MATHEMATICS

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For Preparatory Year Two

First Term

Student's Book

بسم الله الرحمن الرحيم

Dear students:

It is extremely great pleasure to introduce the mathematics book for second preparatory. We have been specially cautious to make your learning to the mathematics enjoyable and useful since it has many practical applications in real life as well as in the other subjects. This gives you a chance to be aware of the importance of learning mathematics, to determine its value and to appreciate the mathematicians roles.

This book sheds new lights on the activities as a basic objective. Additionally, we have tried to introduce the subject simply and excitingly to help attaining mathematical knowledge as well as gaining the patterns of positive thinking which pave your way to creativity .

This book has been divided into units, each unit contains lessons. Colors and pictures are effectively used to illustrate some mathematical concepts and the properties of figures. Lingual level of previous study has been taken into consideration .

Our great interest here is to help you get the information by your self in order to develop your self-study skills.

Calculators and computer sets are used when there's a need for. Exercises, practices, general exams, portfolios, unit test, general tests, and final term tests attached with model answers have been involved to help you review the curriculum completely.

Eventually, we hope getting the right track for the benefits of our students as well as for our dearest Egypt hoping bright future to our dearest students.

Authors

Contents

Unit One : Real Numbers

Unit Two : Relation between two Variables

Unit Three : Statistics

Unit Four: Geometry

Unit Five : Inequality

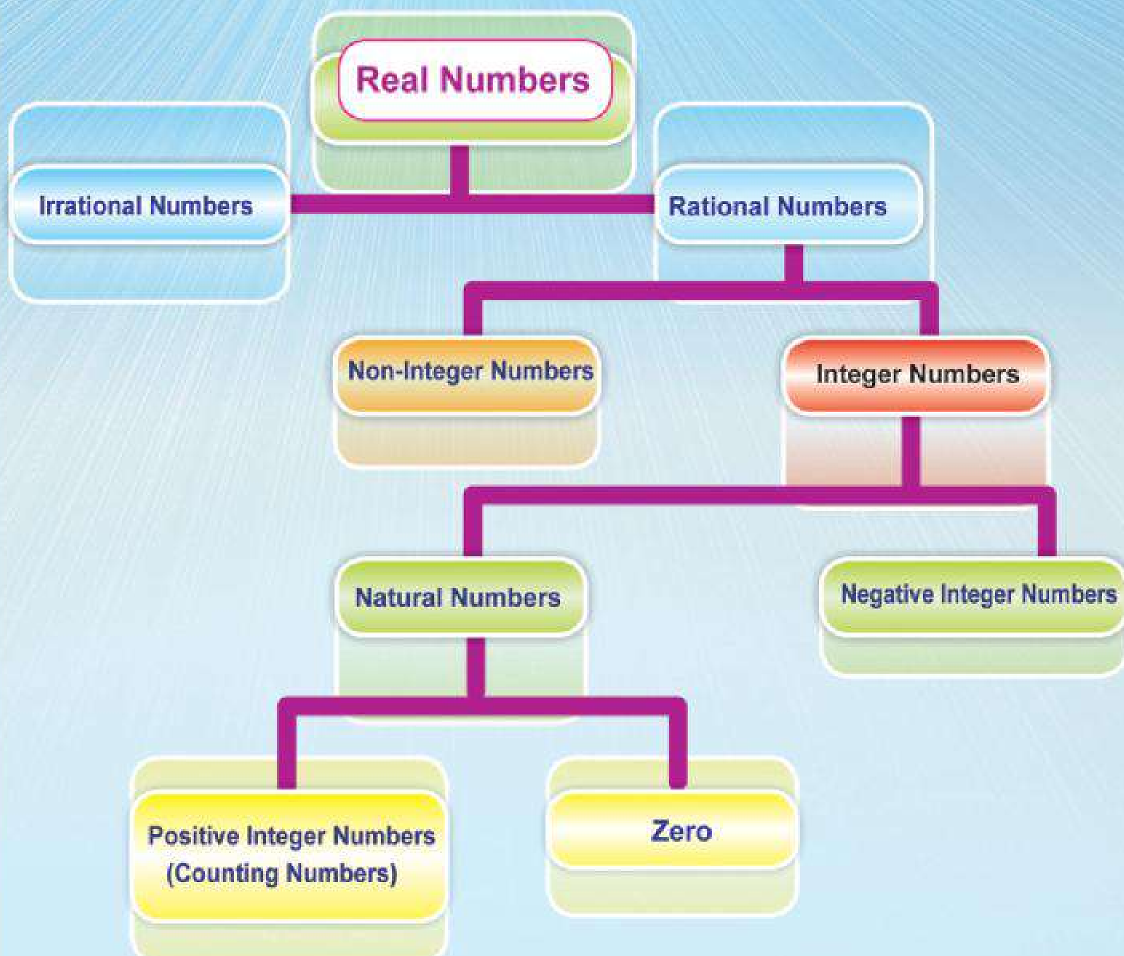
The used Mathematical Symbols

N	The set of natural numbers	$\perp$	perpendicular to
Z	The set of integer numbers	//	parallel to
Q	The set of rational numbers	$\overline{AB}$	Line segment **AB**
Q`	The set of irrational numbers	$\overrightarrow{AB}$	Ray **AB**
R	The set of real numbers	$\overleftrightarrow{AB}$	straight line **AB**
$\sqrt{a}$	Square root of number a	m ($\angle$ L)	measure of angle L
$\sqrt[3]{a}$	Cube root of number a	$\sim$	Similarity
[a , b]	Closed interval	$<$	less than
]a , b[	Open interval	$\leq$	less than or equal to
[a , b[	Half-open (closed) interval	$>$	greater than
]a , b]	Half-open (closed) interval	$\geq$	greater than or equal to
]-∞, a] [a , ∞[	Infinite interval	P(E)	probability of occurring event (E)
$\equiv$	is congruent to		

UNIT ONE

1

Real Numbers



Revision

Think and Discuss

The sets of numbers

The set of Counting numbers $= \{1, 2, 3, ...\}$

The set of Natural numbers : $\mathbf{N} = \{0, 1, 2, 3, ...\}$ = counting numbers $\cup \{0\}$

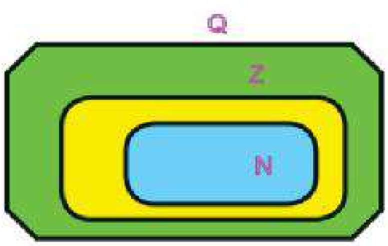
The set of Integers : $\mathbf{Z} = \{ ..., -3, -2, -1, 0, 1, 2, 3, ...\}$

The set of Positive integers $\mathbf{Z}^+ = \{ 1, 2, 3, ...\}$ = Counting numbers

The set of Negative integers $\mathbf{Z}^- = \{ -1, -2, -3, ...\}$

$$Z = Z^+ \cup \{ 0 \} \cup Z^-$$

The set of Rational numbers $Q = \{\frac{a}{b} : a, b \in Z, b \neq 0\}$



$$N \subset Z \subset Q$$

The absolute value of a rational number:

$| -7 | = 7$, $| 3 | = 3$, $| 0 | = 0$, $| -\frac{5}{3} | = \frac{5}{3}$

If $| a | = 5$ then $a = \pm 5$

The Standard form of a rational number is :

$$a \times 10^{n} \text{ where } n \in z, \ 1 \le |a| < 10$$

For example:- The standard form of the number 25.32×10^{4}
$= 2.532 \times 10^{5}$

- The standard form of the number $0.00053 = 5.3 \times 10^{-4}$

The perfect square rational number:

It is that positive number which can be written in the form of a square rational number i.e (rational number)^2

Example 1 , 4 , 25 , $\frac{9}{16}$, $2\frac{1}{4}$, ...

The perfect cube of rational number:

It is that rational number which can be written in the form of a cube rational number. i.e (rational number)^3

Example 1 , 8 , -27 , -216 , $\frac{8}{125}$, ...

The square root of a perfect square rational number

- The square root of the positive rational number a is that number whose square is equal to a.
- ($\sqrt{zero}$ = zero) the square root of zero is zero.
- Every perfect square rational number a has two square roots each one of them is an additive inverse to the other i.e. $\sqrt{a}$, $-\sqrt{a}$

Example $\frac{16}{25}$ has two square roots: $\frac{4}{5}$, $-\frac{4}{5}$

- $\sqrt{9}$ means the positive square root of 9 which is equal to 3
- $\sqrt{\left(\frac{a}{b}\right)^2} = \left|\frac{a}{b}\right|$ i.e

  $\sqrt{(-7)^2} = |-7| = 7$

Practice

Complete the following table

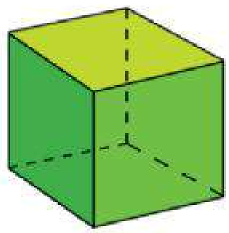
Number	Natural Number	Integer	Rational Number
3	✓	✓	✓
-3			
$\frac{3}{5}$			
$\sqrt{\frac{9}{16}}$			
$\|5 - 7\|$			

Unit One

Lesson One

The cube root of a rational number

Think and Discuss



you have learned that:
The volume of a cube =
the length of its side × itself × itself

Complete

The volume of the cube whose side length is equal to 7 cm
= .... × .... × .... = ... cm^3

Let`s think

If we have a cube of volume 125 cm^3, what is the length of its side?
We search for any three equal numbers of a product equal to 125. Then the number 125 can be factorized into its prime factors
$125 = 5 \times 5 \times 5$

125	5
25	5
5	5
1	

∴ the cube of volume 125cm^3 has a side length = 5cm
Therefore, 5 is called the cube root of 125 and it is written as $\sqrt[3]{125} = 5$.

The cube root of the rational number a is that number whose cube is equal to a

- The cube root for the rational number a is symbolized by $\sqrt[3]{a}$
- The cube root for a positive rational number is also positive
  Ex: $\sqrt[3]{125} = 5$
- - The cube root for a negative rational number is also negative. Ex: $\sqrt[3]{-8} = -2$ why ?
- $\sqrt[3]{zero}$ = zero
- $\sqrt[3]{a^3} = a$

you will learn how

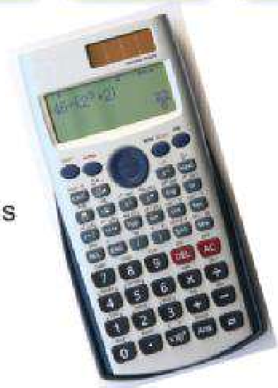
- To find the cube root of a rational number using facorization.
- To find the cube root of a rational number using the calculator.
- To solve equations that include finding the cube root.
- To solve applications on the cube root of a rational number.

Key terms

- Cube root .

To find the cube root of a perfect cube rational number:

- The number can be factorized into its prime factors..
- A calculator can be used.



Remark The perfect cube rational number has one cube root which is also a rational number , why?

Examples

1 **Use factorization to find the value of each $\sqrt[3]{1000}$, $\sqrt[3]{-216}$, $\sqrt[3]{3\frac{3}{8}}$; then check your answer using the calculator.**

Solution

2 {	2	1000
	2	500
	2	250
5 {	5	125
	5	25
	5	5
		1

$\sqrt[3]{1000} = 5 \times 2 = 10$

2 {	2	216
	2	108
	2	54
3 {	3	27
	3	9
	3	3
		1

$\sqrt[3]{-216} = -2 \times 3 = -6$

$3\frac{3}{8} = \frac{27}{8}$

3	27	2	8
3	9	2	4
3	3	2	2
	1		1

$\sqrt[3]{3\frac{3}{8}} = \sqrt[3]{\frac{27}{8}} = \frac{3}{2}$

Use your calculator to check your answer by pressing on $\sqrt[3]{\blacksquare}$

2 **Find the length of the radius of a sphere whose volume is equal to 4851cm³ ($\pi = \frac{22}{7}$)**

Solution

The volume of the sphere $= \frac{4}{3}\pi r^3$

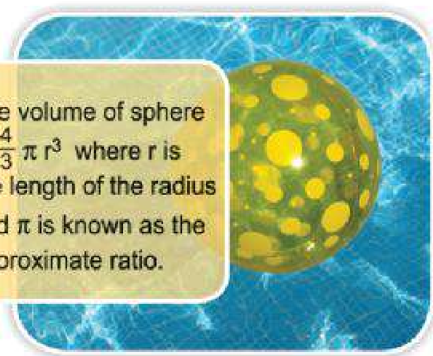
$4851 = \frac{4}{3} \times \frac{22}{7} r^3$

$r^3 = \frac{4851 \times 3 \times 7}{4 \times 22} = \frac{9261}{8}$

$\therefore r^3 = \frac{3^3 \times 7^3}{2^3}$

$\therefore r = \sqrt[3]{\frac{3^3 \times 7^3}{2^3}}$

3	9261
3	3087
3	1029
7	343
7	49
7	7
	1



The volume of sphere $= \frac{4}{3}\pi r^3$ where r is the length of the radius and π is known as the approximate ratio.

$r = \frac{3 \times 7}{2} = \frac{21}{2} = 10.5$ cm

we can use the calculator to find $\sqrt[3]{\frac{9261}{8}}$ directly.

Practice

Find the diameter of the sphere whose volume is 113.04 cm^3 (π = 3.14)

Example

Solve each of the following equations in Q.

A. $x^3 = 8$

B. $x^3 + 9 = 8$

C. $(x - 2)^3 = 125$

D. $(2x - 1)^3 - 10 = 54$

Solution

A. $x^3 = 8$

$x = \sqrt[3]{8} = 2$

$\therefore$ Solution set= {2}

B. $x^3 + 9 = 8$

$x^3 = 8 - 9$

$x^3 = -1$

$x = \sqrt[3]{-1} = -1$ $\therefore$ Solution set = {-1}

C. $(x - 2)^3 = 125$

$x - 2 = \sqrt[3]{125}$

$x - 2 = 5$

$x = 7$

$\therefore$ Solution set = {7}

D. $(2x - 1)^3 - 10 = 54$

$(2x - 1)^3 = 64$

$2x - 1 = \sqrt[3]{64}$

$2x - 1 = 4$

$2x = 5$

$x = \frac{5}{2}$ $\therefore$ Solution set= { $\frac{5}{2}$ }

Practice

Solve the following equations in Q: $(x + 1)^3 = 27$, $(x + 1)^3 = -27$

Unit One

Lesson Two

The set of Irrational numbers Q`

Think and Discuss

you will learn how how
- To define the set of irrational numbers.

key terms
- Irrational number

you have learned that: A rational number is that number which can be put in the form:

$\frac{a}{b}$: where a , b ∈ z, b ≠ 0

for example: when solving the equation $4x^2 = 25$

then $x^2 = \frac{25}{4}$ $\therefore x = \pm\frac{5}{2}$

Remark Each of $\frac{5}{2}$, $-\frac{5}{2}$ is a rational number .

However, there are many numbers which can not be put in the form $\frac{a}{b}$ where a , b ∈ Z , b ≠ 0

for example : when solving the equation $X^2 = 2$, we can not find any rational number whose square is equal to 2

The irrational number It is that number which can not be put in the form $\frac{a}{b}$ where a , b ∈ ℤ, b ≠ 0

the following are examples to irrational numbers.

First : the square roots of the positive numbers which are not perfect squares

Ex : $\sqrt{2}$, $\sqrt{5}$, $-\sqrt{6}$, $\sqrt{7}$

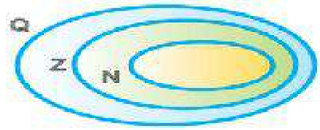
Second: the cube roots of those numbers that are not perfect cubes

Ex : $\sqrt[3]{4}$, $\sqrt[3]{-2}$, $\sqrt[3]{11}$, ...

Third: the pi π (the approximation ratio)

Where it is impossible to find any exact value for any of the previous number. why?

Those numbers and others form a set which is called the set of irrational numbers which is denoted by the symbol Q`.



$Q \cap Q' = \varnothing$

Think : is $\sqrt[3]{-1}$ an irrational number? why?

Unit One

Lesson Three

Finding the approximate value of an Irrational number

Think and discuss

Can you find the two rational numbers which the irrational number $\sqrt{2}$ is located between them.

Remark $\sqrt{2}$ is between $\sqrt{1}$, $\sqrt{4}$ i.e $1 < \sqrt{2} < 2$

i.e. $\sqrt{2} = 1 +$ a decimal fraction

To find the approximate value of $\sqrt{2}$. We check the values of the following numbers:

$(1.1)^2 = 1.21$, $(1.2)^2 = 1.44$, $(1.3)^2 = 1.69$,

$(1.4)^2 = 1.96$, $(1.5)^2 = 2.25$

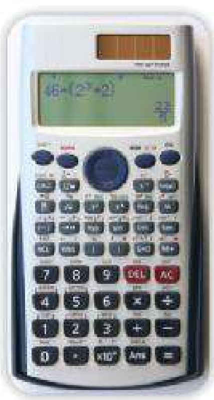
$\because 1.96 < 2 < 2.25$

$\therefore 1.4 < \sqrt{2} < 1.5$

i.e. $\sqrt{2} = 1.4 +$ a decimal fraction

i.e. $1.41 < \sqrt{2} < 1.42$

Use the calculator to check you answer.



You will learn how

- To find the approximate value for an irrational number
- To represent an irrational number on the number line.
- To solve equations in Q'

Representing the irrational number on the number line.

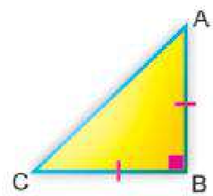
How can the point represents $\sqrt{2}$ be located on the number line?

If we draw the right triangle ABC at B which is an isosceles triangle also.

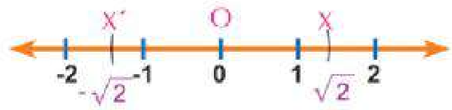
where AB = BC = one unit of length

Then $(AC)^2 = (AB)^2 + (BC)^2 = 1^2 + 1^2 = 2$

$\therefore AC = \sqrt{2}$ unit of length.

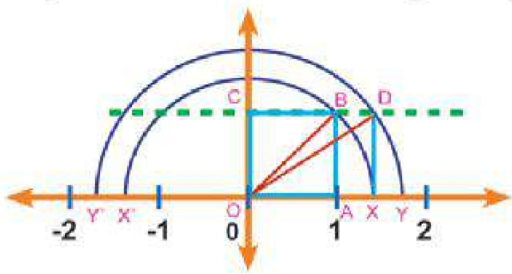


- draw the number line and place the sharp point of the compasses at point O, then adjust the compasses to a length that is equal to $\overline{AC}$ and draw an arc that intersects the number line on the right of o and at the point X, where that point represents $\sqrt{2}$
- Using the same length, we can label the point X` which represent - $\sqrt{2}$ where X` is on the left of the point o.



Think : **Label the point which represents 3 + $\sqrt{2}$ on the number line.**

Activity : **Draw the square O A B C whose side length is equal to one unit of length.**



The length of its diagonal = $\sqrt{1+1}$ = $\sqrt{2}$ unit of length

$\therefore$ OB = $\sqrt{2}$

- Place the sharp point of the compasses at point O and draw a semi-circle whose diameter = the length of $\overline{OB}$ = $\sqrt{2}$.
- $\overleftrightarrow{OA}$ ∩ the semi-circle = {X , X`} where X represents the number $\sqrt{2}$, x` represents the number - $\sqrt{2}$.
- Draw $\overline{XD}$ // $\overline{AB}$ and intersects $\overleftrightarrow{CB}$ at D

  $(OD)^2 = (OX)^2 + (XD)^2 = (\sqrt{2})^2 + (1)^2 = 3$

  $\therefore$ OD = $\sqrt{3}$
- Place the sharp point of the compasses at point O and adjust it to a length which is equal to the length of $\overline{OD}$, then draw semi-circle that intersects with $\overleftrightarrow{OA}$ at points Y, Y`

$\therefore$ OY = $\sqrt{3}$ i.e. point Y represnts $\sqrt{3}$, while point Y` represents - $\sqrt{3}$

- Continue using the same method to represent $\sqrt{4}$, $\sqrt{5}$, $\sqrt{6}$, ...

  also - $\sqrt{4}$, - $\sqrt{5}$, - $\sqrt{6}$, ...

Practice

1 **Find :**

A. Two consecutive integers that $\sqrt{5}$ lies between them.

B. Two consecutive integers that $\sqrt{12}$ lies between them.

C. Two consecutive integers that $\sqrt[3]{10}$ lies between them.

D. Two consecutive integers that $\sqrt[3]{-20}$ lies between them.

2 **Prove that :**

A. $\sqrt{3}$ lies between 1.7 , 1.8 .

B. $\sqrt[3]{15}$ lies between 2.4 , 2.5 .

3 Find the value of $\sqrt{11}$ to the nearest hundredth.

4 Find the value of $\sqrt[3]{2}$ to the nearest tenth.

5 Draw the number line and label the point which represents the irrational number $\sqrt{3}$.

6 Draw the number line and label the point which represents the irrational number $1 + \sqrt{2}$

Example (1)

Find the solutions set for each of the following equations in Q`:

A. $x^2 = 2$ B. $x^3 = 5$ C. $\frac{4}{3} x^2 = 1$ D. $0.001\ x^3 = -8$

Solution

A. $x^2 = 2$

$\therefore x = \pm \sqrt{2}$ Solution set = $\{-\sqrt{2}\ ,\ \sqrt{2}\}$

B. $x^3 = 5$

$\therefore x = \sqrt[3]{5}$ Solution set = $\{\sqrt[3]{5}\}$

C. $\frac{4}{3} x^2 = 1$

$\therefore \frac{3}{4} \times \frac{4}{3} x^2 = \frac{3}{4} \times 1$

$x^2 = \frac{3}{4}$

$\therefore x = \pm \sqrt{\frac{3}{4}} = \pm \frac{\sqrt{3}}{\sqrt{4}} = \pm \frac{\sqrt{3}}{2}$ Solution set $= \{-\frac{\sqrt{3}}{2}, \frac{\sqrt{3}}{2}\}$

D $0.001\ x^3 = -8$

$x^3 = -\frac{8}{0.001} = -8000$

$\therefore x = \sqrt[3]{-8000}$

$= -20 \in Q$ The solution set in $Q' = \varnothing$

Example (2)

Find the length of each of the side and the diagonal of a square whose area is $7cm^2$.

Solution

Let the length of the side be x cm,

then the area $= x \times x = x^2$

Where L is the square diagonal length

$x^2 = 7$

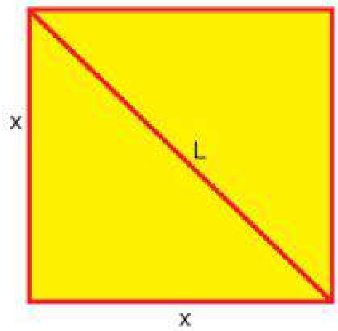
$\therefore x = \pm\sqrt{7}$ cm $\therefore x = \sqrt{7}$ cm why?

To find the diagonal of the square: use pythagorean theorem

$L^2 = x^2 + x^2$ Where L is the square diagonal length.

$\therefore L^2 = 14$

$\therefore L = \pm\sqrt{14}$ cm $\therefore L = \sqrt{14}$ cm why?



Example (3)

Find : the circumference of a circle whose area is $3\pi\ cm^2$

Solution

The area of the circle $= \pi r^2$

$3\pi = \pi r^2$

$\therefore r^2 = 3$

$r = \sqrt{3}$ cm or $r = -\sqrt{3}$ cm (refused)

the circumference $= 2\pi r = 2\pi \times \sqrt{3} = 2\sqrt{3}\pi$ cm.

Unit One

Lesson 4

The set of the Real numbers R

Think and Discuss

You will learn how

- To define the set of real numbers (R).
- To define the realtion among sets of N, Z, Q, Q`, R

Key terms

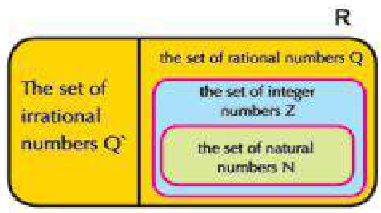
- A real number.

You have learned the set of rational numbers (Q), you have also found that there are other numbers that form the set of irrational number Q\ such as $\sqrt{2}$, $\sqrt[3]{2}$, π,... However, the union of these two sets forms a new set called the set of the real numbers, and it is denoted by the symbol R

$$R = Q \cup Q`$$

Look at the opposite Venn diagram, you find that:

1. $R = Q \cup Q`$
2. Any natural, integer, rational or irrational number is a real number



$N \subset Z \subset Q \subset R$ and so is $Q` \subset R$

Think Give examples from your own to some real numbers which are rational or irrational numbers.

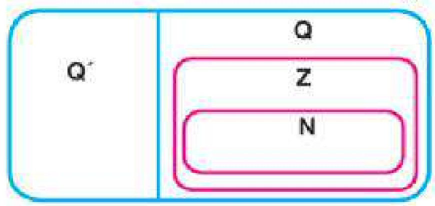
3. Every real number is represented by one point on the number line.

First: zero is represented by the origin O.

Second: the positive real numbers are represented by all the points On the number line that are located on the right side of O

Third: the negative real numbers are represented by all the points on the number line that are located on the left side of O

Practice



1 **Put each of the following numbers in its suitable place on the opposite venn diagram.**

$\frac{1}{2}$, -4 , 9 , $\sqrt{5}$, 0,6 , $\frac{7}{9}$, $\sqrt[3]{-2}$, $\sqrt{16}$, 0 , 5

2 **Label** point A on the number line which represents $\sqrt[3]{-8}$, and point B which represents $\sqrt{9}$, then find the length of $\overline{AB}$.

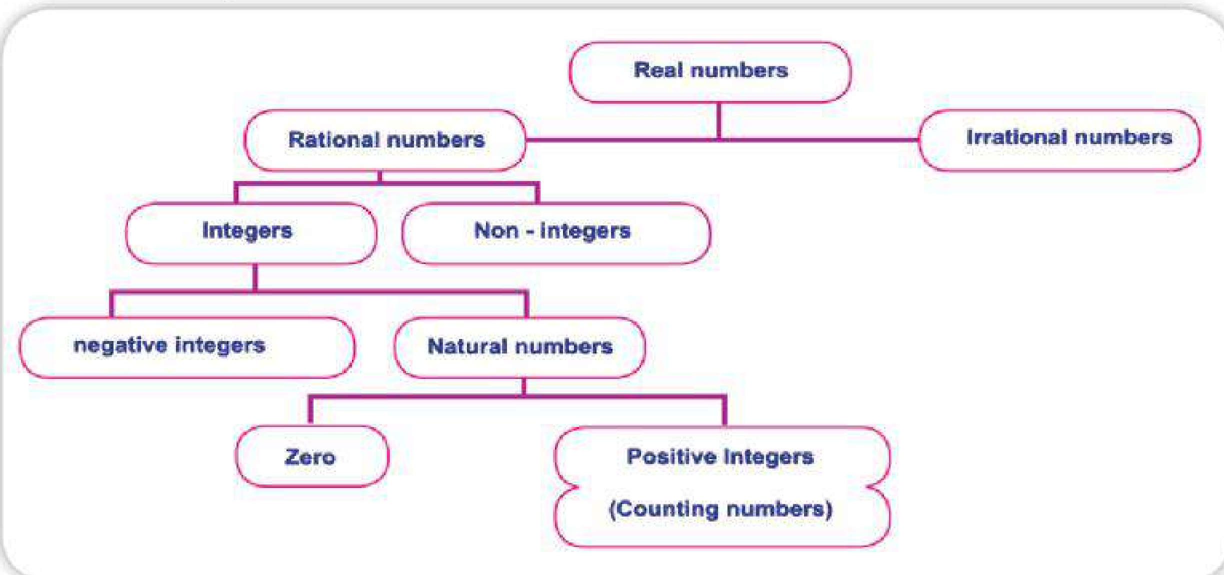
3 **State if each sentence is true or false:**

A Every natural number is a positive real number.

