u+2| + C \Rightarrow \int \cos u - 2| + 18 \ln|x+2| + C|\sin^2| + 18 \ln$$