B Every integer is a real number.

Remark $\sqrt[3]{-1} = -1$ because $-1 \times -1 \times -1 = -1$

While $\sqrt{-1} \notin R$ because there is no real number If multiplyed by it self, the product is -1 .



Discuss with your teacher and classmates: Are there any non- Real number?

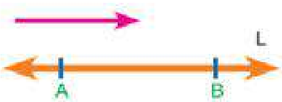
Unit One

Lesson Five

Ordering numbers at R

Think and Discuss

If A, B are two points that belong to the straight line L, and we determined a certain direction as shown by the arrow; then we can say that:



- The point B follows the point A. i.e on its right hand side.
- The point A precedes the point B. i.e on its left hand side.

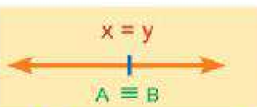
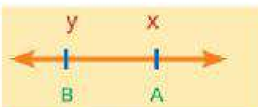
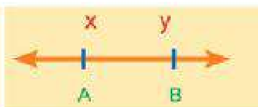
The same applies for all the points on the straight line.

However, If we know that every point on the straight line represent a real number. We can say that :

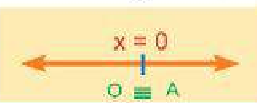
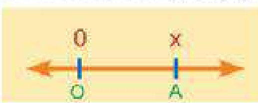
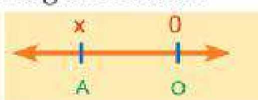
the set of real number is an ordered set.:

The properties of order:

1 If x , y are two real numbers represented on the number line by the two points A, B respectively, the ordering relation can be one of the following three cases:

		
A is congruent to B so x = y	**A follows B** so $x > y$	**A precedes B** so x < y

2 If x is a real number represented by the point A on the number line while O is the origin point which represents the zero, then the ordering relation can be one of the following three cases.

		
A is congruent to O $\therefore x = 0$	**A is on the right of O** $\therefore x > 0$ then x is a positive real number.	**A is on the left of O** $\therefore x < 0$ then x is a negative real number.

You will learn how

- To define the ordering relation in R.

Key terms

- ordering relation .
- more than.
- Less than
- Equal to
- Ascending order
- Descending order .

The set of the positive real numbers: $R^+ = \{x : x \in R ,\ x > 0\}$
The set of the nagative real numbers: $R^- = \{x : x \in R ,\ x < 0\}$

$$R = R^+ \cup \{0\} \cup R^-$$

Remark : The set of non-negative real numbers $= R^+ \cup \{0\} = \{x : x \geqslant 0,\ x \in R\}$
The set of the non - positive real numbers $= R^- \cup \{0\} = \{x : x \leqslant 0,\ x \in R\}$

Example:

Arrange the following numbers ascendingly $\sqrt{27}$, $-\sqrt{45}$, $\sqrt{20}$, 6, 0, $\sqrt[3]{-1}$

Solution

$6 = \sqrt{36}$, $\sqrt[3]{-1} = -1 = -\sqrt{1}$

The ascending order is from the smallest to the greatest.

$-\sqrt{45}$, $-\sqrt{1}$, 0 , $\sqrt{20}$, $\sqrt{27}$, $\sqrt{36}$

i.e. $-\sqrt{45}$, $\sqrt[3]{-1}$, 0 , $\sqrt{20}$, $\sqrt{27}$, 6.

Intervals

Think and Discuss

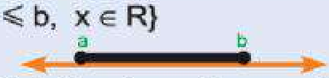
Interval is a subset of the set of real numbers

first: the limited intervals

If a, b ∈ R, a < b, then we can define each of:

The closed inteval [a , b]

$[a, b] = \{ x : a \leqslant x \leqslant b,\ x \in R\}$



[a, b] ⊂ R in which the elements are a, b and all the real numbers between them.

When we draw that interval, we put a shaded circle at each of the two points a and b then, we shade that area between them on the number line.

The open interval]a , b[

$]a, b[= \{ x: a < x < b, x \in R\}$

]a , b[⊂ R in which the elements are all the real numbers between the two numbers a, b

When we draw that interval, we put an unshaded circle at each of the two points which represent the two numbers a and b then, we shade that area between them on the number line.

Practice

Write down each of [3,5],]3,5[using the description method then represent them on the number line.

You will learn how

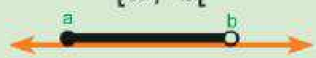
- To define limited intervals.
- To define unlimited intervals.
- To recognize the operations on intervals .

key terms

- Limited interval
- closed interval
- open interval
- half- open interval
- unlimited interval
- union
- intersection
- difference
- complement

Half openor (half closed) intervals

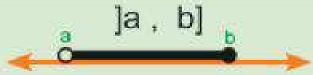
[a , b[



[a , b[= {x : a ⩽ x < b , x ∈ R}

[a, b[⊂ R where its elements are the number a and all the numbers between a and b.

]a , b]



]a , b] = { x: a < x ⩽ b , x ∈ R}

]a , b] ⊂ R where its elements are the number b and all the number between a and b .

Practice

Write down each of the two intervals: [3 , 5[,]3, 5] using the description method, then represent them on the number line.

Examples :

Represent each of the following intervals on the number line: [-1, 4] ,]-1, 4[,]-1, 4], {-1, 4}

Solution

[-1, 4]

Closed interval

]-1, 4[

open interval

]-1, 4]

Half-open interval

{-1, 4}

Set

Discuss with your teacher and your classmates whether the interval is a finite or an infinite set.

Practice

1 **Write** **down the following sets in the form of intervals, then represent them on the number line:**

A) X = {x : 2 < x < 5 , x ∈ R}

B) X = {x : -2 ⩽ x < 3 , x ∈ R}

C) X = {x : 0 ⩽ x ⩽ 4, x ∈ R}

D) X = {x : -3 < x ⩽ -1 , x ∈ R}

2 **Put** The suitable symbol ∈ or ∉ to make each sentence true.

A) 3 .... [-1 , 3[B) -2 ....]-1 , 3[C) $\frac{1}{2}$ ....]0 , 1[

D) $\sqrt{2}$ .... [1 , 2] E) 4 .... [0 , 5[F) $\sqrt[3]{-8}$ .... [-1 , 2]

G) |-5| .... [4 , 6[H) 2.3×10^{-5} ....]0, 1[

3 **Write** down the interval represented by each of the following figures:

A) 

C) 

B) 

D) 

Second: The unlimited intervals

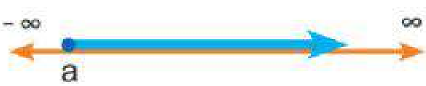
You know that: If the number line of real numbers is expanded on its two direction, we get more positive real numbers at the right direction and more negative real number at the left direction such all those numbers are located on that line.

- The symbol (∞) is read (infinity) and it is more than any imagined real number, ∞ ∉ R
- The symbol (-∞) is read (negative infinity) and it is less then any imagined real number, -∞ ∉ R
- The two symbols ∞ , - ∞ can not be represented by any points on the number line and they are expansions to the number line at its two directions.

If a is a real number, then we can define the following unlimited intervals:

The interval [a , ∞[

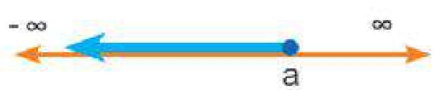
[a, ∞[= { x : x ⩾ a , x ∈ R}



That interval represents the number a and all the real numbers which are more than a

The interval]-∞ , a]

]-∞ , a] = { x : x ⩽ a , x ∈ R}

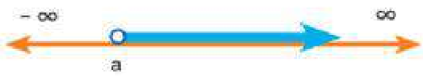


That interval represents the number a and all the real number which are less than a.

Write down each of the following intervals [3, ∞[,]-∞, 3] using the description method, then represent them on the number line.

the interval]a, ∞[

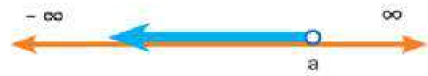
]a,∞[= {x : x > a , x ∈ R}



That interval represents all the real number which are more them a

the interval]-∞ , a[

]-∞ ,a[= { x : x < a , x ∈ R}



that interval represents all the real numbers which are less than a

Write **down the two intervals]3, ∞[,]-∞, 3[using the description method , then represent then on the number line**

Remark : The set of real numbers (R) can be represented in the form of the interval]-∞ , ∞[

The set of the positive real numbers R^+ =] 0, ∞[

The set of the negative real numbers R^- =]-∞, 0[

The set of non-negative real numbers = [0, ∞[

The set of non-positive real numbers =]-∞, 0]

Practice

1 **Write** **down the following sets in the form of intervals, then represent them on the number line.**

- A) X = {x : x ⩾ 2 , x ∈ R}
- B) X = {x : x < 3 , x ∈ R}
- C) X = {x : x > -7 , x ∈ R}
- D) X = {x : x ⩽ $\sqrt[3]{-8}$, x ∈ R}
- E) the set of all the real numbers more than | -3 |

2 **Put** **the suitable symbol ∈ or ∉ or ⊂ or ⊄ To make each statement true:**

- A) 3 ....]-∞, 4[
- B) [1 , 2] ....]-1, ∞[
- C) -5 ....]-∞, -6[
- D)]0 , 2[..............]0, ∞[
- E) 3×10^{10} ....]3, ∞[
- G) [-3, 1] .... [2, ∞[

Operations on intervals

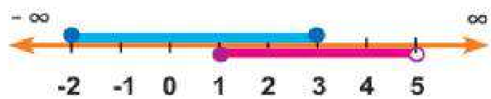
Since all the intervals are subsets of the set of the real number R, The operations of union, intersection, difference and complement can be applied on the intervals. The graphical representation to the intervals on the number line contributes to determine and verify the result of any operation. This can be clarified from the following examples:

Examples

1 **If $X = [-2, 3]$, $Y = [1, 5[$, find the following using the number line:**

A $X \cap Y$ B $X \cup Y$

Solution



A $X \cap Y = [-2, 3] \cap [1, 5[= [1, 3]$

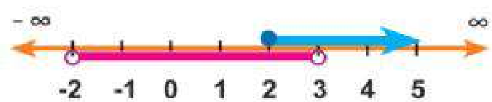
B $X \cup Y = [-2, 3] \cup [1, 5[= [-2, 5[$

2 **If $M = [2, \infty[$, $J =]-2, 3[$, find the following using the number line:**

A $M - J$ B $M \cap J$ C $M \cup J$

D $J \cup \{2, 3\}$ E $M`$ F $J`$

Solution



A $M - J = [2, \infty[-]-2, 3[= [3, \infty[$

B $M \cap J = [2, \infty[\cap]-2, 3[= [2, 3[$

C $M \cup J = [2, \infty[\cup]-2, 3[=]-2, \infty[$

D $J \cup \{2, 3\} =]-2, 3[\cup \{2, 3\} =]-2, 3]$

E $M` =]-\infty, 2[$

F $J` =]-\infty, -2] \cup [3, \infty[$

Practice

Put (✓) on the true sentence and (✗) on the false sentence:

A $[-2, 5] - \{2, 5\} =]-2, 5[$

B $]-1, 3] \cup \{-1, 0\} = [-1, 0]$

C $[2, 5] - \{5\} = [2, 5[$

D $[-1, 3] \cap]1, 4[= [1, 3]$

E $[-2, 5[\cup \{1, 5\} = [-2, 5]$

F $[5, \infty[-]-\infty, 5] =]5, \infty[$

Operations on the real numbers

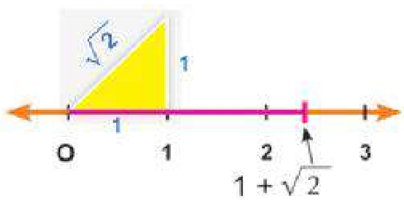
Unit One
Lesson Seven

Think and Discuss

First: The properties of adding the real numbers :

You have determined the location of the point X which represents the number $1 + \sqrt{2}$ on the number line. Since it represents the sum of the two real numbers 1 and $\sqrt{2}$ then the sum of every two real numbers is a real number.

i.e, the set of the real numbers R is closed under the operation of addition.



the closure property If $a \in R$, $b \in R$ then $(a + b) \in R$

for example : each of $2 + 3$, $1 + \sqrt{2}$, $-2 + \sqrt{5}$ and $2 + \sqrt[3]{3}$ are real numbers.

The commutative property If $a \in R$, $b \in R$ then $a + b = b + a$

for example : $2 + \sqrt{3} = \sqrt{3} + 2$, $3 - \sqrt{5} = -\sqrt{5} + 3$

The associative property If $a \in R$, $b \in R$, $c \in R$, then $(a + b) + c = a + (b + c) = a + b + c$

for example : $(3 + \sqrt{2}) + 5 = 3 + (\sqrt{2} + 5)$ associative property

$= 3 + (5 + \sqrt{2})$ commutative property

$= 3 + 5 + \sqrt{2}$ associative property

$= 8 + \sqrt{2}$

You will learn how

- To solve operations on the real numbers .
- To define the properties of operations on the real numbers .

key terms

- Closure property.
- Commutative property.
- associative property.
- Additive neutral.
- Additive inverse.
- multiplicative neutral.
- multiplicative inverse.
- distribution of multiplication on addition or subtraction.

Zero is the additive neutral element: If $a \in R$ then $a + 0 = 0 + a = a$

for example : $\sqrt{5} + 0 = 0 + \sqrt{5} = \sqrt{5}$, $-\sqrt[3]{4} + 0 = 0 + (-\sqrt[3]{4}) = -\sqrt[3]{4}$

Each real number has an additive inverse For a number $a \in R$ there is $(-a) \in R$ where $a + (-a) = (-a) + a =$ zero

for example $\sqrt{3} \in R$, has additive inverse $(-\sqrt{3}) \in R$ where
$\sqrt{3} + (-\sqrt{3}) = (-\sqrt{3}) + \sqrt{3} =$ zero.

Practice

1 **Complete the following to have a true sentence:**

A. $\sqrt{2} + 5 = 5 +$ ....
B. $\sqrt{11} + (-\sqrt{11}) =$ ....
C. $7 + \sqrt{3} = 5 +$ (................ + ...)
D. the additive inverse for $\sqrt[3]{8}$ is ....
E. the additive inverse for $(1 - \sqrt{2})$ is ....
F. $\sqrt{3} + (-\sqrt{3}) =$ ....
G. $7 + \sqrt{5} - 3 =$ ....
H. $(4 + \sqrt{7}) + (3 - \sqrt{7}) =$ ....
I. If $a \in R$, $b \in R$, then $a - b$ means the sum of the number a and .... of the number b.
J. If $a \in N$, $b \in Q$, $c \in R$, then $(a + b + c) \in$ ....

2 **Discuss the following with your teacher and classmates, then give examples:**

A. Is subtraction a commutative operation in R?
B. Is subtraction an associative operation in R?

Second: The properties of multiplying the real numbers

The closure property If $a \in R$, $b \in R$ then $a \times b \in R$

the set of real number is closed under the operation of multiplication.

i.e the product of multiplying every two real number is a real number.

for example : $5 \times \sqrt{2} = 5\sqrt{2} \in R$, $\sqrt{3} \times \sqrt{3} = 3 \in R$

$-2 \times \sqrt[3]{5} = -2\sqrt[3]{5} \in R$, $\frac{2}{3} \times \pi = \frac{2}{3}\pi \in R$

$2\sqrt{3} \times \sqrt{3} = 6 \in R$, $2\sqrt{3} \times 5 = 10\sqrt{3} \in R$

Commutative property If $a \in R$ and $b \in R$, then $a \, . \, b = b \, . \, a$

for example : $\sqrt{2} \times 3 = 3 \times \sqrt{2} = 3\sqrt{2}$

The associative property If $a \in R$, $b \in R$, $c \in R$, then :

$(a \, . \, b) \, . \, c = a \, . \, (b \, . \, c) = a \, . \, b \, . \, c$

for example : $\sqrt{2} \times (5 \times \sqrt{2}) = (\sqrt{2} \times 5) \times \sqrt{2} = (5 \times \sqrt{2}) \times \sqrt{2}$

$= 5 \times \sqrt{2} \times \sqrt{2} = 5 \times 2 = 10$

One is the multiplicative neutral If $a \in \mathbb{R}$, then $a \, . \, 1 = 1 \, . \, a = a$

for example : $2\sqrt{5} \times 1 = 1 \times 2\sqrt{5} = 2\sqrt{5}$

Every real number ≠ 0 has a multiplicative inverse If $a \neq 0$

It exist an real number $\frac{1}{a}$ such that

$a \, . \, \frac{1}{a} = \frac{1}{a} \, . \, a = 1$ (1 is the neutral element of multiplication)

for example: the multiplicative inverse for $\frac{\sqrt{3}}{2}$ is $\frac{2}{\sqrt{3}}$

where $\frac{\sqrt{3}}{2} \times \frac{2}{\sqrt{3}} = \frac{2}{\sqrt{3}} \times \frac{\sqrt{3}}{2} = 1$

Remark : $\frac{a}{b} = a \times \frac{1}{b}$, $b \neq 0$

i.e. $\frac{a}{b} = a \times$ the multiplicative inverse of b .

Discuss with your teacher: is the division operation commutative in R? Is the division operation associative in R?

Examples

Write down each of the following numbers $\frac{6}{\sqrt{2}}$, $-\frac{5}{\sqrt{3}}$, $\frac{15}{2\sqrt{5}}$ where the denominator is an integer .

Solution

Note that the multiplicative neutral is 1 and It can be written in the form $\frac{\sqrt{2}}{\sqrt{2}}$ or $\frac{\sqrt{3}}{\sqrt{3}}$ or $\frac{\sqrt{5}}{\sqrt{5}}$ or ...

$$\frac{6}{\sqrt{2}} = \frac{6}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{6\sqrt{2}}{2} = \frac{3\sqrt{2}}{1} = 3\sqrt{2}$$

$$-\frac{5}{\sqrt{3}} = -\frac{5}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = -\frac{5\sqrt{3}}{3}$$

$$\frac{15}{2\sqrt{5}} = \frac{15}{2\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}} = \frac{15\sqrt{5}}{2\times 5} = \frac{3\sqrt{5}}{2}$$

Practice

1 **Complete** the following to have a true sentence:

A $\sqrt{2} + \sqrt{2} + \sqrt{2} = \qquad \times \sqrt{2} =$

B $3 \times \sqrt{5} = \sqrt{5} \times$

C $\sqrt{7} \times \sqrt{7} =$

D $2\sqrt{5} \times 3\sqrt{5} =$

E The multiplicative neutral in R is the number

F The multiplicative inverse for $\frac{3}{\sqrt{2}}$ is

2 **Write** each of the following numbers such that the denominator is an integer:

A $\frac{15}{\sqrt{6}}$

B $\frac{8}{3\sqrt{2}}$

C $-\frac{6}{\sqrt{3}}$

D $\frac{25}{2\sqrt{10}}$

Distribution of multiplication on addition

For any three real numbers a, b, c.

$a \times (b + c) = (a \times b) + (a \times c) = a\,b + a\,c$

$(a + b) \times c = (a \times c) + (b \times c) = a\,c + b\,c$

Examples

1 **Simplify the following to the simplest form .**

A $2\sqrt{5}\left(3+\sqrt{5}\right)$

B $\left(\sqrt{2}+5\right)\left(3+\sqrt{2}\right)$

C $\left(2-3\sqrt{5}\right)^2$

Solution

A $2\sqrt{5}\left(3+\sqrt{5}\right) = 2\sqrt{5} \times 3 + 2\sqrt{5} \times \sqrt{5}$

$= 2 \times 3 \times \sqrt{5} + 2 \times 5 = 6\sqrt{5} + 10$

B $\left(\sqrt{2}+5\right)\left(3+\sqrt{2}\right) = \sqrt{2}\left(3+\sqrt{2}\right)+5\left(3+\sqrt{2}\right)$

$= \sqrt{2} \times 3 + \sqrt{2} \times \sqrt{2} + 5 \times 3 + 5 \times \sqrt{2}$

$= 3\sqrt{2} + 2 + 15 + 5\sqrt{2}$

$= 3\sqrt{2} + 17 + 5\sqrt{2} = 8\sqrt{2} + 17$

C $\left(2-3\sqrt{5}\right)^2 = (2)^2 + 2 \times 2 \times -3\sqrt{5} + \left(-3\sqrt{5}\right)^2$

$= 4 - 12\sqrt{5} + 9 \times 5$

$= 49 - 12\sqrt{5}$

2 **Give an estimation to the result of $(3 + \sqrt{5}) \times (1 + \sqrt{8})$, then check your answer using the calculator.**

Solution

First: The estimate of $\sqrt{5}$ is 2 $\therefore$ $(3 + \sqrt{5})$ the estimate of 3 + 2 = 5

the estimate of $\sqrt{8}$ is 3 $\therefore$ $(1 + \sqrt{8})$ the estimate of 1 + 3 = 4

$\therefore$ $(3 + \sqrt{5})(1 + \sqrt{8})$ the estimate of $5 \times 4 = 20$

Second: when we use the calculator to find $(3 + \sqrt{5}) \times (1 + \sqrt{8})$

We find that the result is 20.0459

Therefore, the estimate is reasonable.

Unit One

Lesson Eight

Operations on the square roots

Think and Discuss

If a , b are two non-negative real numbers, then

First: $\sqrt{a} \times \sqrt{b} = \sqrt{ab}$

For example : $\sqrt{2} \times \sqrt{3} = \sqrt{2 \times 3} = \sqrt{6}$

$\sqrt{2} \times \sqrt{10} = \sqrt{2 \times 10} = \sqrt{20}$

$\sqrt{15} \times \sqrt{5} = \sqrt{15 \times 5} = \sqrt{75}$

$\sqrt{a \times b} = \sqrt{a} \times \sqrt{b}$

For example : $\sqrt{20} = \sqrt{4 \times 5} = \sqrt{4} \times \sqrt{5} = 2\sqrt{5}$

$\sqrt{75} = \sqrt{25 \times 3} = \sqrt{25} \times \sqrt{3} = 5\sqrt{3}$

Second: $\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$ where $b \neq 0$

For example : $\sqrt{\frac{5}{9}} = \frac{\sqrt{5}}{\sqrt{9}} = \frac{1}{3}\sqrt{5}$

$\sqrt{\frac{16}{3}} = \frac{\sqrt{16}}{\sqrt{3}} = \frac{4}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{4\sqrt{3}}{3}$

Third: $\frac{\sqrt{a}}{\sqrt{b}} = \frac{\sqrt{a}}{\sqrt{b}} \times \frac{\sqrt{b}}{\sqrt{b}} = \frac{\sqrt{ab}}{b}$ $b \neq 0$

For example : $\frac{\sqrt{18}}{\sqrt{2}} = \sqrt{\frac{18}{2}} = \sqrt{9} = 3$

$\frac{\sqrt{84}}{\sqrt{7}} = \sqrt{\frac{84}{7}} = \sqrt{12} = \sqrt{4 \times 3} = \sqrt{4} \times \sqrt{3} = 2\sqrt{3}$

You will learn how

- To conduct operations on the square roots.
- To multiply two conjugates.

key terms

- Square root.
- Two conjugates numbers .

Examples

1 Simplify to the simplest form $\sqrt{32} - \sqrt{72} + 6\sqrt{\frac{1}{2}}$

Solution

$$\sqrt{32} - \sqrt{72} + 6\sqrt{\frac{1}{2}} = \sqrt{16 \times 2} - \sqrt{36 \times 2} + 6 \times \frac{\sqrt{1}}{\sqrt{2}}$$

$$= \sqrt{16} \times \sqrt{2} - \sqrt{36} \times \sqrt{2} + 6 \times \frac{1}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}}$$

$$= 4\sqrt{2} - 6\sqrt{2} + 3\sqrt{2} = \sqrt{2}$$

2 If $x = 2\sqrt{5} - 1$, $Y = 2 + \sqrt{5}$ find the value of $x^2 + y^2$

Solution

$$x^2 = (2\sqrt{5} - 1)^2 = (2\sqrt{5})^2 - 4\sqrt{5} + 1$$

$$= 4 \times 5 - 4\sqrt{5} + 1 = 21 - 4\sqrt{5}$$

$$y^2 = \left(2 + \sqrt{5}\right)^2 = 4 + 4\sqrt{5} + 5 = 9 + 4\sqrt{5}$$

$$x^2 + y^2 = 21 - 4\sqrt{5} + 9 + 4\sqrt{5} = 30$$

Practice

1 put each of the following in the form of $a\sqrt{b}$ where a and b are integers, b is the least possible value:

A $\sqrt{28}$ B $\sqrt{75}$ C $\sqrt{54}$

D $\sqrt{1000}$ E $2\sqrt{72}$ F $\frac{1}{3}\sqrt{162}$

2 Simplify to the simplest form:

A $2\sqrt{18} \times 3\sqrt{2}$ B $\sqrt{5} \times 2\sqrt{10}$ C $3\sqrt{7} \times 2\sqrt{28}$

D $\sqrt{50} + \sqrt{8}$ E $\sqrt{20} - \sqrt{45}$ F $\sqrt{27} + 5\sqrt{18} - \sqrt{300}$

3 **Find** the value of X + Y, X × Y in each of the following cases:

A $x = 3 + \sqrt{5}$, $y = 1 - \sqrt{5}$ B $x = \sqrt{3} - \sqrt{2}$, $y = \sqrt{3} + \sqrt{2}$

C $x = 5 - 3\sqrt{2}$, $y = 5 - 3\sqrt{2}$

Unit 1 : Lesson 8

The two conjugate numbers

If a and b are two positive rational numbers.

Then each of the two number $(\sqrt{a}+\sqrt{b}), (\sqrt{a}-\sqrt{b})$ is a conjugate to the other one.

then, their sum is $= 2\sqrt{a}$ twice the first term

and their product is $= (\sqrt{a}+\sqrt{b}) \cdot (\sqrt{a}-\sqrt{b}) = (\sqrt{a})^2 - (\sqrt{b})^2 = a - b$

= The square of the first term - The square of the second term

The product of two conjugates is always a rational number

If we have a real number whose denominator is written in the form $(\sqrt{a} \pm \sqrt{b})$, we should put it in the simplest form by multiplying both the numerator and denominator by the conjugate of the denominator.

Practice

Complete

A $\sqrt{5} + \sqrt{2}$ their conjugate (...............) and their product is (...............)

B $5 - \sqrt{3}$ their conjugate (...............) and their product is (...............)

C $2\sqrt{3} + \sqrt{2}$ their conjugate (...............) and their product is (...............)

Examples

1 Given $x = \dfrac{8}{\sqrt{5}-\sqrt{3}}$, $y = \dfrac{2-\sqrt{3}}{2+\sqrt{3}}$

Write both of X and Y where the denominator is a rational number, then find X + Y

Solution

$$x = \frac{8}{\sqrt{5}-\sqrt{3}} = \frac{8}{\sqrt{5}-\sqrt{3}} \times \frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}+\sqrt{3}}$$

$$= \frac{8(\sqrt{5}+\sqrt{3})}{(\sqrt{5})^2-(\sqrt{3})^2} = \frac{8(\sqrt{5}+\sqrt{3})}{5-3} = 4(\sqrt{5}+\sqrt{3})$$

$$y = \frac{2-\sqrt{3}}{2+\sqrt{3}} = \frac{2-\sqrt{3}}{2+\sqrt{3}} \times \frac{2-\sqrt{3}}{2-\sqrt{3}}$$

$$= \frac{(2-\sqrt{3})^2}{4-3} = \frac{4-4\sqrt{3}+3}{1} = 7-4\sqrt{3}$$

$$x + y = 4\sqrt{5} + 4\sqrt{3} + 7 - 4\sqrt{3} = 4\sqrt{5} + 7$$

2 Given $x = \frac{4}{\sqrt{7}-\sqrt{3}}$, $y = \sqrt{7} - \sqrt{3}$,

prove that x and y are conjugates, then find the values of:

$x^2 - 2xy + y^2$, $(x - y)^2$. What do you observe?

Solution

$$x = \frac{4}{\sqrt{7}-\sqrt{3}} \times \frac{\sqrt{7}+\sqrt{3}}{\sqrt{7}+\sqrt{3}} = \frac{4(\sqrt{7}+\sqrt{3})}{7-3} = \sqrt{7}+\sqrt{3}$$

$y = \sqrt{7} - \sqrt{3}$ $\therefore$ x , y (two conjugate numbers)

$$x^2 - 2xy + y^2 = (\sqrt{7}+\sqrt{3})^2 - 2(\sqrt{7}+\sqrt{3})(\sqrt{7}-\sqrt{3}) + (\sqrt{7}-\sqrt{3})^2$$

$$= (7 + 2\sqrt{21} + 3) - 2(7 - 3) + (7 - 2\sqrt{21} + 3)$$

$$= 10 + 2\sqrt{21} - 8 + 10 - 2\sqrt{21}$$

$$= 12$$

$$(x - y)^2 = [(\sqrt{7}+\sqrt{3}) - (\sqrt{7}-\sqrt{3})]^2$$

$$= [\sqrt{7} + \sqrt{3} - \sqrt{7} + \sqrt{3}]^2 = (2\sqrt{3})^2$$

$$= 4 \times 3 = 12$$

Remark : $x^2 - 2xy + y^2 = (x - y)^2$

Practice

In the previous example, find the value of each of the following:

A $(x + y)$ B $(x - y)$

C $(x + y)(x - y)$ D $x^2 - y^2$

What do you observe?

Unit One

Lesson Nine

Operations on the cube roots

Think and Discuss

You will learn how

- To carry operations on the cube roots.

key terms

- Cube root.

For any two real numbers a, b :

1. $\sqrt[3]{a} \times \sqrt[3]{b} = \sqrt[3]{a \times b}$

For example : $\sqrt[3]{5} \times \sqrt[3]{2} = \sqrt[3]{5 \times 2} = \sqrt[3]{10}$

$\sqrt[3]{3} \times \sqrt[3]{-4} = \sqrt[3]{3 \times -4} = \sqrt[3]{-12}$

2. $\sqrt[3]{a \times b} = \sqrt[3]{a} \times \sqrt[3]{b}$

For example : $\sqrt[3]{40} = \sqrt[3]{8 \times 5} = \sqrt[3]{8} \times \sqrt[3]{5} = 2\sqrt[3]{5}$

$\sqrt[3]{-128} = \sqrt[3]{-64 \times 2} = \sqrt[3]{-64} \times \sqrt[3]{2} = -4\sqrt[3]{2}$

3. $\frac{\sqrt[3]{a}}{\sqrt[3]{b}} = \sqrt[3]{\frac{a}{b}}$ where $b \neq o$, $a, b \in R$

For example : $\frac{\sqrt[3]{12}}{\sqrt[3]{3}} = \sqrt[3]{\frac{12}{3}} = \sqrt[3]{4}$

4. $\sqrt[3]{\frac{a}{b}} = \frac{\sqrt[3]{a}}{\sqrt[3]{b}}$ where $b \neq o$, $a, b \in R$

For example : $\sqrt[3]{\frac{3}{2}} = \frac{\sqrt[3]{3}}{\sqrt[3]{2}}$

Think : If we multiply both the numerator and the denominator by $\sqrt[3]{4}$, then find the product in its simplest form

Examples :

Simplify to the simplest form:

A $\sqrt[3]{54} + 8\sqrt[3]{\frac{-1}{4}} + 5\sqrt[3]{16}$

B $\sqrt[3]{24} - 6\sqrt[3]{13\frac{8}{9}}$

Solution

A $\sqrt[3]{54} + 8\sqrt[3]{\frac{-1}{4}} + 5\sqrt[3]{16} = \sqrt[3]{27 \times 2} + 8\sqrt[3]{\frac{-1}{4} \times \frac{2}{2}} + 5\sqrt[3]{8 \times 2}$

$= \sqrt[3]{27} \times \sqrt[3]{2} + 8\frac{\sqrt[3]{-2}}{\sqrt[3]{8}} + 5 \times \sqrt[3]{8} \times \sqrt[3]{2}$

$= 3\sqrt[3]{2} + \frac{8 \times (-\sqrt[3]{2})}{2} + 5 \times 2 \times \sqrt[3]{2}$

$= 3\sqrt[3]{2} - 4\sqrt[3]{2} + 10\sqrt[3]{2} = 9\sqrt[3]{2}$

B $\sqrt[3]{24} - 6\sqrt[3]{13\frac{8}{9}} = \sqrt[3]{24} - 6\sqrt[3]{\frac{125}{9}} = \sqrt[3]{8 \times 3} - 6 \times \sqrt[3]{\frac{125}{9} \times \frac{3}{3}}$

$= \sqrt[3]{8} \times \sqrt[3]{3} - 6 \times \frac{5\sqrt[3]{3}}{3} = 2\sqrt[3]{3} - 2\sqrt[3]{10} = -8\sqrt[3]{3}$

Unit One

Lesson Ten

Applications on the real numbers

Think and Discuss

You will learn how

- To solve applications on square and cube roots.

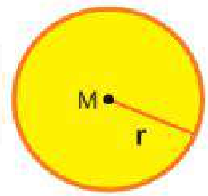
key terms

- Circle
- Cuboid
- Cube
- Right circular cylinder
- Sphere

The circle:

Circumference of a circle = $2 \pi r$ length unit

area of a circle = πr^2 square unit



where r is the length of the radius in a circle,
π is the (approximate ratio).

Examples

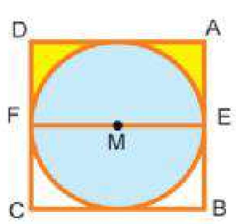
1 **Find** the circumference of a circle whose area is 38.5 cm^2 $(\pi = \frac{22}{7})$

Solution

The area of the circle = πr^2

$38.5 = \frac{22}{7} r^2 \qquad \therefore r^2 = \frac{38.5 \times 7}{22} = \frac{49}{4}$

$\therefore r = \sqrt{\frac{49}{4}} = \frac{7}{2} = 3.5$ cm



2 In the opposite figure, the circle M is inside the square ABCD. If the area of the yellow sector is $10\frac{5}{7}$ cm^2, find the perimeter of the sector$(\pi = \frac{22}{7})$

Solution

We suppose that the length of the raidus in a Circle = r

$\therefore$ The side length of the square = 2r

The area of the yellow color = the area of the rectangle AEFD - the area of semi circle

$$10\frac{5}{7} = r \times 2r - \frac{1}{2} \times \frac{22}{7} r^2$$

$$\frac{75}{7} = 2r^2 - \frac{11}{7} r^2 = \frac{3}{7} r^2$$

$\therefore r^2 = 25 \quad \therefore r = 5$ cm

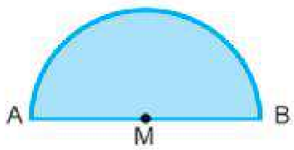
The perimeter of the yellow sectors = (AE + AD + DF) + $\frac{1}{2}$ the circumference of the circle

$$= (5 + 10 + 5) + \frac{1}{2} \times 2 \times \frac{22}{7} \times 5 = 35\frac{5}{7} \text{ cm}$$

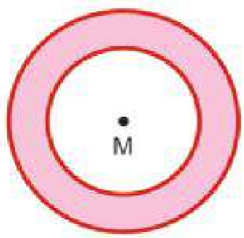
Practice

1. A circle whose area is 64π cm². Find the length of its radius, then find its circumference approximating it to the nearest integer ($\pi = 3.14$).

2. In the figure opposite: $\overline{AB}$ is the diameter of a semi circle. If the area of that region is 12.32cm². Find the circumference of that figure.



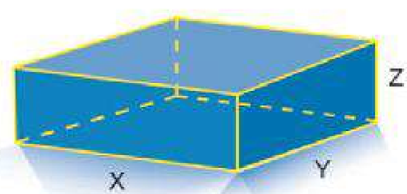
3. In the opposite figure: there are two circles have the same center "concentric" of center M. If the lengths of their radii are 3cm and 5cm. Find the area and the circumference of the colored region in the terms of π.



The cuboid

It is a body whose six faces are of a rectangular shape such that every two opposite faces are congruent.:

If the lengths of its edges were x, y, z, then:



The lateral area = the perimeter of the base × the height

The lateral area = 2 (x + y) × z square unit

The lateral area = the lateral area + 2 × the area of the base

The total area = 2 (xy + yz + Xz) square unit

The volume of the cuboid = the area of the base × the height

The volume of the cuboid = x × y × z cubic unit

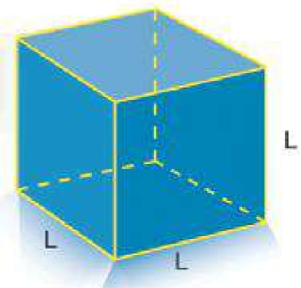
A special case : **the cube**

It is a cuboid whose edges are equal in length. If the length of one edge = L length unit, then:

The area of each face = L^2 square unit

The lateral area of each face = $4L^2$ square unit

The total area = $6L^2$ square unit, the volume of the cube = L^3 cubic unit



Examples

Find the total area of a cube whose volume is 125 cm^3

Solution

The volume of the cube = L^3 $\therefore 125 = L^3$ $\therefore L = \sqrt[3]{125} = 5cm$

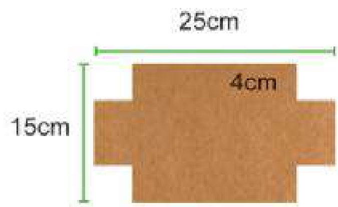
The total area = $6L^2 = 6 \times (5)^2 = 150$ cm^2

Practice

1. Find the total area of a cuboid whose volume is 720cm^3 and height 5cm with a squared shape base.

2. Which is more in volume: A cube of 294 cm^2 area or a cuboid with the following dimensions: $7\sqrt{2}$, $5\sqrt{2}$, 5 cm.

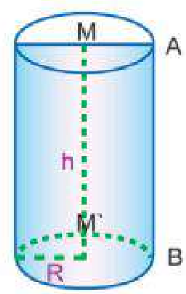
3. A rectangular hard piece of paper has a length of 25 cm and a width of 15 cm. A square whose side = 4 cm was cut from each of its four corners. Then, the projected parts were folded to form a shape of a cuboid. Find the volume and the total area of that cuboid.



The right circular cylinder :

It is a body that has two parallel congruent bases each is a circular shaped surface, while its lateral surface is a curved surface called cylindrical surface.

- If M, M` are the bases of the cylinder, then M M` is the height of cylinder.

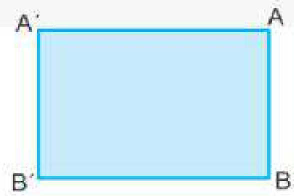


Unit 1 : Lesson 10

Let's think If A $\in$ the circle M, B $\in$ the circle M`,

$\overline{AB} \parallel \overline{MM`}$

- Then, if we cut the lateral cylindrical surface at AB and we stretch That surface, we get the surface of the rectangle A B B` A`



Then, AB = height of cylinder, A A` = the perimeter of the base of the cylinder.

The area of the rectangle A B B` A` = the lateral area of the cylinder

The lateral area of the cylinder = the perimeter of the base × height = $2\pi r h$ (square unit)

the total area of the cylinder = area of lateral surface + sum of the areas of the two bases

$= 2 \pi r h + 2 \pi r^2$ (square unit)

the volume of the cylinder = base area × height = $\pi r^2 h$ (cubic unit)

Example

A piece of paper has shape of a rectangle ABCD in which AB = 10cm, BC = 44cm. It was folded to form a right circular cylinder such that $\overline{AB}$ is congruent to $\overline{DC}$. Find the volume of the resulted cylinder. ($\pi = \frac{22}{7}$).

Solution

The perimeter of the cylinder base = 44 cm.

$2 \pi r = 44$

$2 \times \frac{22}{7} r = 44$

$\therefore r = 7$cm

The volume of the cylinder $= \pi r^2 h$

$= \frac{22}{7} \times (7)^2 \times 10$

$= 1540$ cm^3

Practice

1. Find the volume and the total area of a right circular cylinder in which the length of base radius = 14 cm and the height is 20 cm .

2. Find the total area of a right circular cylinder of volume 7536 cm^3 and height 24 cm ($\pi = 3.14$)
3. Which is more in volume: a right circular cylinder of radius 7cm and height 10 cm or a cube whose edge length is equal to 11cm

The sphere:

It is a body of curved surface in which the points have the same distance (r) from a constant point inside it (the center of the sphere)..

If the sphere is cut by a plane passing by its center, then the resulted section is a circle whose center is the center of a sphere where its radius is the radius of a sphere (r).

Volume of the sphere = $\frac{4}{3}\pi r^3$ cubic units.

area of the sphere = $4\pi r^2$ square units.

Examples

The volume of the sphere is 562.5π cm^3 . Find its surface area.

Solution

the volume of sphere = $\frac{4}{3}\pi r^3$

$562.5\pi = \frac{4}{3} \times \pi r^3$

$\therefore r^3 = 562.5 \times \frac{3}{4} = 421.875$

$r = \sqrt[3]{421.875} = 7.5$cm

the surface area of sphere = $4\pi r^2 = 4 \times \pi (7.5)^2 = 225\pi$ cm^2

Practice

Find the volume and the surface area of a sphere whose diameter is 4.2cm ($\pi = \frac{22}{7}$)

Unit One

Lesson Eleven

Solving Equations and Inequalities of first degree in one variable in R

Think and Discuss

You will learn how

- To solve equation of first degree in one variable in R.
- To solve inequalities of first degree in one variable

key terms

- -equation
- degree of an equation.
- Inequality
- degree of an inequality
- Solution of an equation
- Solution of an inequality

First:Solving Equations of first degree in one variable in R

We know that: The equation 3X - 2 = 4 is called an equation of first degree where the exponent of the (unknown) variable X is 1. To solve that equation in R

$3x - 2 = 4$ **By adding 2 to the sides of the equation**

$3x = 6$ **(we can multiply by the multiplicative inverse of the coefficient of X)**

$\frac{1}{3} \times 3x = \frac{1}{3} \times 6$

$\therefore x = 2$

i.e the solution set { 2 }

This solution can be graphed on the number line as shown in the figure opposite .

Examples

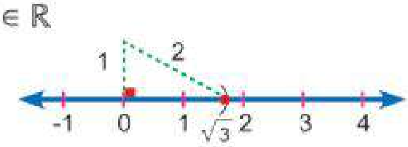
1 **Find the solution set of the equation $\sqrt{3}\,x - 1 = 2$, in R, then graph the solution on the number line.**

Solution

$\sqrt{3}\,x - 1 = 2$ $\quad\therefore \sqrt{3}\,x = 3$

$\therefore x = \frac{3}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$ $\quad\therefore x = \sqrt{3} \in \mathbb{R}$

The solution set is $\{\sqrt{3}\}$

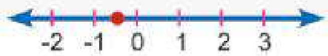


This solution can be graphed on the number line as shown in the figure opposite .

2 **Find** The solution set for the equation $x + \sqrt{2} = 1$, in R, then graph the solution on the number line.

Solution

$x + \sqrt{2} = 1$ $\therefore x = 1 - \sqrt{2} \in \mathbb{R}$



This can be graphed on the number as shown in the figure opposite .

Practice

1 **Find the solution set for the following equations in R, then graph the solution on the line number.**

A) $5x + 6 = 1$ B) $2x + 4 = 3$ C) $2x - 3 = 4$

D) $x + 5 = 0$ E) $\sqrt{2}\,x - 1 = 1$ F) $x - 1 = \sqrt{5}$

Second : Solving inequalities of the first degree in one variable in R, graphing the solution on the number line.

The following properties are used to solve the inequality in R. The solution set is written in the form of an interval

If A , B، C were real number where A< B, then:

1	$A + C < B + C$.	addition property.
2	If $C > 0$ then $A \times C < B \times C$.	property of multiplication by a positive real number
3	If $C < 0$ then $A \times C > B \times C$.	property of multiplication by negative real number.

Examples

1 **Find** the solution set for the inequality $2x - 1 \geq 5$ in R and represent the solution set graphically.

Solution

By adding 1 to the sides of the inequality it becomes $2x \geq 6$

by multiplying the side of the in equality by ($\frac{1}{2} > 0$) $x \geq 3$

$\therefore$ The solution set in R is $[3 , \infty[$

and it is graphed by green color ray on the number line.

2 **Find** the solution set for the inequality 5 - 3 x > 11 , in R, then represent the solution graphically.

Solution

By adding (-5) to the sides of the inequality then -3 x > 6

by multiplying the sides of the inequality by ($-\frac{1}{3} < 0$) we get :

$\therefore$ x < -2

i.e., the solution set in R is est]- ∞ , -2[

and it is represented by the green color ray on the number line.

3 **Find** the solution set for the in equality -3 ⩽ 2x -1 < 5 in R and represent the solution graphically.

Solution

by adding (1) to the sides of the inequality-3 + 1 ⩽ 2x -1 + 1 < 5 + 1

Namely,-2 ⩽ 2 x < 6, and by multiplying the sides of the inequality by ($\frac{1}{2} > 0$) -1 ⩽ x < 3

$\therefore$ the solution set in R is [-1, 3[and it is graphed on the number line by the green color.

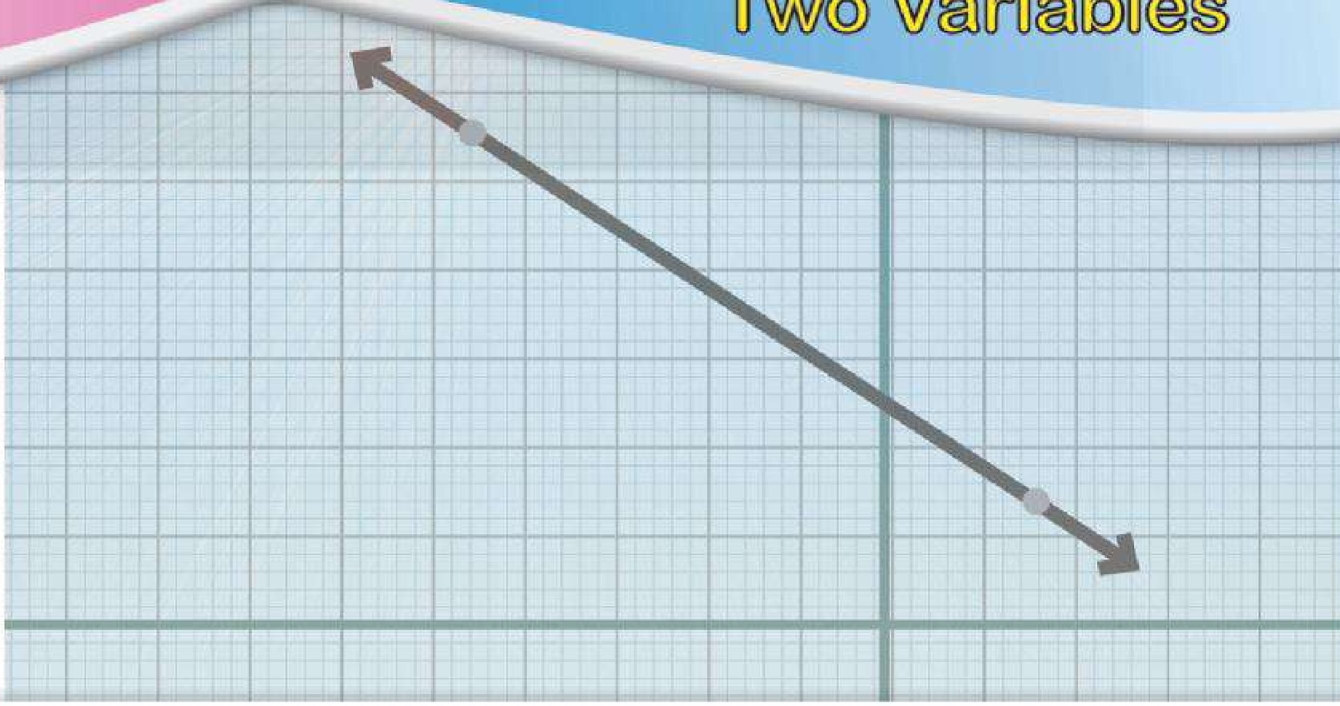
in example 3 What is the solution set for the inequality in N?

What is the solution set for the inequality in Z?

UNIT TWO

2

Relation Between Two Variables



Unit TWO

Lesson One

Linear Relation of two variables

You will learn how

- The linear Relations of two variables
- To graph the linear relations of two variables

key terms

- Variable
- Relation
- Linear equation

Think and Discuss

A person has some bills of LE 50 and LE 20. He bought an electrical apparatus for LE 390.

Think: How many bills of each type does he give to the seller?

Suppose : x represents the number of fifties bills, then the value of what he has of these bills is L.E 50x, y represents the number of Twenties bills, then the value of what he has of these bills is L.E 20y.

Required is to know: x and y that verify the equation:

$$50x + 20y = 390$$

This relation represents a linear equation in two variables. Dividing both sides over 10 produces the following equivalent equation:

$$5x + 2y = 39$$

$$\therefore y = \frac{39 - 5x}{2}$$

Note that : x and y are natural numbers. Therefore, x should be an odd number.

The following table can be created to know the different possibilities of giving bills to the seller: a bill of L.E50 and 17 bills of L.E 20, or 3 bills of L.E 50 and 12 bills of L.E 20, or 5 bills of L.E 50 and 7 bills of L.E 20, or 7 bills of 50 and 2 bills of L.E 20.

x	y	(x , y)
1	17	(1 , 17)
3	12	(3 , 12)
5	7	(5 , 7)
7	2	(7 , 2)
9	negative	refused

Practice

1. A person has some bills of L.E 5 and some of L.E 20. He bought some goods from a shopping center for L.E 75. What are the different possibilities of paying this amount in the two types of bills which he has?

2. The perimeter of an isosceles triangle is 19cm. What are the different possible lengths of its sides? Side length $\in Z_+$

Remember : The sum of the lengths of any two sides of a triangle is greater than the length of the third side .

The Relation of two variables

$a x + by = c$ where $a \neq 0$, $b \neq 0$ is called a linear relation of two variable **x** and **y** and can be described by a set of ordered pairs (**x**, **y**) verifying this relation.

Example:

Refer to the relation $2x - y = 1$

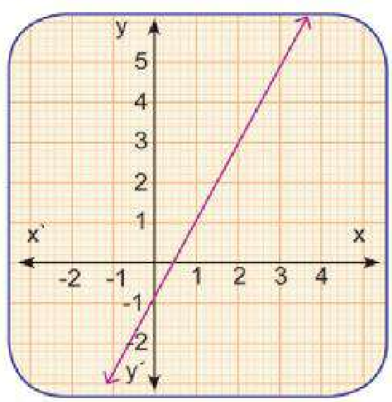
If x = 1, $\therefore$ y = 1 $\therefore$ (1 , 1) satisfies the relation

If x = 0, $\therefore$ y = -1 $\therefore$ (0 , -1) satisfies the relation

If x = 3, $\therefore$ y = 5 $\therefore$ (3 , 5) satisfies the relation

If x = -1, $\therefore$ y = -3 $\therefore$ (-1 , -3) satisfies the relation

Thus, there are an infinite number of ordered pairs satisfying the relation.



Note that:

- The linear relation $2x - y = 1$, can be represented graphically by using any of the ordered pairs obtained before.
- Each point $\in$ the straight line (in red) is represented by an ordered pair whose elements satisfy the linear relation $2x - y = 1$.

Practice

1. **Find four ordered pairs satisfy each linear relation and represent it graphically:**

   A. $x + y = 3$ B. $x - 2y = 5$ C. $y = 2$ D. $x = 1$

2. Find the value of b, where (-3, 2) satisfies the relation $3x + b\,y = 1$.

3. Find the value of k, where (k, 2k) satisfies the relation $x + y = 15$.

Graphing the Relation of two Variables

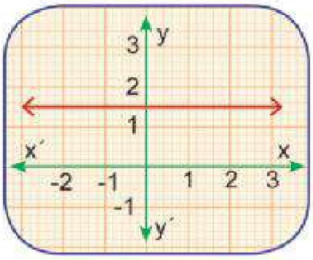
The relation $ax + by = c$ where a and b or both are not equal zero.
is called a linear relation of two variables x and y and can be represented graphically by a straight line.

for $a = 0$

The relation is represented by a straight line parallel to x-axis.

Example : $2\,y = 3$

i.e. : $y = \frac{3}{2}$
is represented by the red line which passes through the point $(0. \frac{3}{2})$ and is parallel to x-axis.



Special case:

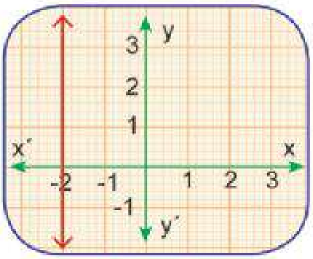
the relation y = 0 represents the x-axis

for $b = 0$

The relation is represented by a straight line parallel to y-axis.

Example : $x = -2$

is represented by the red line which passes through the point (-2, 0) and is parallel to y-axis.



Special case:

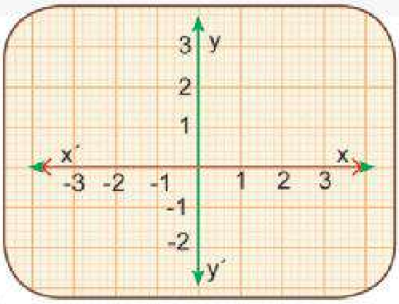
the relation x = 0 represents the y-axis.

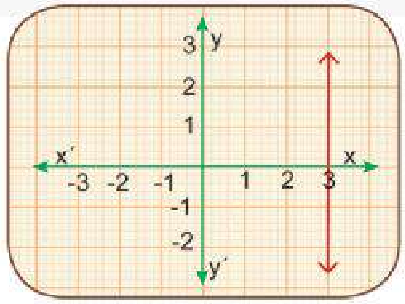
Practice

1. **Graph each relation of the following:**

   A. $2\,x = 5$ B. $y + 1 = 0$

2 Find the relation that is represented by the red line in each figure below:





Example :

Graph the relation: $x + 2y = 3$

Solution

Choose some ordered pairs that satisfy the relation:

Example : For **y = 2** $\therefore x = -1$ (-1 , 2) **satisfies the relation**

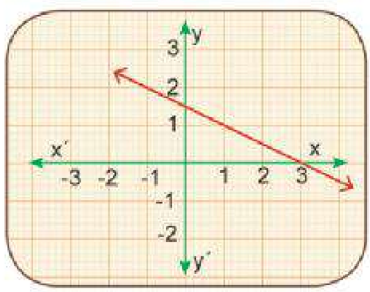
y = 0 $\therefore x = 3$ (3 , 0) **satisfies the relation**

y = -1 $\therefore x = 5$ (5 , -1) **satisfies the relation**

and so on ....

The following table lists these data:

x	-1	3	5	0
y	2	0	-1	$\frac{3}{2}$



The red line represents this relation.

Discuss with your teacher:

1. What happens to the value of y when increasing the value of x?
2. When does the line representing the relation ax + by = c pass through the origin 0?

Unit TWO

Lesson Two

The Slope of a line and real-life Applications

Think and Discuss

You will learn how

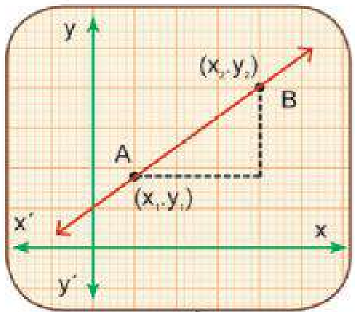
- The slope of a line .
- Real-life applications on the slope of a line.

key terms

- Slope.
- Positive slope.
- Negative slope.
- Zero-slope.
- Undefined slope.

When observing the motion of a point on a straight line from the location A (x_1, y_1) to the location B (x_2, y_2), where $x_2 > x_1$ and A, B $\in$ line, then:

1. the change in x-coordinate $= x_2 - x_1$, and is called the horizontal change.
2. the change in y-coordinate $= y_2 - y_1$ is called the vertical change and may be positive, negative or zero.



The slope of a line = $\dfrac{\text{change in y-coordinate}}{\text{change in x-coordinate}} = \dfrac{\text{vertical change}}{\text{horizontal change}}$

$$S = \frac{y_2 - y_1}{x_2 - x_1} \quad \text{where } x_2 \neq x_1$$

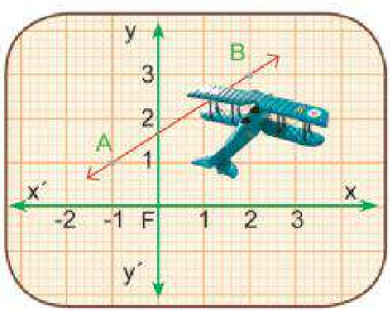
In the following examples you will learn different cases of the vertical change $(y_2 - y_1)$:

Example (1)

If: A (-1, 1) and B (2, 3),

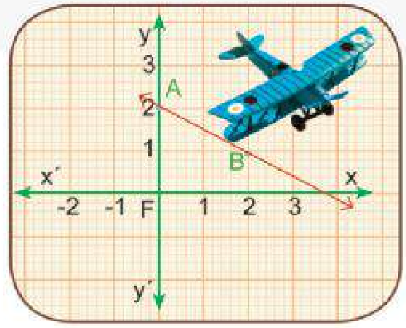
then: the slope of $\overleftrightarrow{AB}$

$$= \frac{3 - 1}{2 - (-1)} = \frac{2}{3}$$



Note that :

1. The point A moves on the line upwards to the point B.
2. $y_2 > y_1$
3. The slope of the line is positive.



Example (2) :

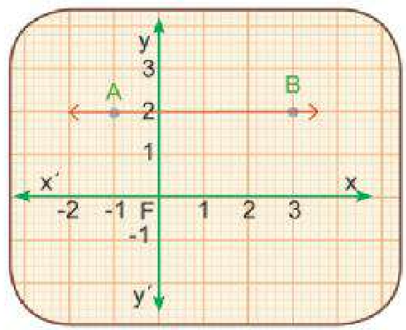
If: A (0, 2), B (2, 1);

then: the slope of $\overleftrightarrow{AB} = \frac{1-2}{2-0} = -\frac{1}{2}$

Not that :

The point A moves on the line downwards to the point B

2. $y_2 < y_1$
3. The slope of the line is negative.



Example (3) :

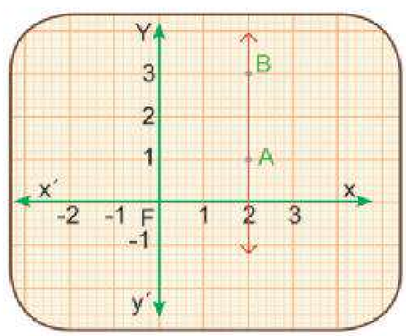
If: A (-1, 2) and B (3, 2),

then: the slope of the line

$\overleftrightarrow{AB} = \frac{2-2}{3-(-1)} = \frac{0}{4} = 0$

Not that :

1. The point A moves horizontally to point B.
2. $y_2 = y_1$
3. The slope of the line = zero



Example (4) :

If: A = (2, 1) and B(2, 3) then: we can not calculate the slope. Because the definition of the slope is conditioned to have a change in the x-coordinate i.e. $x_2 - x_1 \neq 0$

Not that :

1. The point A moves vertically to point B.
2. $x_2 = x_1$
3. The slope of the line is an underfined number.

Practice :

1 Find the slope of the straight line $\overleftrightarrow{AB}$ in each of the following cases:

- **A** A (1, 2), B (5, 0).
- **B** A (2, -1), B (4, -1).
- **C** A (-1, 3), B (2, 1).
- **D** A (3, -1), B (3, 2).

2 Find the slope of $\overleftrightarrow{AB}$, $\overleftrightarrow{BC}$ and $\overleftrightarrow{AC}$, where A (2, -1), B (3, 2), and C (4, 5) and represent each line graphically. What do you observe?

3 **Choose the true answer:**

First: The following table shows the relation between x and y as follows:
(y = x + 4 or y = x + 1 or y = 2x -1 or y = 3x -2)

x	1	2	3	4	5
y	1	3	5	7	9

Second: If (2, -5) satisfies the relation 3x –y + c = 0, then c = (1, -1, 11, -11)

Third: (3, 2) does not satisfy the relation (y +x = 5, 3y –x=3, y + x = 7, y – x = 1)

Fourth: An irrigation machine consumes 2.47Litres of diesels to work for 3 hours. If the machine works for 10 hours, it consumes.... litres. (7.2, 8, 8.4, 9.6)

4 **Find the slope of the line $\overleftrightarrow{AB}$, where A(-1, 3) and B (2, 5).**

Is the point c (8, 1) ∈ $\overleftrightarrow{AB}$?

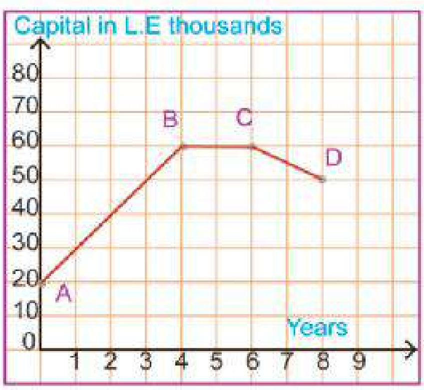
Real-life Applications on the slope of a line.

Application (1) :

The opposite figure shows capital change of a company during 8 years.

A Find the slope of $\overleftrightarrow{AB}$, $\overleftrightarrow{BC}$ and $\overleftrightarrow{CD}$
What is the meaning of each?

B Find the starting capital of the company.



Solution

A (0 , 20), B (4 , 60), C (6 , 60) , D (8 , 50)

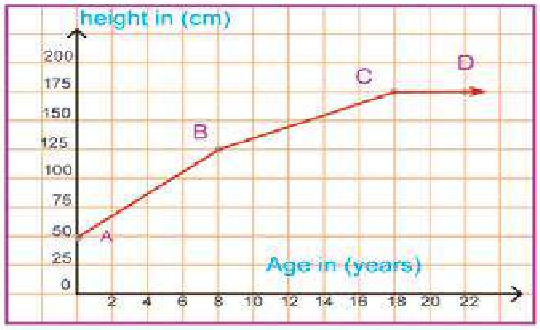
First: The slope of = $\overleftrightarrow{AB} = \frac{60-20}{4-0} = 10$, shows the increasing of the capital during the first four years with a rate of 10 thousand pound.

The slope of $\overleftrightarrow{BC} = \frac{60-60}{6-4} = 0$, means that the capital was constant during the fifth and sixth years.

The slope of $\overleftrightarrow{CD} = \frac{50-60}{8-6} = -5$ shows the decreasing of thecapital during the last two years with a rate of 5 Thousand pound.

Second: Starting Capital = the y-coordinate of the point A = LE 20,000

Practice:



The opposite figure shows the relation between the height of a person (in cm) and his age (in years).

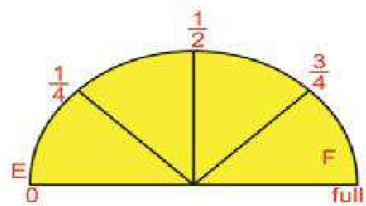
First: Find the slope of $\overleftrightarrow{AB}$, $\overleftrightarrow{BC}$ and $\overleftrightarrow{CD}$

What is the meaning of each?

Second: Calculate the difference between the height of this person as he was 8 years old and his height as he was 30 years old.

Application (2) :

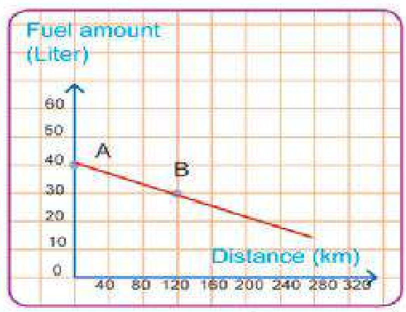
Hazem filled up the 40 Litres tank of his car. As covering a distance of 120 km, the fuel gage shows the rest of fuel is $\frac{3}{4}$ of the tank. Draw a diagram to show the relation between the amount of fuel in the tank and coverd distance (This relation is linear).Calculate the coverd distance as the tank is totally getting empty.



Solution

On the starting point: A (0, 40)

traveled distance — the amount of used fuel



After covering 120 km B = (120, 30)

The slope of $\overleftrightarrow{AB} = \frac{30-40}{120-0} = \frac{-1}{12}$

This slope means the fuel amount decreases with a rate of 1L per 12 km ,which means 1L is enough to cover a distance of 12 km.

The coverd distance that make the tank empty = $\frac{\text{Fuel Amount}}{\text{Decreasing Rate}} = \frac{40}{\frac{1}{12}}$

$= 40 \times \frac{12}{1} = 480$km.

Note that : $\overrightarrow{AB}$ intersects the distance-axis in the point (480, 0) **which gives the required distance.**

UNIT THREE

3

Statistics

UNIT THREE

Lesson One

Collecting and Organizing data

Think and Discuss

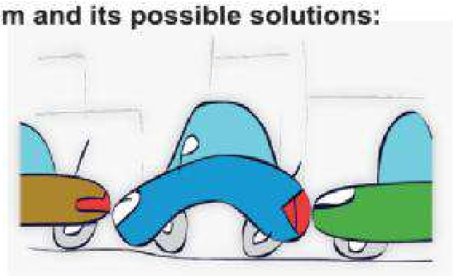
You will learn how

- To collect and organize data
- Using frequency tables with sets

key terms

- Collecting data
- Organizing data.
- Frequency table with sets

If you study the traffic jam problem and its possible solutions:



- What are the sources of your data?
- How can you collect data about such a problem?
- What are the statistical methods you will use to analyze the data?
- Can you explain the results you collected?
- What do you suggest to solve that problem and improve traffic fluidity?

Collecting data

Let's work together Cooperate with your classmates on collecting data from their sources through distribution of roles:

A **Group 1:** Collects primary data about the problem under discussion through a survey that asks about (the means of transportation - Roads conditions - time of traffic jam - Existence of traffic signs - existence of security).

B **Group 2:** Collects secondary data about the problem under discussion from the traffic reports - the internet - the mass media).

C **Group 3:** Observes the crowdest roads, the drivers` behavior and their obedience to traffic rules the pedestrians` commitment to the virtues to the road as well as crossing the roads at safe places.

Organizing and Analyzing data

Cooperate with your classmates on making afrequancy table that represents the means of transportation used by your classmates..

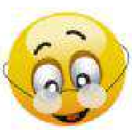
Means of trans-portation	Subway	bus	Private Car	Taxi	bicycle	on foot	total
Frequency	....	....	....	....	....	....	....

Determine the most used means of transportation (The mode)

1. Is that means suitable? does it help solving the traffic jam problem? why?

2. What do you suggest to solve this problem according to the results you have collected?

Organizing data and representing them in frequency tables



Example

Below are the scores of 30 students in an examination

7	10	7	4	5	8	6	7	13	12
2	9	11	12	11	9	15	12	13	9
5	14	19	3	9	14	3	13	8	17

Required: forming a frequency table with sets that represents that data .

Solution

To form a frequency table with sets, follow the following steps:

First: find the highest and the lowest values of the collected data?

let the previous collected data be X

then: X = {x : 2G x G 19}

i.e: X values begins with 2 and ends in 19

i.e: the range = the highest value - the lowest value = 19 – 2 = 17

Second: divide set X into a number of separate subsets each of them is equal in range.

let them be 6 sets. ⊠ The range of the set = $\frac{17}{6}$ i.e approximated to 3

Third: the subsets are as follow.

The first set	2 –	the third set	8 –	
The second set	5 –	The Fourth set	11 –	and so on

Remark : 2- means the set of data greater than or equal to 2 and less than 5 and so on.

Fourth: Record the data in the following table:

Set	tally	frequency
2 –	////	4
5 –	~~////~~ /	6
8 –	~~////~~ //	7
11 –	~~////~~ ///	8
14 –	///	3
17 –	//	2
Total		30

Fifth: Delete the tally column from the table to get the frequency table with sets. It can be written either verticaly or horizontaly. The following is the horizontal form of the table:

Sets	2 –	5 –	8 –	11 –	14 –	17 –	total
Frequency	4	6	7	8	3	2	30

UNIT THREE

Lesson Two

The Ascending and Descending Cumulative Frequency Table and Their Graphical Representation

Think and Discuss

You will learn how

- To Form both ascending and descending cumulative frequency tables.
- To represent both ascending and descending cumulative frequency tables graphically.

key terms

- The frequency distribution.
- The frequency table.
- The ascending cumulative frequency table.
- The descending cumulative frequency table.
- The ascending cumulative frequency curve.
- The descending cumulative frequency curve.

First: Ascending cumulative frequency table and its graphical representation.

Examples

The following table shows the frequency distribution for the heights of 100 students in a school in centimeters.

Tall (sets) in c.m	115–	120–	125–	130–	135–	140–	145–	Total
Number of students (frequency)	8	12	19	23	18	13	7	100

1. How many students are with height less than 115cm?
2. How many students are with height less than 135cm?
3. How many students are with height less than 145cm?

Form the ascending cumulative frequency table for these data and represent them graphically.

Solution

- Are there students with height less than 115c.m? **No**
- Are there students with height less than 135c.m? How many? **yes, 62 student.**
- How can you calculate the number of students with height less than 145 cm? **Add the number of students in the sets of height less than the set 145.**

Now, to answer the previous questions in an easier way, form an ascending cumulative frequency table as follows:

Upper boundaries of sets	Ascending cumulative frequency
Less than 115	0
Less than 120	0 + 8 = 8
Less than 125	8 + 12 = 20
Less than 130	20 + 19 = 39
Less than 135	39 + 23 = 62
Less than 140	62 + 18 = 80
Less than 145	80 + 13 = 93
Less than 150	93 + 7 = 100

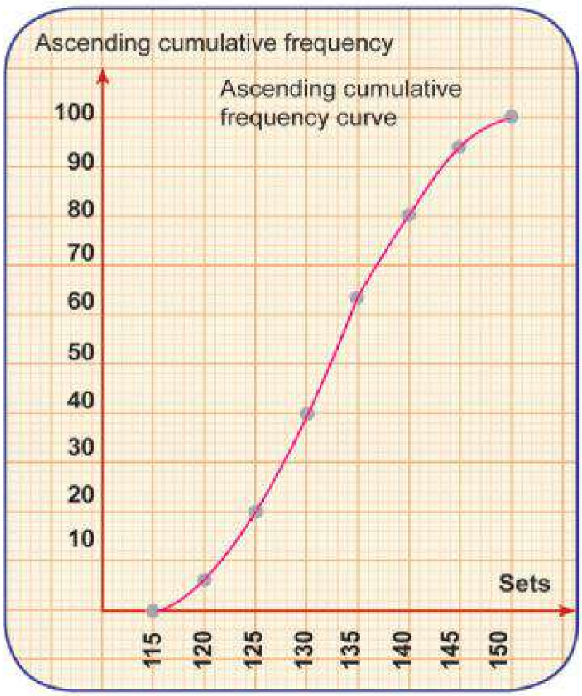
i.e.

ascending cumulative frequency table

Upper boundaries of sets	Ascending cumulative frequency
Less than 115	zero
Less than 120	8
Less than 125	20
Less than 130	39
Less than 135	62
Less than 140	80
Less than 145	93
Less than 150	100

To represent the ascending cumulative frequency table graphically:

1. Specify the horizontal axis to the sets and the vertical axis to the ascending cumulative frequency
2. Choose a drawing scale to draw the vertical axis such that the ascending cumulative frequency axis can hold the number of elements in a set
3. Represent the ascending cumulative frequency for each set and draw its line graph successively.



Second: The descending cumulative frequency table and its graphical representation. :

Of the previous frequency distribution which shows the heights of 100 students in a school in centimeters.

Find: The number of students with heights of 150cm and more..

The number of students with heights of 140cm and more..

The number of students with heights of 125cm and more..

Form the descending cumulative frequency table and represent it graphically..

Solution There are no students with heights of 150cm and more .

The number of students with heights of 140cm and more is 7 + 13 = 20 students.

The number of students with heights of 125cm and more is

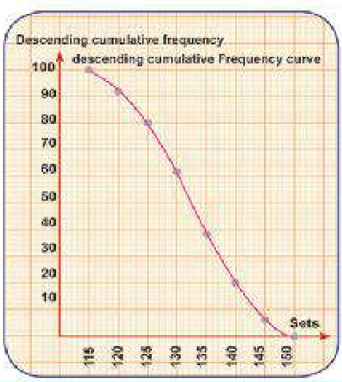
complete: 19 + ... + ... + ... + ... = ...

To answer these questions in an easier way, form the descending cumulative frequency table as follows :

Descending cumulative frequency table	
Lower limits of sets	**Ascending cumulative frequency**
115 and more	100
120 and more	92
125 and more	80
130 and more	61
135 and more	38
140 and more	20
145 and more	7
150 and more	zero

Lower limits of sets	descending cumulative frequency
115 and more	92 + 8 = 100
120 and more	80 + 12 = 92
125 and more	61 + 19 = 80
130 and more	38 + 23 = 61
135 and more	20 + 18 = 38
140 and more	7 + 13 = 20
145 and more	0 + 7 = 7
150 and more	0

To represent this table graphically, follow the steps of representing the ascending cumulative frequency to get the following graphical representation:



UNIT THREE

Lesson Three

Arithmetic Mean, Median and Mode

Think and Discuss

First: the mean

You have learned to find the mean for a set of values and learned that:

$$\text{The arithmetic mean} = \frac{\text{The sum of values}}{\text{Number of values}}$$

Example: If the ages of 5 students are 13، 15، 16، 14، and 17 years old, then

The mean of their ages $= \frac{13+15+16+14+17}{5}$

$= \frac{75}{5} = 15$ years

Remark: $15 \times 5 = 13 + 15 + 16 + 14 + 17$

The mean: is the simplest and most commonly used type of averages, It's that value given to each item in a set, then the total of these new values is the same total of the original values. It can be calculated by adding up all values, then divide the sum by the number of values.

Finding the mean of data from the frequency table with sets:

How can you find the mean of the following frequency distribution:

Sets	10 –	20 –	30 –	40 –	50 –	Total
Frequency	10	20	25	30	15	100

Remark: To find the mean for a frequency distribution with sets, follow the following steps:

You will learn how

- To find the mean from a frequency table with sets.
- To calculate the median from a frequency table.
- To calculate the mode from a frequency table with sets.

key terms

- Mean.
- Median.
- Frequency histogram.
- Mode.

1 Determine the centers of sets:

The center of the first set = $\frac{20+10}{2}$ = 15 . The center of the second set = $\frac{30+20}{2}$ = 25 ... and so on

Since the ranges of the subsets are equal and each = 10

We consider the upper limit of the last set = 60 and then :

$$\text{its center} = \frac{50+60}{2} = 55$$

2 Form the following vertical table:

Sets	Centre of the sets (X)	Frequency	Centre of the sets × frequency X × F
10 –	15	10	150
20 –	25	20	500
30 –	35	25	875
40 –	45	30	1350
50 –	55	15	825
Total		**100**	**3700**

3 The mean $= \frac{\text{The total of } (F \times X)}{\text{the total of } F}$

$$= \frac{3700}{100} = 37$$

Practice

1 If the mean of the scores of a student during the first 5 months is 23.8. What is the score of the 6th month If the mean of his scores is 24 marks?

2 The following table shows the frequency distribution of the weights of 30 children in kg.

Weight in (kg)	6–	10–	14–	18–	22–	26–	30–	Total
frequency	2	3	....	8	6	4	2	30

Complete the table, then find the mean of such a distribution.

Second: the median

The median is the middle value in a set of values after arranging it ascendingly or descendingy such that the number of values which are less than it is equal to the number of values which are greater than it.

Finding the median of a frequency distribution with sets graphically:

1. Draw the ascending or descending cumulative frequency table, then draw the cumulative frequency curve of it .
2. **Determine the order of the median =** $\frac{\text{The total of frequency}}{2}$.
3. Determine point A on the vertical axis (frequency) which represents the order of the median.
4. Draw a horizontal straight line from point A to intersect the curve at a point. Form this point, draw a vertical straight line on the horizontal axis to intersect it at a point that represants the median.

Example (1)

The following table shows the frequency distribution for the scores of 60 students in an exam.

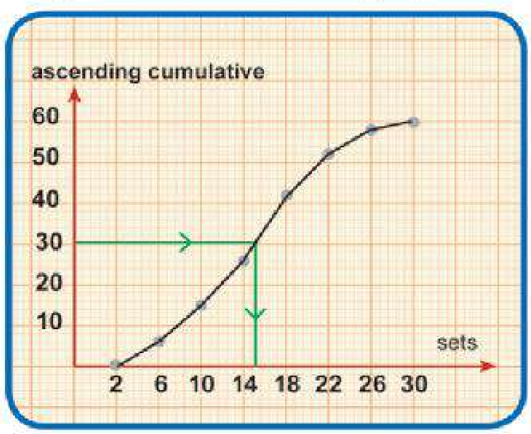
Sets	2–	6–	10–	14–	18–	22–	26–	Total
Frequency	6	9	12	15	10	5	3	60

Find the median of the distribution using the ascending cumulative frequency table.

Solution

1. Draw an ascending cumulative frequency table.
2. Find the order of the median = $\frac{60}{2} = 30$
3. Draw the ascending cumulative frequency curve, and get the median form the graph.

The upper limits of the sets	The ascending cumulative frequency
Less than 2	0
Less than 6	6
Less than 10	15
Less than 14	27
Less than 18	42
Less than 22	52
Less than 26	57
Less than 30	60



From the graph, the median = 14.8 mark

Think up Can you find the median using the descending cumulative frequency table? Is the value of the median different in such a case?.

Example (2)

The following table shows the daily wages of 100 workers in a factory..

daily wages in LE (sets)	15–	20–	25–	30–	35–	40–	Total
Number of workers (frequency)	10	15	22	25	20	8	100

Required:

1. Graph the ascending and descending cumulative frequency curves on one figure.
2. Can you find the median wage from this curve?

Solution

Upper boundaries of sets	Cumulative frequency
Less than 15	zero
Less than 20	10
Less than 25	25
Less than 30	47
Less than 35	72
Less than 40	92
Less than 45	100

Lower boundaries of sets	Cumulative frequency
15 and more	100
15 and more	90
15 and more	75
15 and more	53
15 and more	28
15 and more	8
15 and more	zero

Remark:

The ascending cumulative frequency curve intersects with the descending cumulative frequency curve at one point which is m .

The y-coordinate for the point M = 50

$= \frac{100}{2}$

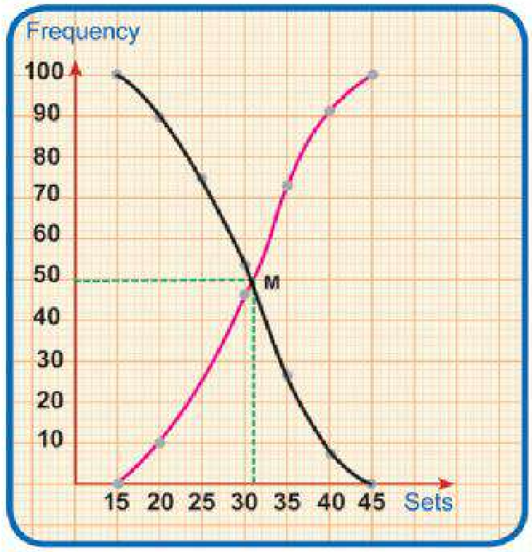
= the order of the median

∴ The X-coordinate of the point M determines the median

every 10 mm of the x coordinate represents L.E 5

Complete: 2 mm represents ....

The median wage = 30 + $\frac{2 \times 5}{10}$ = LE 31 .



practice

Draw the descending cumulative frequency curve for the following frequency distribution, then find the value of the median.

Sets	5 –	10 –	15 –	20 –	25 –	30 –	total
Frequency	4	6	10	17	10	3	50

Third: the mode

The mode is the most common value in the set or in other words, it is the value which is repeated more than any other values.

Example

The following table shows the frequency distribution for the scores of 40 students in an examination.

Sets	2–	6–	10–	14–	18–	22–	26–
Frequency	3	5	8	10	7	5	2

Find the mode of this distribution graphically

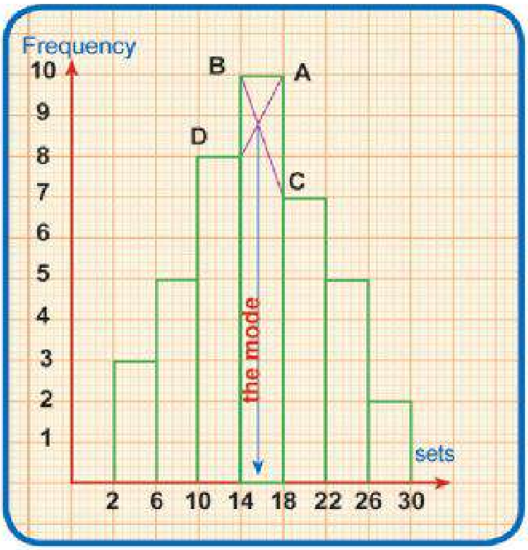
Solution

You can find the mode of this distribution graphically using the histogram as follows:

First: draw a histogram.

1. Draw two perpendicular axes: one horizontal to represent sets and the other vertical to represent the frequency of each set.

2. Divide the horizontal axis into a number of equal parts using a suitable drawing scale to represent sets.
3. Divide the vertical axis into a number of equal parts using a suitable drawing scale such that the greatest frequency among sets can be represented..
4. Draw a rectangle whose base is set (2-) and height is equal to the frequency (3).
5. Draw another rectangle adjacent to the first one whose base is set (6-) and height is equal to the frequency (5).
6. Repeat drawing the rest of adjacent rectangles till the last set (26-).



Second: Finding the mode from the histogram, to find the mode from the histogram, we observe that: the most repeated set is (14-), and it is called the mode set, why?

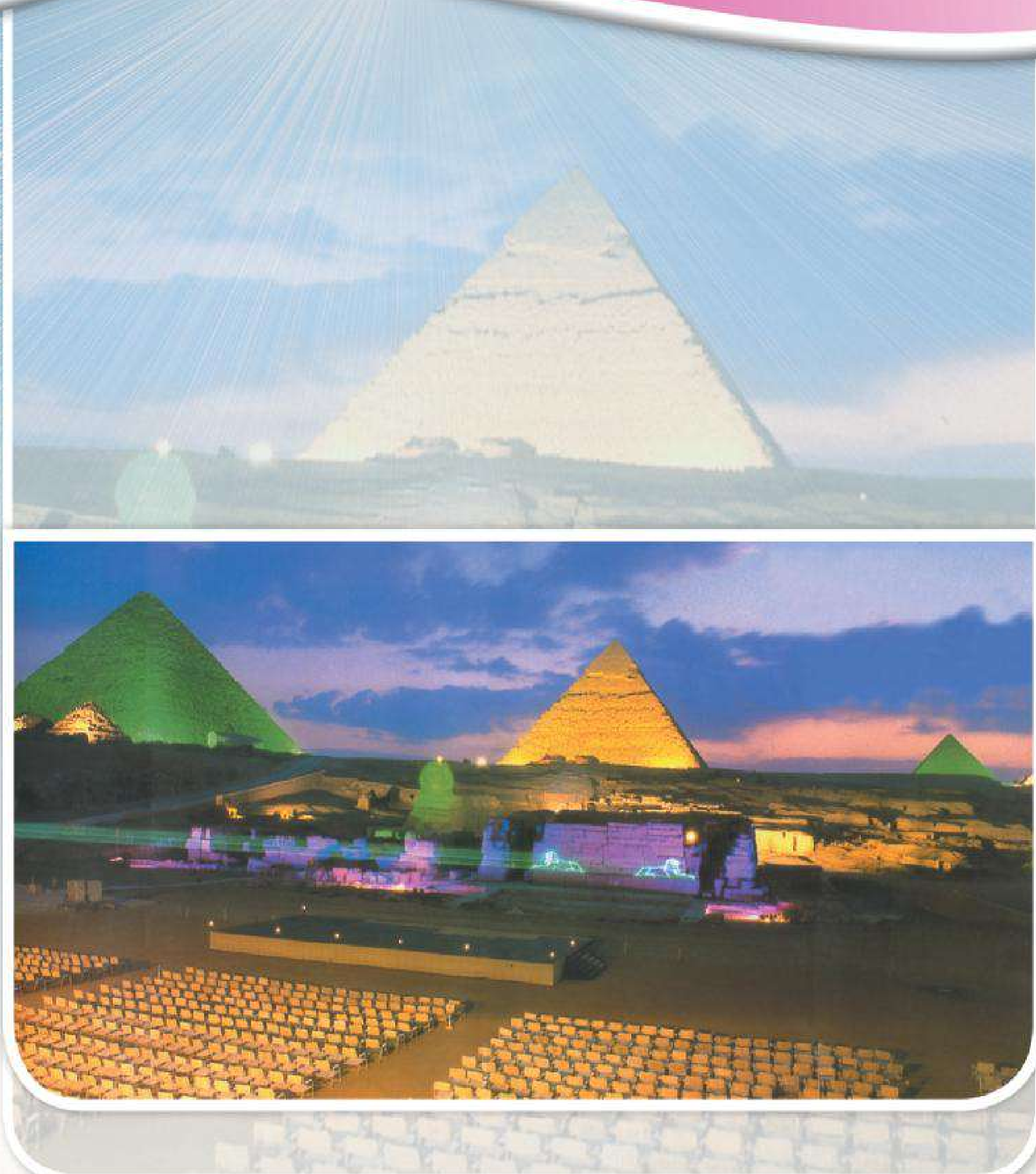
Define the intersection point of $\overline{AD}$, $\overline{BC}$ from the graph, and from this point, drop a vertical line on the horizontal axis to define the sequential value within that distribution.

From the graph, what's the mode value?

UNIT FOUR

4

Geometry



UNIT FOUR

Lesson One

The Medians Of Triangle

Think and Discuss

You will learn how

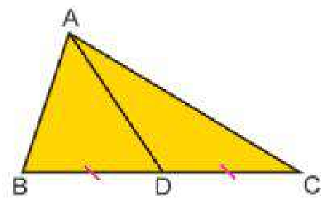
- Medians of the triangle.
- A 30° – 60° – 90° triangle.

key terms

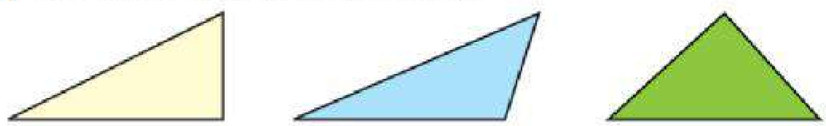
- Median of the triangle.
- A 30° – 60° – 90° triangle.
- Point of Concurrence

The medians of a triangle is the line segment drawn from the triangle vertex to the middle of the opposite side of this vertex.

ABC is a triangle where the point D bisects $\overline{BC}$.
So $\overline{AD}$ is a triangle Median.



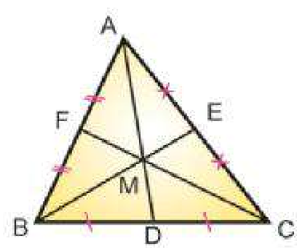
- How many medians does the trianglle have?
- Draw the medians in each triangle.



Theorem 1

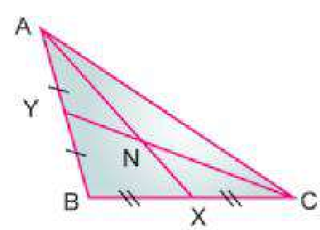
The medians of a triangle are concurrent

ABC is a triangle where point D bisects $\overline{BC}$, point E bisects $\overline{AC}$, point F bisects $\overline{AB}$, then $\overline{AD}$, $\overline{BE}$ and $\overline{CF}$ all intersect in one point (M)



Practice

In the Figure opposite: ABC is a triangle where point x bisects $\overline{BC}$, point y bisects $\overline{AB}$, and $\overline{AX} \cap \overline{CY} = \{N\}$.



1 Draw $\overrightarrow{BN}$ to intersect $\overline{AC}$ at point Z, then find the lengths of $\overline{AZ}$ and $\overline{CZ}$ is AZ = CZ? Reason your answer.

2 Measure, then complete.

$\frac{NX}{NA} = \frac{\dots\dots}{\dots\dots} = \frac{\dots\dots}{\dots\dots}$, $\frac{NY}{NC} = \frac{\dots\dots}{\dots\dots} = \frac{\dots\dots}{\dots\dots} = \frac{NZ}{NB} = \frac{\dots\dots}{\dots\dots} = \frac{\dots\dots}{\dots\dots}$

- If your measurements are accurate, then $\frac{NX}{NA} = \frac{1}{2}$, $\frac{NY}{NC} = \frac{1}{2}$ and $\frac{NZ}{NB} = \frac{1}{2}$

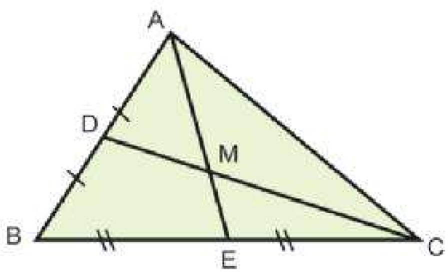
Theorem 2

The point of concurrence of the medians of the traingle divides each median in the ratio of 1:2 from its base

Practice

Complete

A

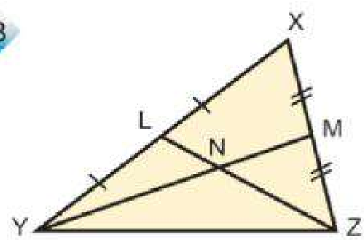


ME = 3cm, MC = 8cm

MA = ..., MD = ...

ME = ... AE, MC = ... CD

B



LZ = 15cm, YM = 18cm, XY = 20cm

NL = ..., NY = ...,

Perimeter of Δ NLY= ....,

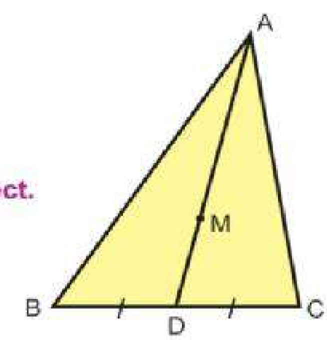
Fact

$\overline{AD}$ is a median in Δ ABC, M $\in$ $\overline{AD}$

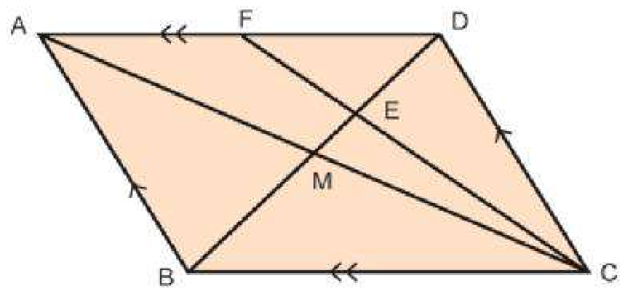
if AM = 2 MD,

then

then **M is the point where the medians of the triangle intersect.**



Example (1)



In the figure opposite: ABCD is a parallelogram where its two diagonals intersect at point M, point $E \in \overline{DM}$ and DE = 2 EM.

$\overrightarrow{CE}$ is drawn and intersected $\overline{AD}$ at point F.

Prove that: AF = FD

Proof: In the parallelogram ABCD

$\because \overline{AC} \cap \overline{BD} = \{M\}$

$\therefore$ M bisects $\overline{AC}$

the triangle DAC

$\because$ M bisects $\overline{AC}$

$\therefore \overline{DM}$ is a median of the triangle.

$\because E \in \overline{DM}$, DE = 2 EM.

$\therefore$ E is the intersecting point of the triangle's medians.

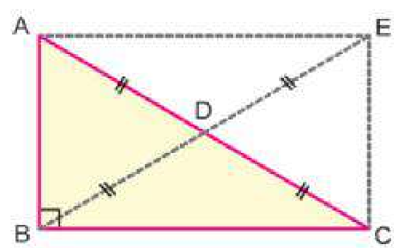
$\because E \in \overline{CF}$

$\therefore \overline{CF}$ is a median of the triangle and point F bisects $\overline{AD}$

Theorem 3

In the right - angled triangle, the length of the median from the vertex of the right angle equals half the length of the hypotenuse.

Given data: ABC is a triangle where m ($\angle$ B) = 90°, $\overline{BD}$ is a median in $\triangle$ ABC.



Required: Prove that: $BD = \frac{1}{2} AC$.

Construction: Draw $\overrightarrow{BD}$, let point $E \in \overrightarrow{BD}$ where BD = DE.

Proof:

$\because$ in the Figure ABCE, $\overline{AC}$, $\overline{BE}$ bisect each other.

∴ the Figure ABCE is a parallelogram.

∵ m (∠ B) = 90°

∴ ABCE is a rectangle.

∴ BE = AC.

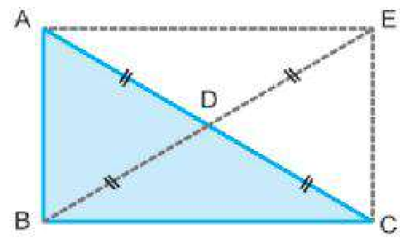
∵ $BD = \frac{1}{2} BE$ ∴ $BD = \frac{1}{2} AC$ Q.E.D.

The converse of the theorem 3

If the length of the median drawn from a vertex of a triangle equals half the length of the opposite side to this vertex, then the angle at this vertex is right.

Given: ABC is a triangle where $\overline{BD}$ is a median,

BD = DA = DC



Required: Prove that: m (∠ ABC) = 90°.

Construction : Draw $\overrightarrow{BD}$ and let point E ∈ $\overrightarrow{BD}$ where BD = DE.

Proof: ∵ $BD = \frac{1}{2} BE = \frac{1}{2} AC$.

∴ BE = AC.

In the Figure ABCE, AC and BE are equal in length and bisect each other.

∴ the Figure ABCE is a rectangle.

∴ m (∠ ABC) = 90°) Q.E.D.

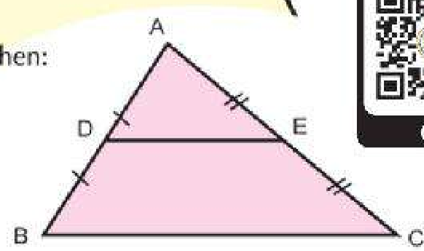
Corollary

The length of the side opposite to the angle of measure 30° in the right - angled triangle equals half the length of the hypotenuse.

Remember:

If the point D bisects $\overline{AB}$ and the point E bisects $\overline{AC}$, Then:

1. $DE = \frac{1}{2} BC$
2. $\overline{DE} // \overline{BC}$



UNIT FOUR

Lesson Two

The Isosceles Triangle

Think and Discuss

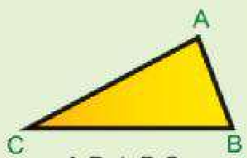
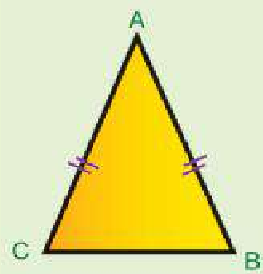
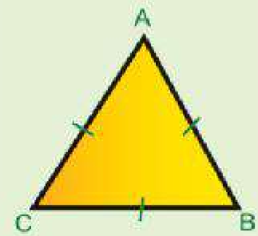
You will learn how

- To define the properties of the isosceles triangle.
- To define the classifications of the isosceles triangle..

key terms

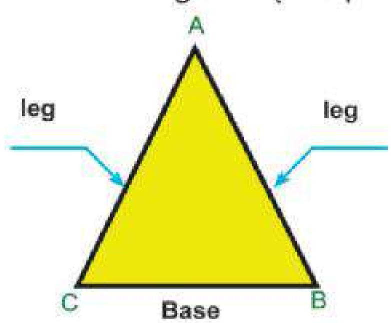
- The isosceles triangle.
- The equilateral triangle.
- The scalene triangle.

You have learnt that triangles are classified according to the lengths of their sides into three types:

The scalene triangle	The isosceles triangle (two sides are congruent)	The equilateral triangle (three sides are congruent)
 $AB \neq BC$ $AB \neq AC$ $BC \neq AC$	 $AB = AC$	 $AB = AC = BC$

In the figure opposite :

Remark : the two sides $\overline{AB}$, $\overline{AC}$ are congruent (of equal lengths), so the triangle ABC is called isosceles triangle while the point A is called the vertex. $\overline{BC}$ is the base, and the two angles B and C are the base angles of the triangle.



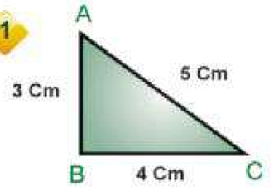
The properties of isosceles triangle

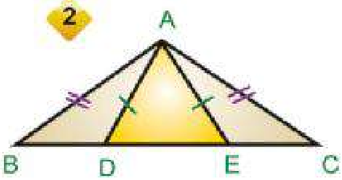
In any isosceles triangle

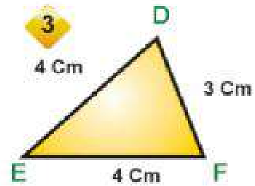
- What is the type of the base angles? (acute - right - obtuse)
- What is the type of the vertex angle?

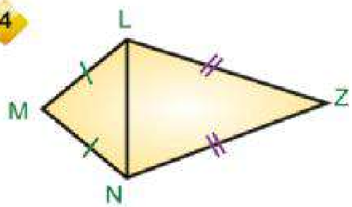
Practice

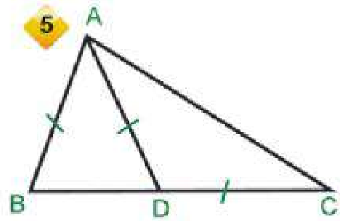
In each of the following figures, state the isosceles triangles and define their bases, then notice the type of the two base angles and the vertex angle.

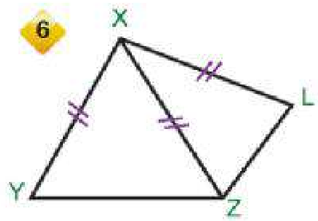
1


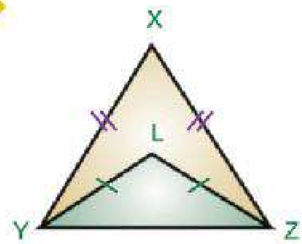
2


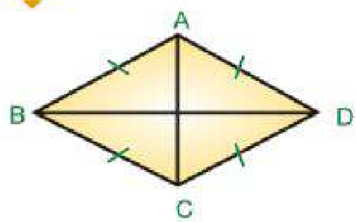
3


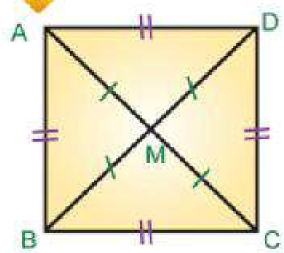
4


5


6


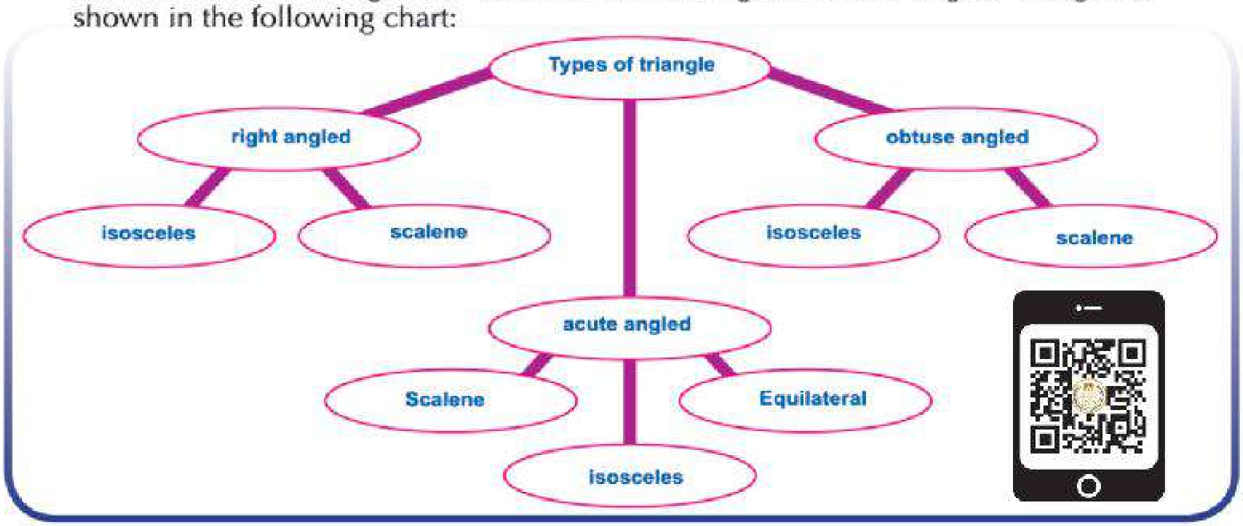
7


8


9


Remark :

1. Both of the base angles in the isosceles triangle are acute.
2. The vertex angle in the isosceles triangle can be either acute, right or obtuse. So, the isosceles triangle can be either obtuse, right or acute angled triangle as shown in the following chart:



UNIT FOUR

Lesson Three

The Isosceles Triangle Theorems

Think and Discuss

You will learn how

- To define the relation between the base angles in the isosceles triangle.
- To define the relation among the measures of the angles in the equilateral triangle.
- To define the relation between two sides opposite to two equal angles in a triangle.
- To know that if the angles in a triangle are congruent, then the triangle is equilateral.

key terms

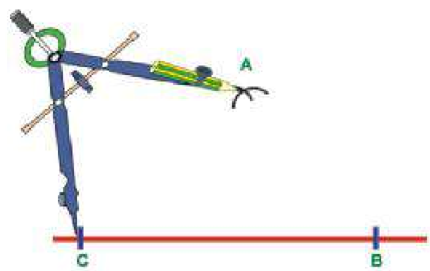
- The isosceles triangle.
- The base angles.

Is there a relation among the measures of the two base angles in the isosceles triangle?

to know that, let's conduct the next activity:

Activity

Using the compass



1. Draw several isosceles triangles as shown in the opposite figure Where AB = AC.
2. **Find** using a protractor, the measure of the two base angles ∠ ABC and ∠ ACB
3. Write down the data you got in a table as follows, then compare the measures in each case.

Number of the triangle	m (∠ ABC)	m (∠ ACB)
1		
2		
3		

4. Keep your activity in the portfolio.

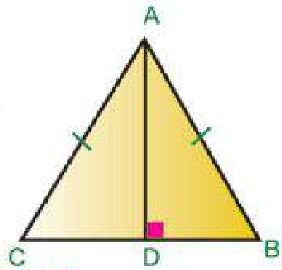
Theorem 1

(the isosceles triangle theorem) the base angles of the isosceles triangle are congruent.

Given: ABC is triangle in which $\overline{AB} \equiv \overline{AC}$

R.T.P: ∠ B ≡ ∠ C

Construction : draw $\overline{AD} \perp \overline{BC}$



Proof : The two triangles ADB and ADC are right angled in which.

$\left\{ \begin{array}{l} \overline{AB} \equiv \overline{AC} \\ \overline{AD} \end{array} \right.$ (a given)

(a common side)

$\therefore \Delta ADB \equiv \Delta ADC$ (a hypotenuse and a side)

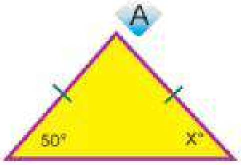
from the congruency , we deduce that

$\angle B \equiv \angle C$ Q.E.D.

practice

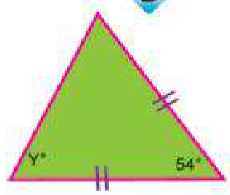
1 **In each of the following figures, find the value of the symbol that is used to measure the angle:**

A



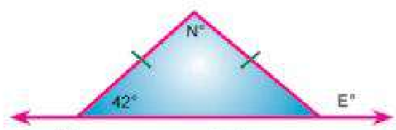
x = ...

B



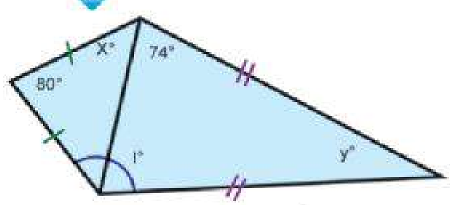
y = ...

C



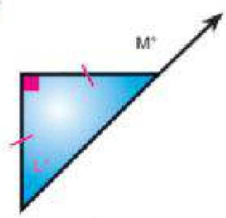
N = ... E = ...

D



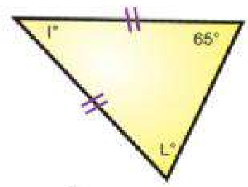
x = ... , y = ... , l = ...

E



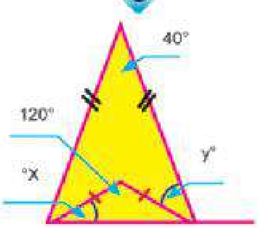
L = ... , M = ...

F



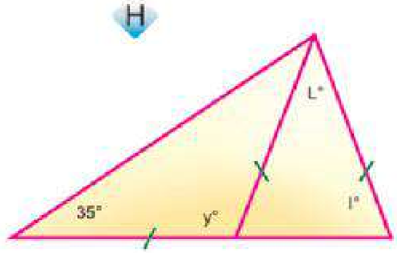
L =...... , l = ...

G



x = ... , y = ...

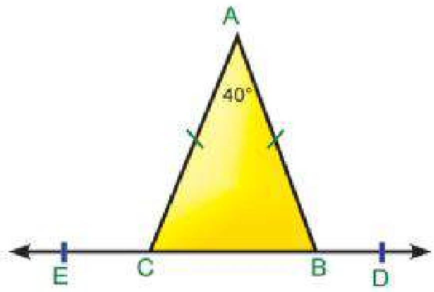
H



y = ... , L = ... , l = ...

2 **In the figure opposite,** ABC is an isosceles triangle in which AB = AC

$m(\angle A) = 40°$, $D \in \overrightarrow{CB}$, $E \in \overrightarrow{BC}$.



First: **Find** $m(\angle ABC)$

Second: **Prove that** $\angle ABD \equiv \angle ACE$

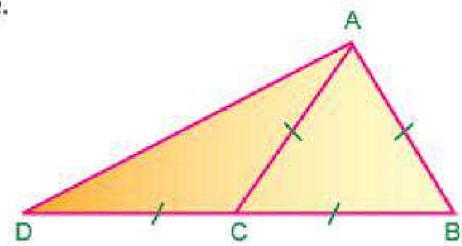
Think: Are the supplementary angles to congruent angles congruent?

Corollary

If the triangle is equilateral, then it is equiangular where each angle measure 60°.

Example (1)

In the figure opposite: ABC is an equilateral triangle. $D \in \overrightarrow{BC}$ such that BC = CD



prove that $\overline{AB} = \overline{AD}$

Given: $AB = BC = CA = CD$, $D \in \overrightarrow{BC}$

R.T.P: Prove that $\overline{BA} = \overline{AD}$

Proof : $\because \Delta$ ABC is an equilateral triangle.

$\therefore m(\angle ACB) = m(\angle BAC) = m(\angle B) = 60°$ **(corollary)**

$\because D \in \overrightarrow{BC}$

$\therefore \angle BCA$ is an exterior angle of the $\triangle ACD$

$m(\angle BCA) = m(\angle CAD) + m(\angle CDA) = 60°$ **(1)**

In $\triangle ACD$

$\because CA = CD$ $\therefore m(\angle CAD) = m(\angle CDA)$ **(2)**

from (1), (2) we deduce that: $m(\angle CAD) = m(\angle CDA) = 30°$

$\because$ m (∠ BAD) = m (∠ BAC) + m (∠ CAD)

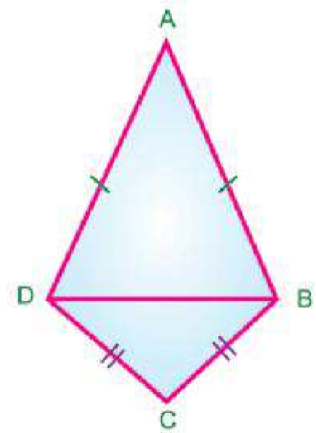
$\therefore$ m (∠ BAD) = 60° + 30° = 90°

$\therefore$ $\overline{BA} \perp \overline{AD}$ Q.E.D

Remark:

The measure of an exterior angle of a triangle is equal to the sum of the measures of the two non - adjacent interior angles.

Example (2)



2 **In the figure opposite:** AB = AD, BC = CD

Prove that ∠ ABC ≡ ∠ ADC

Given: AB = AD, BC = CD

R.T.P: **prove that** ∠ ABC ≡ ∠ ADC

Proof : In △ ABD

$\because$ AB = AD

$\therefore$ m (∠ ABD) = m (∠ ADB) (1)

in △ CBD

$\because$ CB = CD

$\therefore$ m (∠ CBD) = m (∠ CDB) (2)

By adding (1) and (2) we deduce that

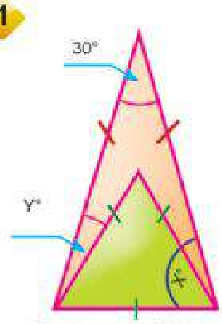
m (∠ ABD) + m (∠ CBD) = m (∠ ADB) + m (∠ CDB)

$\therefore$ m (∠ ABC) = m (∠ ADC)

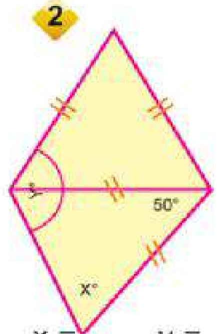
∠ ABC ≡ ∠ ADC **Q.E.D.**

practice

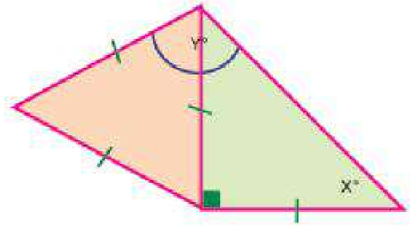
In each of the following figures, find the value of the symbol that is used to measure the angle:

1


x = ... , y = ...

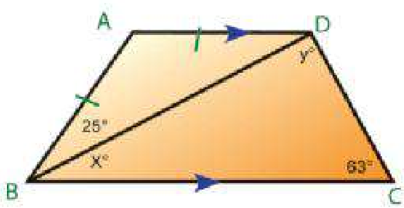
2


x = ... , y = ...

3


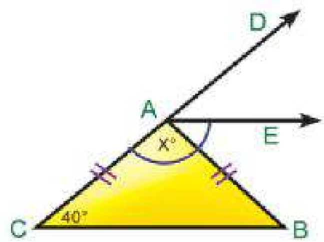
x = ... , y = ...

4 $\overline{AD}$ // $\overline{BC}$



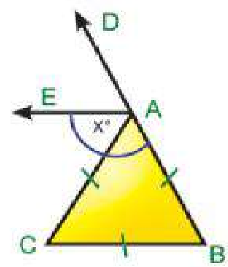
x = ... , y = ...

5 $\overrightarrow{AE}$ // $\overline{BC}$



x = ...

6 $\overrightarrow{AE}$ Bisects ∠ CAD



x = ...

Activity

Draw the triangle ABC in which BC = 7 cm, m (∠ B) = m (∠ C) = 50°, then measure the lengths of both $\overline{AB}$ and $\overline{AC}$. Repeat the activity using other measures for the length of $\overline{BC}$ and the measures of angles B and C, then fill in the table:

Number of the triangle	BC	m (∠ B)	m (∠ C)	AB	AC
1	7cm	50°	50°	....	....
2	....	....	....	....	....
3	....	....	....	....	....
4	....	....	....	....	....

1 Are $\overline{AB}$ and $\overline{AC}$ equal in length?

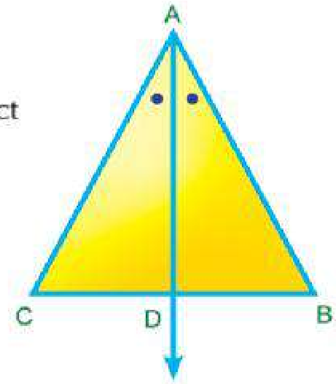
2 Is $\overline{AB}$ ≡ $\overline{AC}$?

3 How can you explain such corollaries geometricaly?

Theorem (2)

If two angles of a triangle are congruent, then the two sides opposite to these two angles are congruent and the triangle is isosceles.

ΔABC, $\angle B \equiv \angle C$



R.T.P: Prove that: $\overline{AB} \equiv \overline{AC}$

Construction : bisect $\angle BAC$ with the bisector $\overrightarrow{AD}$ to intersect $\overline{BC}$ at D

Proof : $\because \angle B \equiv \angle C$

$\therefore m(\angle B) = m(\angle C)$

$\because \overrightarrow{AD}$ bisects $\angle BAC$

$\therefore m(\angle BAD) = m(\angle CAD)$

$\because$ the sum of the measures of interior angles of a triangle is = 180 °

$\therefore m(\angle ADB) = m(\angle ADC)$

$\therefore$ In the two triangles ADB, ADC

$\overline{AD}$ is a common side
$m(\angle BAD) = m(\angle CAD)$
$m(\angle ADB) = m(\angle ADC)$

$\therefore \Delta ADB \equiv \Delta ADC$

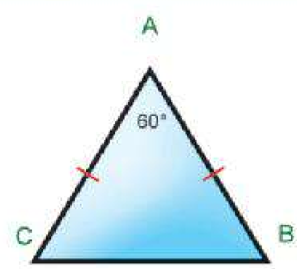
Form the congruency, we deduce that $\overline{AB} \equiv \overline{AC}$

Therefore, ΔABC is an isosceles triangle

Corollary

If the angles of a triangle are congruent, then the triangle is equilateral.

In the figure opposite ABC is an isosceles triangle in which:
AB = AC , m ($\angle$ BAC) = 60°



Complete m ($\angle$ ABC) = m ($\angle$ ACB) = ...

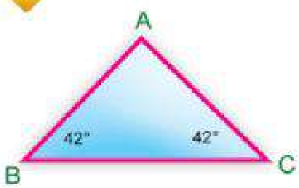
i.e: $\angle$...... $\equiv \angle$ .... $\equiv \angle$ ....

$\therefore \Delta ABC$ is .... triangle

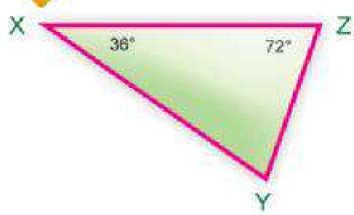
Remark: In an isosceles triangle, If any angle has a measure of 60°, then the triangle is an equilateral triangle.

practice

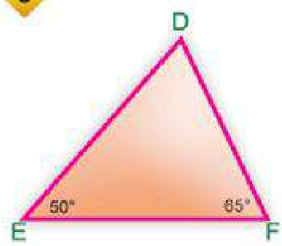
In each of the following figures, define the triangle`s sides that are equal in length as shown in example 1 :

1


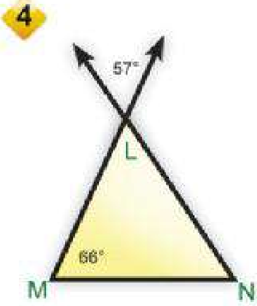
AB = AC

2


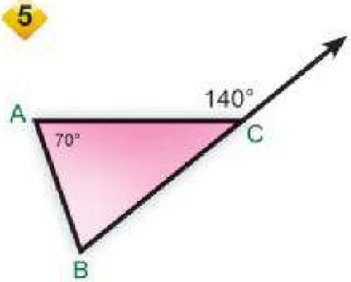
.... = ...

3


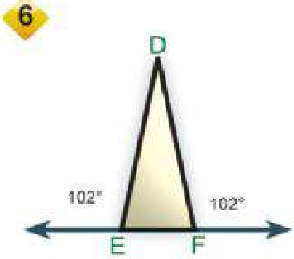
.... = ...

4


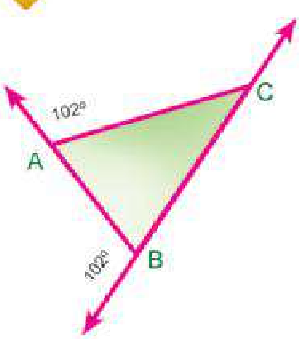
.... = ...

5


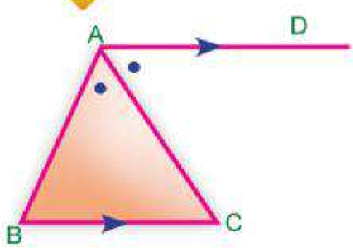
.... = ...

6


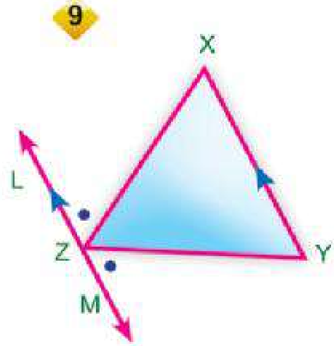
.... = ...

7


.... = ...

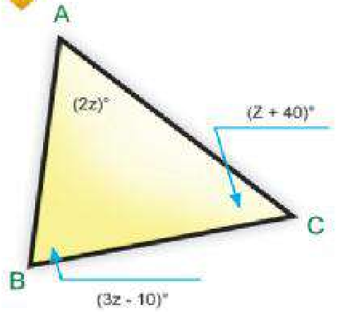
8


.... = ...

9


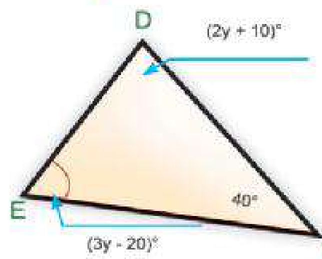
.... = ...

10



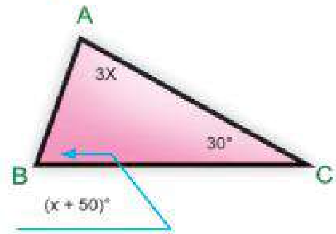
.... = ...

11



.... = ...

12

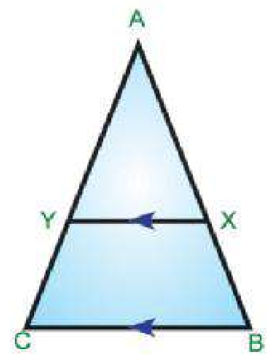


.... = ...

Examples

1 **In the figure opposite:** ABC is a triangle in which AB = AC, $\overline{XY}$ // $\overline{BC}$

prove that Δ AXY is an isosceles triangle



Given: AB = CA , $\overline{XY}$ // $\overline{BC}$.

Required: prove that AX = AY

Proof: In $\triangle$ ABC $\therefore$ AB = AC

$\therefore$ m ($\angle$ ABC) = m ($\angle$ ACB) (1)

$\because$ $\overline{XY}$ // $\overline{BC}$, $\overleftrightarrow{AB}$ a transversal

$\therefore$ m ($\angle$ AXY) = m ($\angle$ ABC) correspondingly (2)

The same $\overline{XY}$ // $\overline{BC}$, $\overleftrightarrow{AC}$ a transversal

$\therefore$ m ($\angle$ AYX) = m ($\angle$ ACB) correspondingly (3)

from (1), (2), (3) we deduce that :

m ($\angle$ AXY) = m ($\angle$ AYX)

In Δ AXY

$\because$ m ($\angle$ AXY) = m ($\angle$ AYX)

$\therefore$ AX = AY

i.e. the triangle AXY is an isosceles triangle Q.E.D

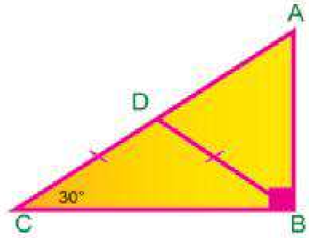
Think : **Can we deduce that XB = YC ? Explain your answer,**

2 **In the figure opposite :**

ABC is a right angled triangle at B, m (∠ C) = 30°,

D ∈ $\overline{AC}$ where DB = DC

prove that Δ ABD is an equilateral triangle.



Given: m (∠ A B C) = 90° , m (∠ C) = 30° , D B = D C

R.T.P: prove that A B = B D = AD

proof: In △ D B C ∵ D B = D C

∴ m (∠ D B C) = m (∠ C) = 30°

in △ A B C ∵ m (∠ AB C) =90° , m (∠ D B C) = 30°

∴ m (∠ BAD) =90 - 30 = 60° **(1)**

∵ ∠ ADB is an exterior angle of △ B D C

∴ m (∠ A D B) = m (∠ DBC) + m (∠ DCB)

m (∠ ADB) = 30° + 30° = 60° **(2)**

In Δ ABD ∵ the sum of the measures of the interior angles of a triangle = 180°

∴ m (∠ ABD) = 180° - (60° + 60°) = 60° **(3)**

from (1), (2), (3) ∴ m (∠ ABD) = m (∠ ADB) = m (∠ A)

i.e. ∠ ABD ≡ ∠ ADB ≡ ∠ A

∴ the triangle ABD is equilateral i.e. AB = B D =AD

UNIT FOUR

Lesson Four

Corollaries of isosceles triangle theorems

Think and Discuss

You will learn how

- The corollaries on the theorems of isosceles triangles.

key terms

- The isosceles triangle
- The bisector of a vertex angle
- The bisector of a triangle base.
- The axis of symmetry for a line segment..

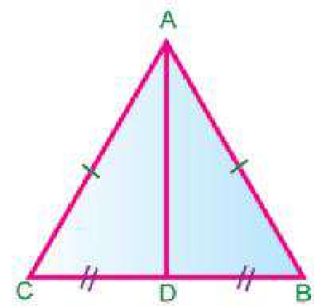
Corollary (1)

The median of an isosceles triangle from the vertex angle bisects it and is perpendicular to the base.

In the figure opposite:

In ΔABC , $AB = AC$, $\overline{AD}$ is a median

then: $\overrightarrow{AD}$ **bisects** $\angle BAC$. $\overline{AD} \perp \overline{BC}$



Remark: $\Delta ADB \equiv \Delta ADC$. Why?

Corollary (2)

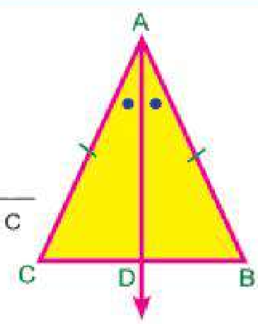
The bisector of the vertex angle of an isosceles triangle bisects the base and is perpendicular to it.

In the figure opposite:

In ΔABC , $AB = AC$,

$\overrightarrow{AD}$ bisects $\angle BAC$

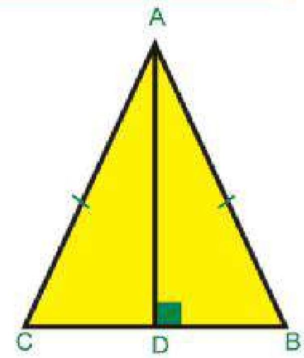
then D is a midpoint of $\overline{BC}$ and $\overline{AD} \perp \overline{BC}$



Remark: $\Delta ADB \equiv \Delta ADC$. why?

Corollary (3)

The straight line drawn passing through the vertex angle of an isosceles triangle perpendicular to the base bisects each of the base and the vertex angle.



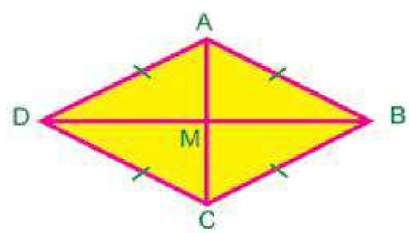
In the figure opposite :

In ΔABC, $AB = AC$, $\overline{AD} \perp \overline{BC}$

then D bisects $\overline{BC}$, $m(\angle BAD) = m(\angle CAD)$

Remark : $\Delta ADB \equiv \Delta ADC$. why?

Think



In the figure opposite :

ABCD is a quadrilateral in which all sides are equal in length, this figure is called rhombus, its diagonals are $\overline{AC}$ and $\overline{BD}$. they intersect at point M

Remark : $\Delta ABD \equiv \Delta CBD$. why?

$\therefore m(\angle ABD) = m(\angle CBD)$

in ΔABC, $AB = BC$, $\overrightarrow{BM}$ bisects $\angle ABC$

$\therefore \overline{BM} \perp$ ...., M is the midpoint of $\overline{AC}$

in ΔBAD, $AB = AD$, $\overline{AM} \perp \overline{BD}$

$\therefore \overrightarrow{AM}$ bisects $\angle$ ...., M is the midpoint of $\overline{BD}$

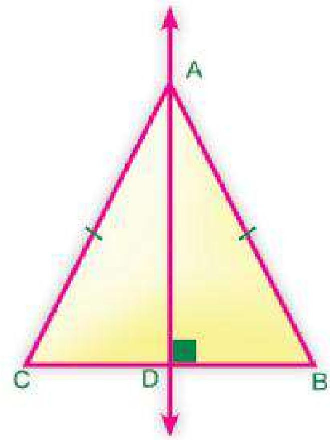
Are the two diagonals of the rhombus perpendicular?

Do the two diagonals of the rhombus bisect each other?

Does the diagonal of the rhombus bisect the vertex angles which it connects?

Write down your answer.

Axes of symmetry



First: axes of symmetry in the isosceles triangle:

The axis of symmetry of the isosceles triangle is the straight line drawn from the vertex angle perpendicular to its base.

In the figure opposite:

Δ ABC in which A B = A C, $\overline{AD} \perp \overline{BC}$

then $\overleftrightarrow{AD}$ is the axis of symmetry in the isosceles triangle ABC.

Discuss:

Does the isosceles triangle has more than one axis of symmetry?

- How many axes of symmetry are there in the equilateral triangle?

- Are there any axes of symmetry in the scalene triangle?

Second: Axis of symmetry of a line segment :

The straight line perpendicular to a line segment at its middle is called the axis of symmetry for that line segment in brief it is known as the axis of a line segment.

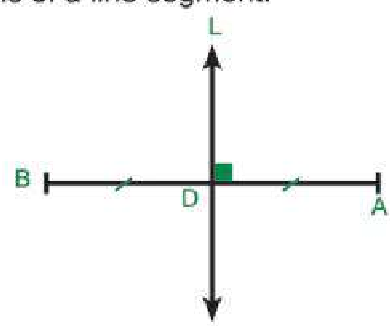
In the figure opposite:

If D the midpoint of $\overline{AB}$ and

The straight line L $\perp$ $\overline{AB}$

Where D $\in$ L, then the straight line L

Is the axis of $\overline{AB}$

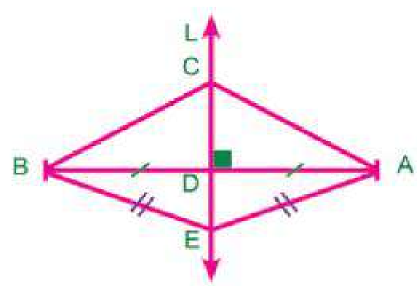


Important property

Any point at the axis of symmetry of a line segment is at equal distances from its end points.

Remark:

1. If C $\in$ L then AC = B C
2. If EA = E B then E $\in$ L. why?



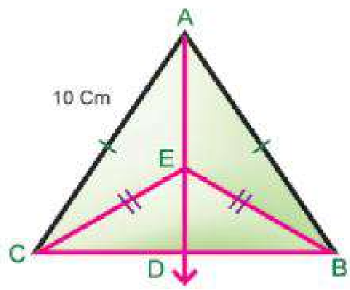
Examples

1 **In the figure opposite**

$AB = AC = 10$ cm , $EB = EC$

$\overrightarrow{AE} \cap \overline{BC} = \{D\}$

If $BC = 6$ cm, find the length of $\overline{CD}$ and $\overline{AD}$



Given : $AB = AC$, $EB = EC$

R.T.P : Find CD and AD

Proof : $\because AB = AC$ $\therefore$ A is on the axis of $\overline{BC}$

$\because EB = EC$ $\therefore$ E is on the axis of $\overline{BC}$

$\therefore \overleftrightarrow{AE}$ is the axis of $\overline{BC}$

D is the midpoint of $\overline{BC}$, $\overline{AD} \perp \overline{BC}$

$\because$ D is the midpoint of $\overline{BC}$, $BC = 6$cm $\therefore CD = 3$cm

$\because \overline{AD} \perp \overline{BC}$

$\therefore$ In Δ ADC that is right angled triangle at D

$(AD)^2 = (AC)^2 - (CD)^2$

$(AD)^2 = 100 - 9$

$\therefore AD = \sqrt{91}$ cm

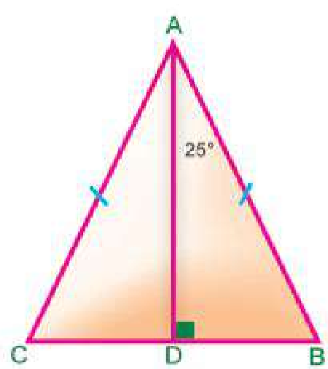
2 **In the figure opposite**

ABC is a triangle in which $AB = AC$,

$\overline{AD} \perp \overline{BC}$, $m(\angle BAD) = 25°$,

$BC = 4$ cm. Find:

A $m(\angle DAC)$ **B** the length of $\overline{DC}$



Solution

Given: $AB = AC$,

$\overline{AD} \perp \overline{BC}$, $m(\angle BAD) = 25°$, $BC = 4$ cm

R.T.P: $m(\angle DAC)$, and the length of $\overline{DC}$.

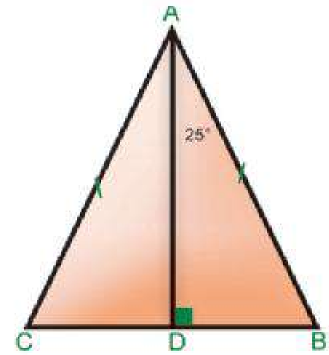
Proof : in △ ABC

$\because AB = AC$, $\overline{AD} \perp \overline{BC}$

$\therefore \overrightarrow{AD}$ bisects both of the base $\overline{BC}$ and ∠ BAC

$\therefore$ m (∠ DAC) = m (∠ DAB) = 25°,

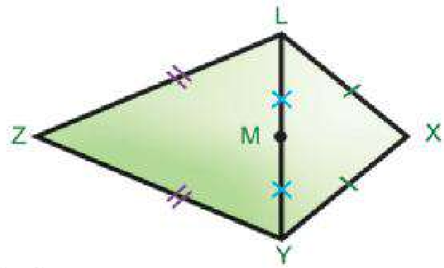
$DC = \frac{1}{2} BC = \frac{4}{2} = 2$ cm.



practice

1 **In the figure opposite**

x y = x L , Zy = Z L , L M = Y M



Prove that X, M and Z are on the same straight line

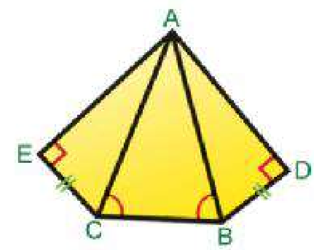
2 **In the figure opposite:**

BD = CE

m (∠ ABC) = m (∠ ACB)

m (∠ D) = m (∠ E) = 90°

Prove that : m (∠ DAB) = m (∠ CAE)



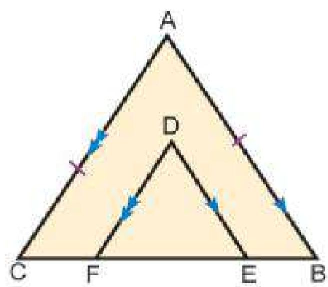
3 **In the figure opposite:**

AB = AC, $\overline{DE} \parallel \overline{AB}$

$\overline{DF} \parallel \overline{AC}$

Prove that: DE = DF

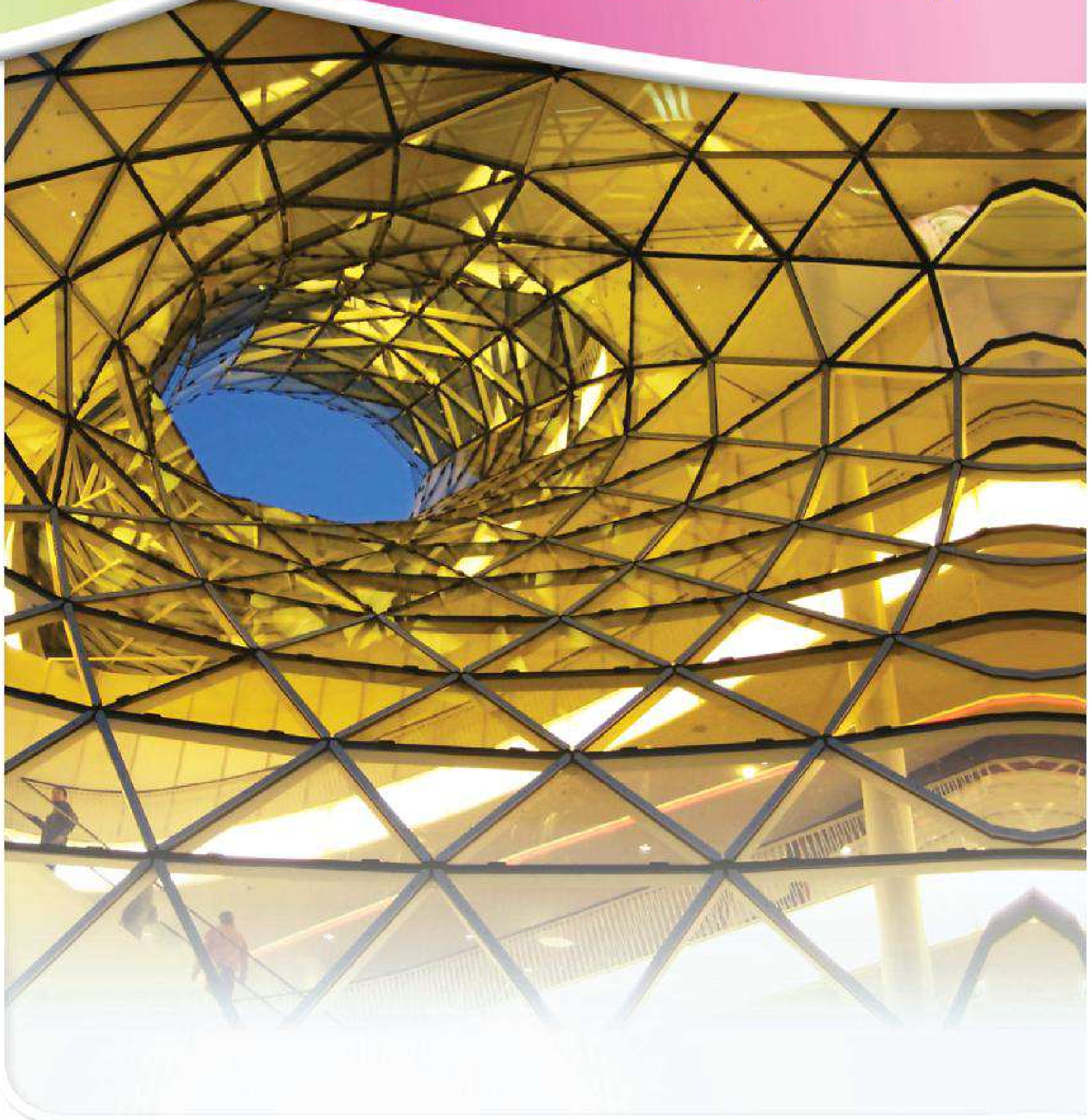
Second: m (∠ BAC) = m (∠ E D F)



UNIT FIVE

5

Inequality



UNIT FIVE

Lesson One

Inequality

Think and Discuss

You will learn how

- To define the concept of inequality.
- To define axioms of inequality.

key terms

- Inequality.
- axioms.
- greater than >.
- Less than <.
- equal to

The concept of inequality:

1. Do all the students in your class have the same height?
2. Are there any differences among the measures of acute, right and obtuse angles?

What does this difference mean?

Remark :

An Inequality means that there is a difference in the heights of the students and in the measures of the angles. This difference is represented by the relation of inequality which is used to compare two different numbers.

Examples

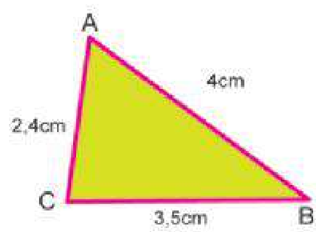
1. **If:** $\angle ABC$ is an acute angle **then:** $m(\angle ABC) < 90^\circ$.
2. **In the figure opposite ,** ABC is a triangle in which:

   AB = 4cm, BC = 3.5cm,

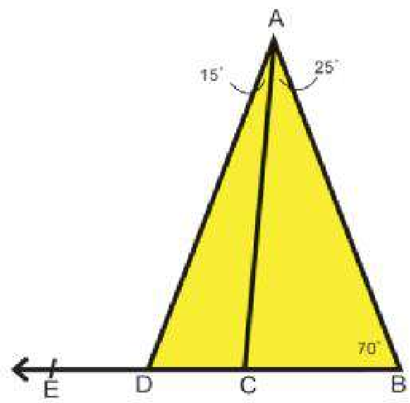
   A C = 2.4cm

   then: AB > BC , BC > AC

   i.e AB > BC > AC



practice:



In the figure opposite , find: m (∠ ACB), m (∠ ACD) and m (∠ ADE) then complete by using > or < :

m (∠ ADE) .... m (∠ CAD)

m (∠ ADC) .... m (∠ ACB)

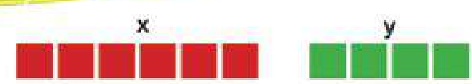
m (∠ ACD) .... m (∠ ABC)

m (∠ ACD) .... m (∠ ADE)

Remark : All the previous relations are called inequalities.

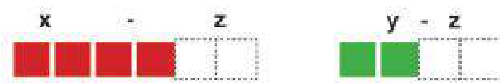
Axioms of inequality

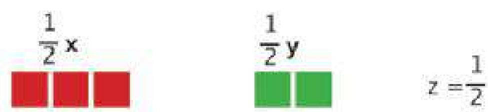
For any given three numbers x, y and z:



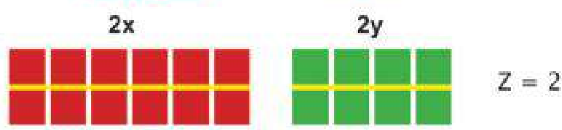
1 **If:** $x > y$
then: $x + z > y + z$

2 **If:** $x > y$
then: $x - z > y - z$

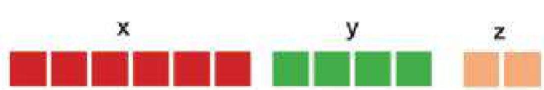




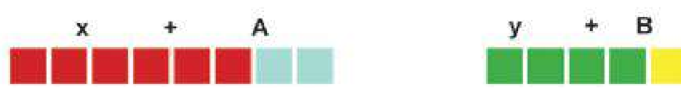
3 **If:** $x > y$, z **is a positive number**
then: $x z > y z$



4 **If:** $x > y$, $y > z$
then: $x > z$.



5 **If:** $x > y$, $A > B$
then: $x + A > y + B$

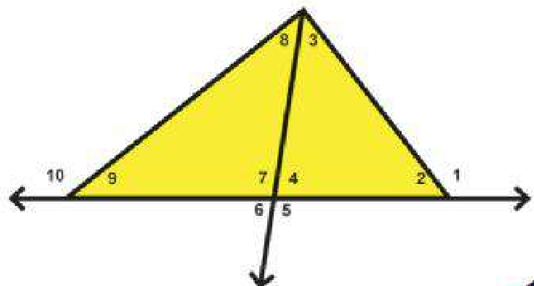


Remember: The measure of any exterior angle of a triangle is greater than the measure of any interior angle except for the adjacent angle.

practice

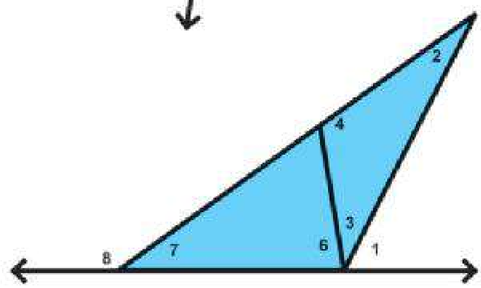
1 In the figure opposite: which of the following angles has the greatest measure?

- **A** ∠1 , ∠3 , ∠4
- **B** ∠4 , ∠8 , ∠9
- **C** ∠2 , ∠3 , ∠7
- **D** ∠7 , ∠8 , ∠10

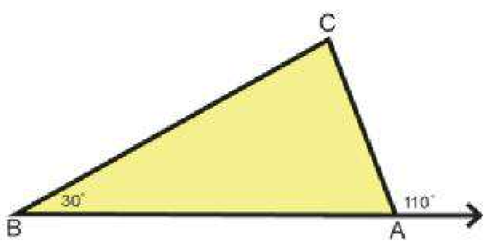


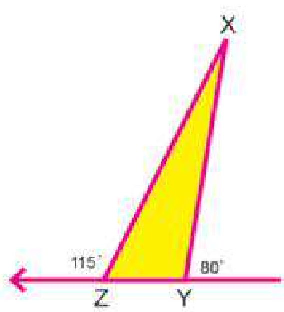
2 In the figure opposite , find:

- **A** All angles of measures less than m (∠ 1)
- **B** All angles of measures greater than m (∠ 6)
- **C** All angles of measures less than m (∠ 4)



3 Order the measures of the angles in the triangle ABC in an ascending order and the measures of the angles in the triangle X Y Z in a descending order.



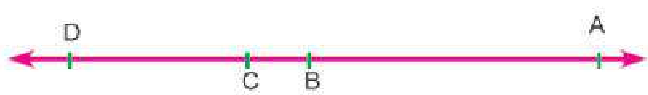


m (∠ ....) < m (∠ ....) < m (∠ ....)   m (∠ ....) > m (∠ ....) > m (∠ ....)

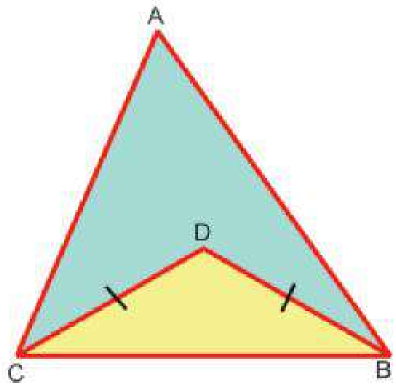
4 In the figure opposite: $C \in \overleftrightarrow{AB}$, $D \in \overleftrightarrow{AB}$

If: A B > C D

then: A C .... B D



Example



In the figure opposite :

$m(\angle ACB) > m(\angle ABC)$, $DB = DC$

Prove that : $m(\angle ACD) > m(\angle ABD)$

Given: $m(\angle ACB) > m(\angle ABC)$, $DB = DC$

Required to prove: $m(\angle ACD) > m(\angle ABD)$

R.T.P: $\because DB = DC$

$\therefore m(\angle DCB) = m(\angle DBC)$ **(1)**

$\because m(\angle ACB) > m(\angle ABC)$ **(2)**

$\therefore$ By subtracting **(1)** from **(2)** , we get:

$m(\angle ACB) - m(\angle DCB) > m(\angle ABC) - m(\angle DBC)$

$\therefore m(\angle ACD) > m(\angle ABD)$ **Q.E.D**

UNIT FIVE

Lesson Two

Comparing the measures of the angles of a triangle

Think and Discuss

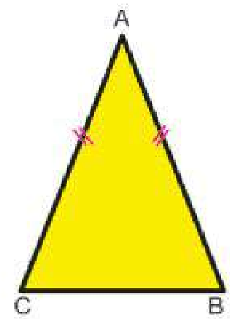
You will learn how

- To compare the measures of angles in a triangle.

key terms

- Angle.
- Measure of an angle.
- The greatest angle in a triangle.
- The smallest angle in a triangle.
- The largest side of a triangle.
- The smallest side of a triangle...

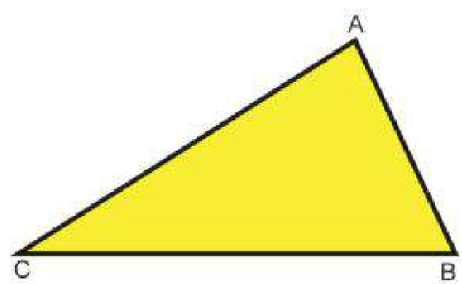
Activity



1 **In the figure opposite:** ABC is an isosceles triangle in which AB = AC

- Fold the triangle to make the vertex B congruent to vertex C. What do you observe regarding to the measures of the angles B, C which are opposite to the two equal sides $\overline{AC}$, $\overline{AB}$?
- Fold the triangle to make the vertices A, C congruent. what do you observe regarding to the measures of the two angles opposite to the two unequal sides $\overline{BC}$, $\overline{AB}$?
- Does the difference in the lengths of the two sides in a triangle lead to a difference in the measures of their two opposite angles?

2 Draw the scalene triangle. ABC Flip the triangle to make the vertex A coincide the vertex B. What do you observe regarding to the measures of the two angles A, and B that are opposite to the two unequal sides, $\overline{BC}$, $\overline{AC}$.



- Repeat the previous steps to make the vertex B coincide vertex C. what do you observe?

Are there any equal angles in measures in that triangle?

Notice that : In a triangle, if the sides are unequal in length, the measures of the opposite angles are unequal.

Activity

Draw the scalene triangle ABC, then measure the lengths of its 3 sides and the measures of the opposite angles, then complete the following table::

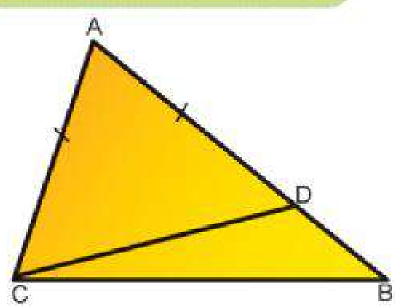
Lengths of sides	Measures of the opposite angles
AB = ... cm	$m(\hat{C}) = ...°$
BC = ... cm	$m(\hat{A}) = ...°$
CA = ... cm	$m(\hat{B}) = ...°$

What do you observe?

Theorem (3)

(Angle - Comparison Theorem)

In a triangle, if two sides have unequal lengths, then the longer is opposite to the angle of the greater measure.



Given: ABC in which A B > A C

R.T.P: $m(A\hat{C}B) > m(A\hat{B}C)$

Construction: take D ∈ $\overline{AB}$ where AD = A C

proof: in △ ACD , A D = A C

∵ $m(A\hat{C}D) = m(A\hat{D}C)$ **(1)**

A DC is an exterior angle of △ BDC

$m(A\hat{D}C) > m(\hat{B})$ **(2)**

from **(1)**, **(2)**

$m(A\hat{C}D) > m(\hat{B})$

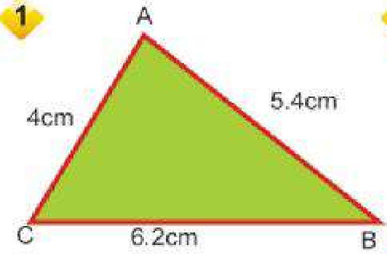
$m(A\hat{C}B) > m(A\hat{C}D)$

$m(A\hat{C}B) > m(A\hat{B}C)$ **Q.E.D**

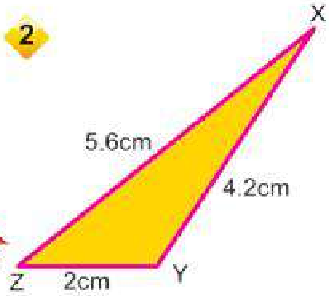
Practice

In each of the following figures, complete using (>, <))

1



2



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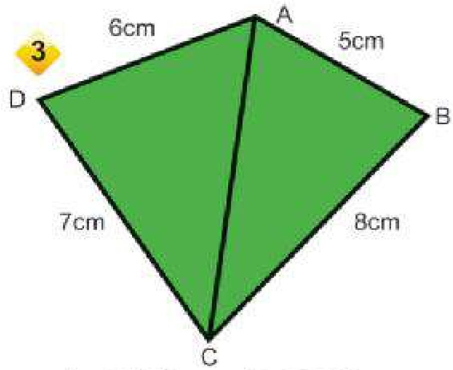
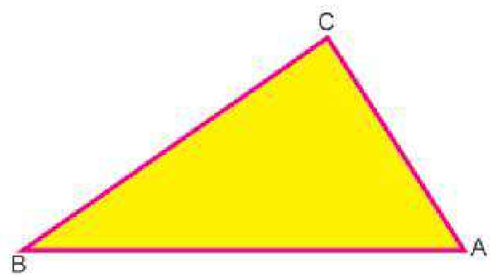


m (∠ A)	m (∠ B)	m (∠ z)	m (∠ y)	m (∠ BAC)	m (∠ BCA)
m (∠ A)	m (∠ C)	m (∠ x)	m (∠ y)	m (∠ DAC)	m(∠ DCA)
m (∠ B)	m (∠ C)	m (∠ z)	m (∠ x)	m (∠ BAD)	m (∠ BCD)

Remark : The measure of the greatest angle in the triangle > 60°

The measure of the smallest angle in the triangle is < 60° why?

Example



In the figure opposite :

ABC is a triangle in which AB > BC > CA

Prove that: m (∠ C) > m (∠ A) > m (∠ B)

Given: AB > BC > CA

R.T.P: m (∠ C) >m (∠ A) > m (∠ B)

Proof: In Δ A B C

∴ A B > B C ∴ m (∠ C) > m (∠ A) (1)

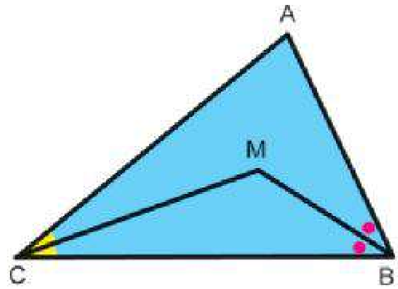
∴ B C > C A ∴ m (∠ A) > m (∠ B) (2)

from (1), (2) and using the axioms of inequality :

m (∠ C) > m (∠ A) > m (∠ B)

Remember : In a triangle, the longest side in length is opposite to the greatest angle in measure while the shortest side in length is opposite to the smallest angle in measure.

Example



In the figure opposite :

ABC is a triangle where $\overrightarrow{BM}$ bisects ∠ A B C, and $\overrightarrow{CM}$ bisects ∠ ACB If: M C > MB

Prove that: m (∠ ABC) > **m** (∠ ACB)

Given : $\overrightarrow{BM}$ bisects ∠ A B C, $\overrightarrow{CM}$ bisects ∠ ACB , MC > MB.

R.T.P: Prove that (∠ ABC) > **m** (∠ ACB)

Proof: in Δ MBC

∵ MC > MB ∴ **m** (∠ MBC) > **m** (∠ MCB) (1)

In Δ ABC

∵ $\overrightarrow{BM}$ bisects ∠ ABC ∴ **m** (∠ MBC) = $\frac{1}{2}$ **m** (∠ ABC) (2)

∵ $\overrightarrow{CM}$ bisects ∠ ACB ∴ **m** (∠ MCB) = $\frac{1}{2}$ **m** (∠ ACB) (3)

∴ from **(1), (2), (3) :** $\frac{1}{2}$ **m** (∠ ABC) > $\frac{1}{2}$ **m** (∠ ACB) Using the axioms of inequality

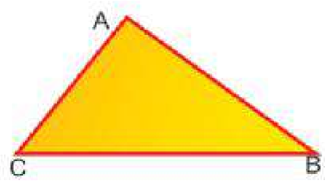
∴ m (∠ ABC) > m (∠ ACB) Q.E.D

UNIT FIVE
Lesson Three

Comparing the lengths of sides of a triangle

Think and Discuss

Activity 1 **The figure opposite:** ABC is a triangle of unequal measures of angles.



- Fold the triangle to make the vertex A coincide vertex B what do you observe regarding to the lengths of the two sides $\overline{BC}$ and $\overline{AC}$, which are opposite to the two unequal angles A and B?
- Repeat the same previous steps to make vertex B congruent to vertex C. What do you observe?
- When vertex C is coincide to vertex A, what do you observe?
- Are there any equal sides in lengths in that triangle?

Remark : **If the measures of the angles in a triangle are unequal, then the lengths of its sides which are opposite to the angles are unequal.**

Activity 2 Draw the triangle ABC where its angles are unequal in measure then measure the lengths of opposite sides to the angles and complete the following table:

the measures of the angles	the lengths of the opposite sides
m (∠ A) = ...°	BC = ... cm
m (∠ B) = ...°	CA = ... cm
m (∠ C) = ...°	AB = ... cm

What do you observe?

- Is the greatest angle in measure opposite to the longest side in length? Is the smallest angle in measure opposite to the shortest side in length?
- Is it possible to order the lengths of the sides in the triangle in an ascending or descending order in terms of the measures of the opposite angles?

You will learn how

- To compare the measures of sides in a triangle..

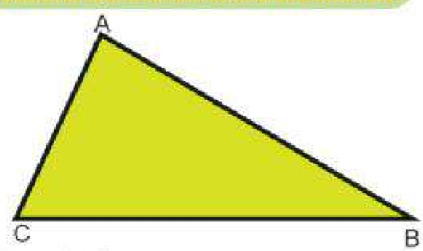
key terms

- The longest side of a triangle.
- The shortest side of a triangle.
- the greatest angle of a triangle.
- the smallest angle of a triangle.
- The perpendicular line segment.

Theorem (4)

(Side - Comparison Theorem)

In a triangle, if two angles are unequal in measure, then the greater angle in measure is opposite to a side greater in length than that opposite to the other angle.



Given : In Δ A B C m (∠ C) > m (∠ B)

R. T. P : A B > A C

Proof : ∴ $\overline{AB}$, $\overline{AC}$ are line segments

∴ one of the following cases should be verified:

(1) A B < A C (2) A B = A C (3) A B > A C

If not A B > A C

Either A B = A C **or** A B < A C

if A B = A C, then **m** (∠ C) = **m** (∠ B)

Again this contradicts the given where m (∠ C) > m (∠ B)

and if A B < A C, then m (∠ C) < m (∠ B). According to the theorem above.

Again this contradicts the given, where

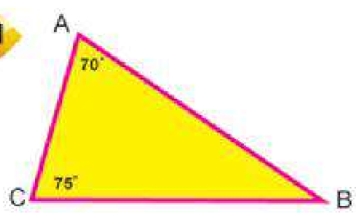
m (∠ C) > m (∠ B)

∴ A B > A C **Q.E.D**

practice

In the following figures , complete using > , < or =:

1

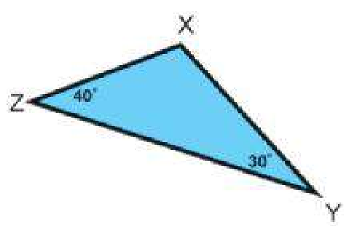


A B ...... A C

A B ...... B C

A C ...... B C

2

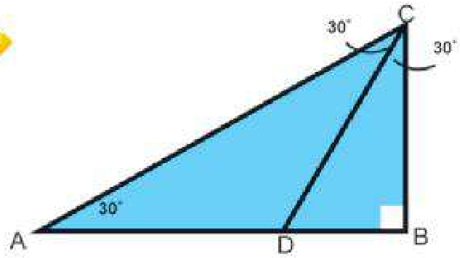


XY ...... XZ

YZ ...... XY

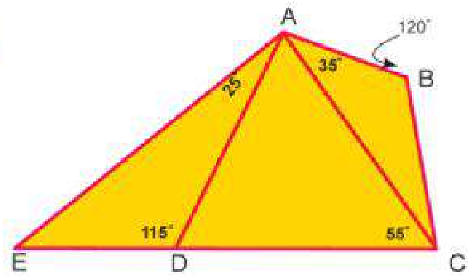
YZ ...... XZ

3



AC .... BC
BC .... DB
AC .... BD
CD .... AC

4



BC .... AB
CD .... CA
AD .... AE
CD .... AD

Corollaries :

Corollary (1)

In the right - angled trinagle, the hypotenuse is the longest side.

In the figure opposite Δ ABC is a right - angled triangle at B.

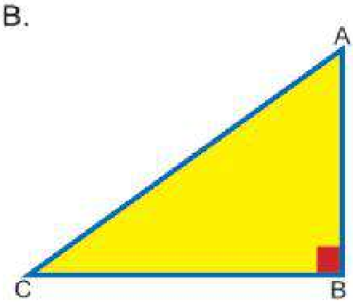
$\because \angle A$ acute $\quad \therefore m(\angle B) > m(\angle A)$

AC > BC

$\because \angle C$ acute $\quad \therefore m(\angle B) > m(\angle C)$

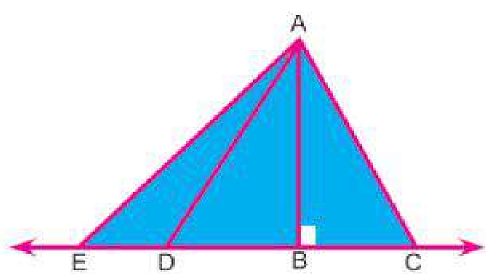
AC > AB

$\therefore \overline{AC}$ is the longest side Q.E.D.



Remark : In the obtuse angled triangle, the side opposite to the obtuse angle is the longest side in the triangle .

Let's think



AC > AB. Why?

AD > AB. Why?

AE > AB. Why?

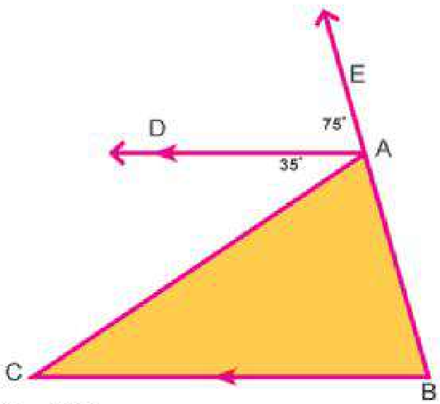
Is the length of the right leg in the right angled-triangle is shorter than the length of the hypotenuse? Why?

Corollary (2)

The length of the perpendicular line segment drawn from a point outside a straight line to this line is shorter than any line segment drawn from this point to the given striaght line.

Definition : The distance between any point and a given straight line is the length of the perpendicular line segment drawn from the point to the given line.

Example



in the figure opposite: ABC is a triangle , E ∈ $\overrightarrow{BA}$

$\overrightarrow{AD}$ \\ $\overline{BC}$, **m** ($\angle$ C A D) = 35°

m ($\angle$ DAE) = 75°

Prove that : AC > A B

Given that: $\overrightarrow{AD}$ // $\overline{BC}$, **m** ($\angle$ E A D) = 75°, **m** ($\angle$ D A C) = 35°

R.T.P: A C > A B

Proof: ∵ $\overrightarrow{AD}$ // $\overline{BC}$, $\overleftrightarrow{AB}$ is a transversal

∴ **m** ($\angle$ B) = m ($\angle$ E A D) = 75° — **Corresponding angles (1)**

∵ $\overrightarrow{AD}$ // $\overline{BC}$, $\overleftrightarrow{AC}$ is a transversal

∴ m ($\angle$ A C B) = m ($\angle$ D A C) = 35° — **alternate angles (2)**

From **(1) and (2)** :

in $\triangle$ A B C

m ($\angle$A B C) = 75° , m ($\angle$ A C B) = 35°

i.e. m ($\angle$ A B C) > m ($\angle$ A C B)

∴ AC > AB — **Q.E.D**

UNIT FIVE

Lesson Four

Triangle inequality

Think and Discuss

Activity

By using your ruler and compass, try to draw the triangle ABC where :

1. AB = 4 cm , BC = 5 cm , AC = 6 cm
2. AB = 6 cm , BC = 3 cm , AC = 2 cm
3. AB = 9 cm , BC = 4 cm , AC = 3 cm
4. AB = 8 cm , BC = 3 cm , AC = 5 cm

In which of the previous cases were you able to draw the triangle? What do you conclude?

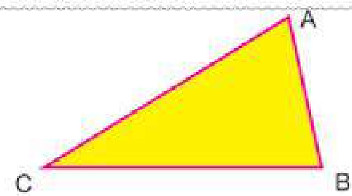
Fact : For any triangle, the sum of the lengths of any two sides is greater than the length of the third side.

i.e. : In any triangle ABC :

$AB + BC > AC$

$BC + CA > AB$

$AB + AC > BC$

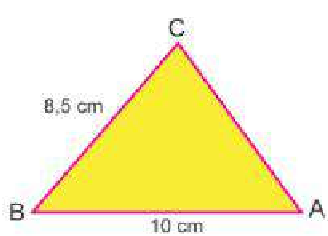


for example: the numbers 5, 3 and 9 are not valid to be the lengths of a triangle because the sum of the smallest two numbers = 3 +5 = 8 , 8< 9. Therefore, the inequality of the triangle is not verified.

Examples

ABC is a triangle . If AB = 10 cm, BC = 8.5 cm

Find the interval which the length of side $\overline{AC}$ belongs to.



You will learn how

- To define the triangle inequality .

key terms

- inequality .
- triangle inequality.

Solution

$AC < AB + BC$ $\qquad$ $AC < 18.5$ $\quad$ **(1)**

However, $AC + BC > AB$ $\qquad$ **triangle inequality**

$AC > AB - BC$ $\qquad$ $AC > 1.5$ $\quad$ **(2)**

From (1) , (2) $\quad 18.5 > AC > 1.5$

$AC \in]1.5 \;,\; 18.5[$

Practice

Find the interval which the third side belongs to in each of the follwing triangles. If the lengths of the other two sides were as follows:

A) 6 cm, 9 cm $\quad$ B) 5 cm, 12 cm $\quad$ C) 7 cm, 15 cm $\quad$ D) 2.9 cm, 3.2 cm

Second Term

Contents

Unit One: Factorization

Unit Two: Non-Negative and negative integer Powers in R and the Operation on them

Unit Three: Propability

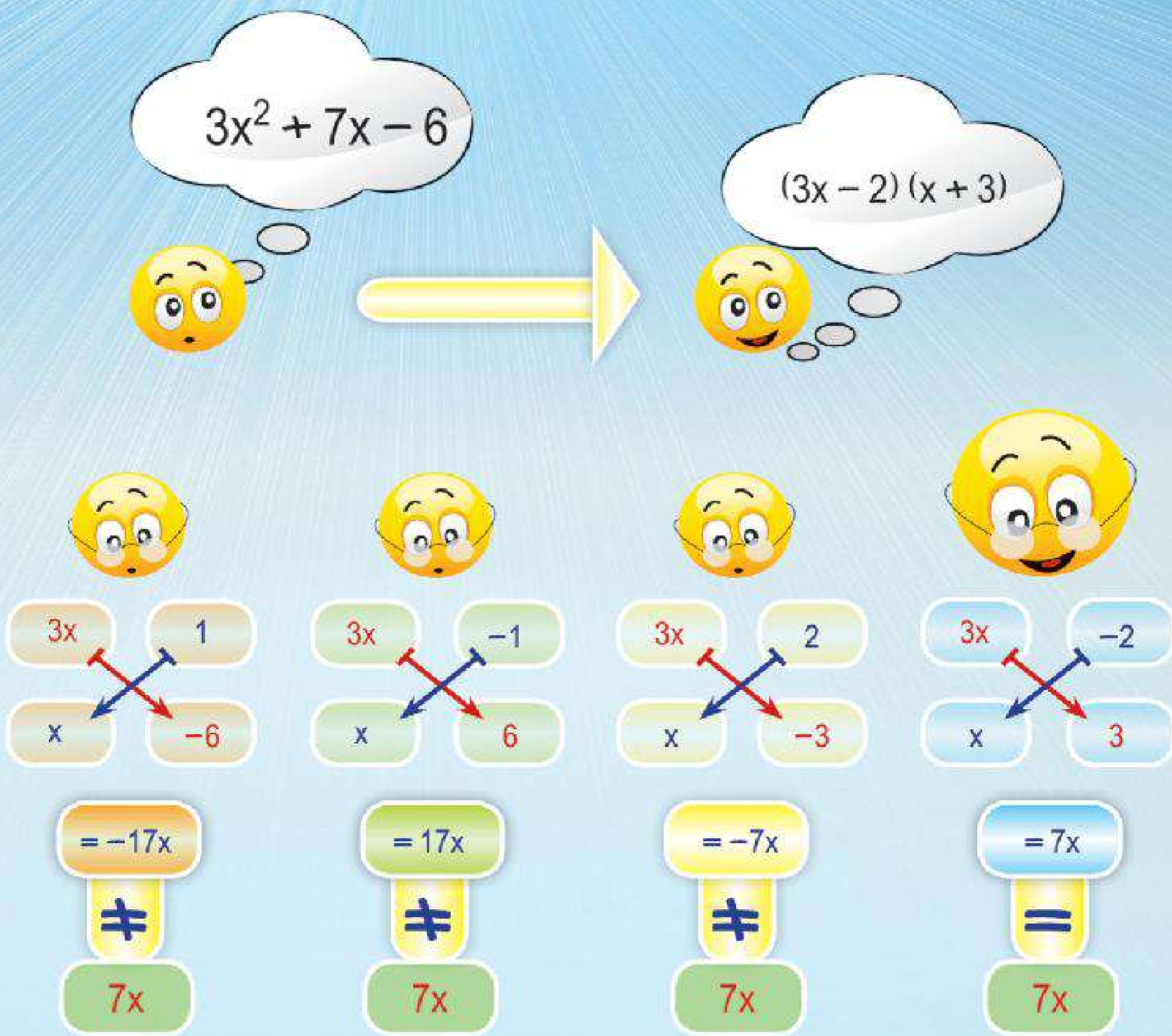
Unit Four: Areas

Unit Five: similarity and Projextions

UNIT ONE

1

FACTORIZATION



Unit One

Lesson One

Factorizing Trinomials

Think and Discuss

You will learn

- The meaning of factorizing an algebraic expression
- factorizing a trinomial

Key-Terms

- Factorizing.
- An algebraic expression
- A trinomial

You have learned that :

Factorizing an integer means to write it as a product of two or more factors.

For example:

$12 = 3 \times 4$, $12 = -3 \times -4$ or $12 = 2 \times 6$ or.........

We have learned before to factorize by taking out the highest common factor H.C.F:

For example:

$6x^2y^2 - 9x^3y = 3x^2y(2y - 3x)$

Practice

Factorize using H.C.F for all the following terms:

1. $2 \times (m + 3) - 4y(m + 3)$
2. $a(a-b) - b(b - a)$

You know that: $(x + 3)(x + 4) = x(x + 4) + 3(x + 4)$

$= x^2 + 4x + 3x + 3 \times 4$

$= x^2 + (4 + 3)x + 12$

$= x^2 + 7x + 12$

The algebraic expression $(X^2 + 7X + 12)$ **is often called a trinomial.**

By using the previous multiplication properties. Can you factorize the expression $(x^2 + 7x + 12)$ into two factors?

First: Factorize x^2 into $x \times x$

Product	sum
1×12	13
-12×-1	-13
2×6	8
-2×-6	-8
3×4	7
-3×-4	-7

Second: Guess and check two numbers whose product is 12 and whose sum is 7. They are 3 and 4. Thus, $x^2 + 7x + 12 = (x + 3)(x + 4)$

Practice

1. Find two numbers whose product is 20 and whose sum is 9
2. Find two numbers whose product is 12 and whose sum is -8
3. Find two numbers whose product is -24 and whose sum is 5
4. Find two numbers whose product is -15 and whose sum is -14

First: Factorizing Quadratic Trinomials in the form $x^2 + b x + c$

Factorize this expression into two linear factors:

- the first term in each factor is x.
- the last two terms are two numbers whose product is C and whose sum is b.

Exemples :

1 **Factorize the expression:** $x^2 - 5x + 6$

Look for two numbers whose product is 6 and whose sum is -5.
They are -2 and -3.

Thus, $x^2 - 5x + 6 = (x - 2)(x - 3)$

2 **Factorize the expression:** $x^2 - 5x - 6$

Look for two numbers whose product is -6 and whose sum is -5. They are 1 and -6.

Thus, $x^2 - 5x - 6 = (x + 1)(x - 6)$

3 **Factorize the expression:** $3y^2 - 48 + 18y$

1. The expression should be orderd according to the descending exponenets of y.
The expression will be: $3y^2 + 18y - 48$
2. Note that H.C.F for all of the terms then take out H.C.F which is 3.
The expression will be: $3(y^2 + 6y - 16)$
3. Look for two numbers whose product is -16 and whose sum is 6.
They are -2 and 8.

$\therefore$ **The expression = $3(y - 2)(y + 8)$**

4 **Factorize the expression:** $m^4 - 6m^2n + 5n^2$

Solution

1. m^4 is factorized as $m^2 \times m^2$
2. Look for two numbers whose product is $(5n^2)$ and whose sum is $(-6n)$. They are $-n$ and $-5n$.

Thus, the expression = $(m^2 - n)(m^2 - 5n)$

Practice

Factorize each expression of the following:

1. $x^2 + 11x + 10$
2. $x^2 - 7x + 10$
3. $x^2 - 3x - 10$
4. $x^2 - 7x + 12$
5. $x^2 + 4x - 12$
6. $x^2 - x - 12$
7. $x^2 - 8x + 12$
8. $y^2 - 20y + 51$
9. $y^2 - 50y - 51$
10. $5x^2 - 10x - 15$
11. $x^4 - 9x^2 + 20$
12. $x^3 - 3x^2 - 28x$
13. $x^2 - 5xy - 24y^2$
14. $b^2 + 3bc - 10c^2$
15. $-x^2 + 2x + 63$

Seconed: Factorizing Quadratic Trinomials of the form $ax^2 + bx + c$, where $a \neq \pm 1$

You know that:

$(2x - 3)(5x + 4) = 10x^2 + (8x + (-15x)) + (-12)$

- $10x^2$: $2x \times 5x$
- $(8x + (-15x))$: Product of inner terms + product of outer terms
- (-12): -3×4

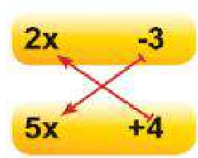
i.e. $(2x - 3)(5x + 4) = 10x^2 - 7x - 12$

Reverse the process to factorize the quadratic trinomial $10x^2 - 7x - 12$ let's do some trials. The opposite diagram will help you to factorize the expression.

Middle Term $= (2x)(4) + (-3)(5x)$

$= -7x$

$\therefore 10x^2 - 7x - 12 = (2x - 3)(5x + 4)$



Exemple (1)

Factorize the expression $3x^2 + 7x - 6$

Solution

Note that $3x^2 = (3x)(\times)$ while -6 is factorized as follows

1×-6, -1×6, 2×-3 or -2×3. Observe the following trials to get a true answer:

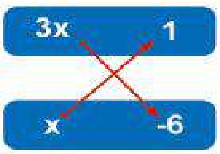


Fig. (1)

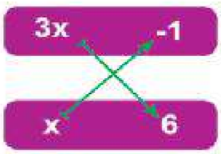


Fig. (2)

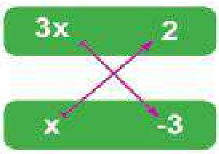


Fig. (3)

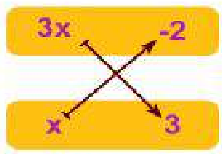


Fig. (4)

In fig. (1): : $3x \times -6 + x \times 1 = -17x \neq$ Middle Term (False).

In fig.(2): : $3x \times 6 + x \times -1 = 17x \neq$ Middle Term (False).

In fig.(3): : $3x \times -3 + x \times 2 = -7x \neq$ Middle Term (False).

In fig. (4): : $3x \times 3 + x \times -2 = 7x =$ Middle Term (True).

$\therefore 3x^2 + 7x - 6 = (3x - 2)(x + 3)$

Exemple (2)

Factorize the expression $15x^4 - 21z^2 - 6x^2z$

Solution

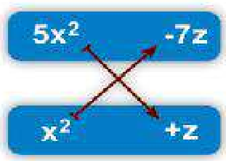
1. The expression after the order is : $15x^4 - 6x^2z - 21z^2$. Note that H.C.F = 3 :

   the expression $= 3(5x^4 - 2x^2z - 7z^2)$

2. $\therefore$ The third term is negative

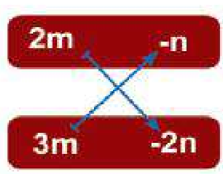
   $\therefore$ The Factors of $-7z^2$ have opposite signs.

   $\therefore$ **The expression** $= 3(5x^2 - 7z)(x^2 + z)$



Exemple (3)

Factorize the expression $6m^2 + n(2n - 7m)$



Solution

The given Expression $= 6m^2 + 2n^2 - 7nm$

$= 6m^2 - 7nm + 2n^2 = (2m - n)(3m - 2n)$

Note that: You can check your answer by testing multiplication visually to get the original expression, before the factorize.

Unit One

Lesson Two

Factorizing the Perfect-Square Trinomials

Think and Discuss

You have learned before:

$(2x - 3)^2 = 4x^2 - 12x + 9$

$(5y + 7x)^2 = 25y^2 + 70xy + 49x^2$

$(L^2 - 5m)^2 = L^4 - 10L^2m + 25m^2$

Trinomials like $4x^2 - 12x + 9$, $25y^2 + 70xy + 49x^2$, $L^4 - 10L^2m + 25m^2$ **are called perfect-squares.**

You will learn

- To factorize perfect-square trinomials

Key-Terms

- a perfect-square.

Note that

1. Each of the first and third terms are perfect squares.
2. The middle term = ± 2 × square root of first term × square root of third term.
   The factorization of a perfect-square trinomial is written in the form:

Perfect-Square Trinomial =
$$\left(\sqrt{\text{First term}} \pm \sqrt{\text{Third term}}\right)^2$$
(± : the sign of the middle term)

Ex. $9x^2 - 30x + 25 = (\sqrt{9x^2} - \sqrt{25})^2 = (3x - 5)^2$

$L^4 + 14L^2m + 49m^2 = (\sqrt{L^4} + \sqrt{49m^2})^2 = (L^2 + 7m)^2$

Note that

1. Take out H.C.F, if existed.
2. To order the terms descendingly according to the exponent of one variable.

Exemple (1)

Determine which of the following trinomial expressions is a perfect-square, then factorize it in the form of a perfect-square:

A) $25x^2 - 30x + 9$ B) $m^2 + 4m - 4$ C) $49a^2 + 70ab^2 + 25b^4$

Solution

A) $25x^2 = (5x)^2$ and $9 = (3)^2$ the first and third terms are perfect-squares The Middle Term $= 2(5x)(3) = 30x$.

$\therefore$ The expression $25x^2 - 30x + 9$ is a perfect square and the expression $= (5x - 3)^2$

B) $m^2 + 4m - 4$ is not a perfect-square, where the third term is negative.

C) The first term $49a^2 = (7a)^2$ is a perfect-square and the third term $= 25b^4 = (5b^2)^2$ is a perfect-square and the middle term $= 2(7a)(5b^2) = 70ab^2 =$ Middle Term.

$\therefore$ The expression $49a^2 + 70ab^2 + 25b^4$ is a perfect-square trinomial and the expression $= (7a + 5b^2)^2$

Exemple (2)

Complete the missing term to make a perfect-square in each of the following expressions then factorize each expression.

A) $4y^2 \pm \ldots\ldots + 121$ B) $25a^2 - 30ab \ldots.$

Solution

A) The middle term $= \pm 2(\sqrt{\text{first term}} \times \sqrt{\text{third term}}) = \pm 2(2y)(11) = \pm 44y$

$\therefore$ The perfect-square trinomial of the expression $= 4y^2 \pm 44y + 121$ and the expression $= (2y \pm 11)^2$

$25a^2 = (5a)^2$

B) The middle term $= -30ab = 2(5a) \times$ square root of the third term

The square root of the third term $= \dfrac{-30ab}{2 \times 5a} = -3b$

The third term $= (-3b)^2 = 9b^2$

$\therefore$ The perfect-square trimnial $= 25a^2 - 30ab + 9b^2$. the expression $= (5a - 3b)^2$

Practice

Complete the missing term in the expression .... + 12 x^2 +36 to make a perfect-square trinomial, then factorize it:

Exemple (3)

Use factorization to evaluate: $(7.3)^2 + 2 \times 7.3 \times 2.7 + (2.7)^2$

Solution

We notice that the numerical expression is in the form of a perfect-square trinomial, so it can be written in the form The expression, $= (7.3 + 2.7)^2 = (10)^2 = 100$

Practice

Use factorization to evaluate: $(574)^2 - 2 \times 574 \times 573 + (573)^2$

Exemple (4)

Factorize each of the following expressions:

A $5x^3 + 50x^2 + 125x$

B $40a^2b - 50a^4 - 8b^2$

Solution

A Take out H.C.F:

$\therefore$ The expression $= 5x(x^2 + 10x + 25) = 5x(x + 5)^2$

B The expression $= 2(20a^2b - 25a^4 - 4b^2)$ descending order according to the exponent of a:

$= -2(25a^4 - 20a^2b + 4b^2)$

$= -2(5a^2 - 2b)^2$

Factorizing theDifference of two Squares

Unit One

Lesson Theree

Think and Discuss

You have learned before:

$$(x + y)(x - y) = x^2 - y^2$$

The algebraic expression $x^2 - y^2$ is called the difference of two squares.

The difference of two square quantities = the sum of the two quantities × the difference of the two quantities.

$$x^2 - y^2 = (x + y)(x - y)$$

You will learn

- To factorize the difference of two squares.

Key-Terms

- Difference of two squares

Exemple (1)

Factorize each of the following expressions:

A $49x^2 - 25$ B $(2y - 3)^2 - 1$

C $27m^3 - 48mn^6$ D $(x + y)^2 - (x - y)^2$

Solution

A $49x^2 - 25 = (7x + 5)(7x - 5)$

B $(2y - 3)^2 - 1 = [(2y - 3) + 1][(2y - 3) - 1]$
$= (2y - 2)(2y - 4)$
$= 2(y - 1) \times 2(y - 2) = 4(y - 1)(y - 2)$

C $27m^3 - 48mn^6 = 3m(9m^2 - 16n^6)$
$= 3m(3m + 4n^3)(3m - 4n^3)$

D $(x + y)^2 - (x - y)^2 = [(x + y) + (x - y)][(x + y) - (x - y)]$
$= 2x \times 2y = 4xy$

Exemples

2 **Use factorization to evaluate the value of each of the following:**

A $(763)^2 - (237)^2$ B $(999)^2 - 1$

Solution

A The algebraic expression = $(763 + 237)(763 - 237) = 1000 \times 526 = 526000$

B The algebraic expression= $(999 + 1)(999 - 1) = 1000 \times 998 = 998000$

3 **Factorize the expression $81x^4 - 16y^4$**

Solution

$81x^4 - 16y^4 = (9x^2 + 4y^2)(9x^2 - 4y^2)$
$= (9x^2 + 4y^2)(3x + 2y)(3x - 2y)$

Factorizing the Sum and the Difference of two Cubes

Unit One
Lesson Four

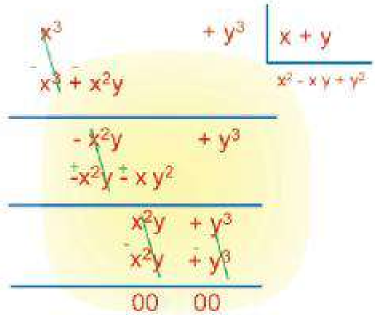
Think and Discuss

You will learn
- To factorize the sum of two cubes.
- To factorize the difference between two cubes.

Key-Terms
- The sum of two cubes.
- The difference between two cubes.

First: factorizing the sum of two cubes

The Teacher asked a student:	Can you factorize the expression : $x^3 + y^3$?
The student answered:	I expect that $(x + y)$ is one of its factors.
The teacher said:	Can you find the other factor in x^3+y^3?
The student answered:	To know the other factor in x^3+y^3 we divide $(x^3 + y^3) \div (x + y)$ using the long division which you have learned previously. The quotient will be $x^2 - x y + y^2$



The algebraic expression $x^3 + y^3$ is called the sum of two cubes **and can be factorized as follows:**

$$x^3 + y^3 = (x + y)(x^2 - x y + y^2)$$

Exemple :

$8x^3 + 27 = (2x)^3 + (3)^3 = (2x + 3)\,[(2x)^2 - 2x \times 3 + (3)^2]$

$\qquad = (2x + 3)(4x^2 - 6x + 9)$

Second: factorizing the difference between two cubes:

The algebraic expression $x^3 - y^3$ is called the difference between two cubes **and can be factorized as follows:**

$x^3 - y^3 = x^3 + (-y)^3$

$\qquad = \big(x + (-y)\big)\,[x^2 - x(-y) + (-y)^2]$

$$\therefore x^3 - y^3 = (x - y)(x^2 + x y + y^2)$$

$125\,a^3 - b^6 = (5a)^3 - (b^2)^3$

$= (5a - b^2)(25a^2 + 5\,ab^2 + b^4)$

Exemples :

1 **Factorize each of the following expressions:**

A $x^3 + 343y^3$ B $40a^3 + 135\,b^3$

C $(x + z)^3 - x^3$ D $x^6 - 64\,y^6$

Solution

A $x^3 + 343\,y^3 = (x)^3 + (7y)^3$

$= (x + 7y)(x^2 - 7\,xy + 49\,y^2)$

B $40\,a^3 + 135\,b^3 = 5\,(8a^3 + 27\,b^3) = 5\,[(2a)^3 + (3\,b)^3]$

$= 5\,(2a + 3b)\,(4\,a^2 - 6\,ab + 9\,b^2)$

C $(x + z)^3 - x^3 = [(x + z) - x][(x + z)^2 + x(x + z) + x^2]$

$= z\,(x^2 + 2\,xz + z^2 + x^2 + xz + x^2)$

$= z\,(3x^2 + 3\,xz + z^2)$

D $x^6 - 64\,y^6$

Note that The algebraic expression $x^6 - 64y^6$ can be factorized as a difference between two cubes and as a difference between two squares as well. You must start factorizing it first as a difference between two square, then factorize the resulting factors.

$x^6 - 64y^6 = (x^3 + 8y^3)(x^3 - 8y^3)$

$= (x + 2y)(x^2 - 2xy + 4\,y^2)(x - 2y)(x^2 + 2xy + 4\,y^2)$

2 Si $x^2 - y^2 = 20$, $x - y = 2$ et $x^2 - xy + y^2 = 28$, **Find the value of $x^3 + y^3$**

Solution

$x^2 - y^2 = 20$ $\therefore (x - y)(x + y) = 20$

$x - y = 2$ $\therefore 2(x + y) = 20$ $\therefore x + y = 10$

$x^3 + y^3 = (x + y)(x^2 - x\,y + y^2)$

$= 10 \times 28 = 280$

Unit One

Lesson Five

Factorizing by Grouping

Think and Discuss

You will learn

- To factorize by grouping.

Key-Terms

- To factorize by grouping.

To factorize an algebraic expression made up of more than three terms like:

$$2ax + ay + 2bx + by$$

We notice that there is no H.C.F and it doesn't have any of the previous forms that, we have learned before. Therefore, we try to make groups having an H.C.F.

The expression $= 2ax + ay + 2bx + by$ divide into two groups

$= a(2x + y) + b(2x + y)$ H.C.F of each group

$= (2x + y)(a + b)$, $(2x+y)$ is an H.C.F of the two groups

Note that:

There is another way to regroup the expression:

The expression $= 2ax + 2bx + ay + by$ commutative property

$= 2x(a + b) + y(a + b)$

$= (a + b)(2x + y)$

Exemple

Factorize each of the following expressions:

A) $x^3 + 2x^2 - x - 2$ B) $16x^2 - a^2 + 6ab - 9b^2$

C) $1 - x^2 - 4xy - 4y^2$

Solution

A) The expression $= x^3 + 2x^2 + (-x - 2)$

$= x^2(x + 2) - (x + 2)$

$= (x + 2)(x^2 - 1)$

$= (x + 2)(x + 1)(x - 1)$

B We notice that there is no relation between the first term and all other terms. Therefore, it can be grouped this way:

$= 16x^2 - (a^2 - 6ab + 9b^2)$

$= 16x^2 - (a - 3b)^2$

$= [4x + (a - 3b)][4x - (a - 3b)] = (4x + a - 3b)(4x - a + 3b)$

C The expression $= (1) - (x^2 + 4xy + 4y^2)$

$= 1 - (x + 2y)^2$

$= (1 - x - 2y)(1 + x + 2y)$

Unit One

Lesson Six

Factorizing by completing the square

Think and Discuss

You will learn

- To factorize by completing the square.

Key-Terms

- Completing the square.

You have learned before:

A perfect square has the form $a^2 \pm 2ab + b^2$ and can be factorized in the form $(a \pm b)^2$.

There are many algebraic expression which are not in the form of a perfect square, but can be completed to have the form of a perfect square.

Exemple 1

Factorize the expression: $x^4 + 4y^4$

Solution

This expression can not be factorized according to what you have learned previously.

To factorize it, the term $2 \times \sqrt{x^4} \times \sqrt{4y^4} = 4x^2y^2$

is needed to have the form of a perfect square.

Thus $= x^4 + 4x^2y^2 + 4y^4 - 4x^2y^2$

$= (x^2 + 2y^2)^2 - 4x^2y^2$

$= [(x^2 + 2y^2) - 2xy][(x^2 + 2y^2) + 2xy]$

$= (x^2 - 2xy + 2y^2)(x^2 + 2xy + 2y^2)$

Exemple 2

Factorize the expression: $9a^4 - 13a^2b^2 + 4b^4$

Solution

The expression $= (3a^2)^2 - 13a^2b^2 + (2b^2)^2$ To be, in the form of a perfect square it should be as follows:

The middle term should be $= \pm 2 \times 3a^2 \times 2b^2 = \pm 12a^2b^2$

The expression $= (3a^2)^2 - 12\,a^2b^2 + (2b^2)^2 - a^2b^2$

$= (3a^2 - 2b^2)^2 - a^2b^2$

$= (3a^2 - 2b^2 - ab)(3\,a^2 - 2b^2 + ab)$

$= (3a^2 - ab - 2\,b^2)(3\,a^2 + ab - 2\,b^2)$

$= (3a + 2b)\,(a - b\,)(3a - 2\,b)\,(a + b)$

Autre solution

The expression $9a^4 - 13a^2b^2 + 4b^4$ can be factorized as a trinomial.

The expression $= (9a^2 - 4b^2)\,(a^2 - b^2)$

$= (3a + 2b)\,(3a - 2b)\,(a + b)\,(a - b)$

Using the commutative property, you get the same answer.

Unit One

Lesson Seven

Solving Quadratic Equations in one Variable

Think and Discuss

You will learn
- To solve a quadratic equation in one variable.

Key-Terms
- a quadratic equation in one variable .
- Real roots of a quadratic equation.
- Solution of a quadratic equation.

You have learned before:

For any real numbers a and b, if ab = 0, then a = 0 or b = 0.

Exemple If (x - 5) (x + 2) = 0 (1)

then: $x - 5 = 0$ or $x + 2 = 0$

$\therefore x = 5$ or $x = -2$

Note that:

1. Each of 5 and -2 is called a root of the equation (1)
2. The solution set is {5, -2}

Exemple 1

Find in R, the solution set of $2x^2 - 5x - 3 = 0$

Solution

By factorizing the left hand side, the equation will be in the following form.

$$(2x + 1)(x - 3) = 0$$

$2x + 1 = 0$ or $x - 3 = 0$

$2x = -1$ or $x - 3 = 0$

$x = \frac{-1}{2}$ or $x = 3$

$\therefore$ **The solution set** $\{\frac{-1}{2}, 3\}$

Note that:

You can check your answer by substituting the value of x in the given equation:

For $x = \frac{-1}{2}$, L.H.S $= 2\left(\frac{-1}{2}\right)^2 - 5\left(\frac{-1}{2}\right) - 3$

$= 2 \times \frac{1}{4} + \frac{5}{2} - 3 = 3 - 3 = 0 =$ R.H.S.

For $x = 3$ L.H.S $= 2(3)^2 - 5(3) - 3$

$= 18 - 15 - 3 = 0 =$ R.H.S.

$\therefore$ **This means each of $\frac{-1}{2}$, and 3 verify the equation**

Exemple 2

Find in R the solution set of $2x^3 = 18x$

Solution

Rewrite the equation in the form

$2x^3 - 18x = 0$, then factorize.

$2x(x^2 - 9) = 0$ or $2x(x - 3)(x + 3) = 0$

$\therefore 2x = 0$ or $x - 3 = 0$ or $x + 3 = 0$

$\therefore x = 0$ or $x = 3$ or $x = -3$

$\therefore$ **S. S. = {0, 3, -3}, check your answer.**

Exemple 3

Find the real number whose double is increased by 1 than its multiplicative inverse.

Solution

Let the number be = x $(x \neq 0)$

The double of the number $= 2x$

The multiplicative inverse $= \frac{1}{x}$

$\because$ The double of the number is increased by 1 than its multiplicative inverse

$\therefore 2x - \frac{1}{x} = 1$

Multiply the both sides of the equation by x

$2x^2 - 1 = x$

$2x^2 - x - 1 = 0$

$$(2x + 1)(x - 1) = 0$$

$2x + 1 = 0$ or $x - 1 = 0$

$2x = -1$

$x = \frac{-1}{2}$ or $x = 1$

Verification:

The double of the number = -1

The multiplicative inverse = -2

Verification:

The double of the number = 2

The multiplicative inverse = 1

In both cases, it is clear that the double of the number is 1 more than the multiplicative inverse.

Exemple 4

Find the dimensions of a rectangle whose length is 4cm more than its width and whose area is 21cm².

Solution

Let the width of the rectangle = x cm

∴ The length of the rectangle = (x + 4) cm

$\therefore x(x + 4) = 21$

$\therefore x^2 + 4x - 21 = 0$

$\therefore (x - 3)(x + 7) = 0$

$x - 3 = 0$ or $x + 7 = 0$

$x = 3$ or $x = -7$ (refused because it is a negative number)

∴ The width of the rectangle = 3cm, and the length of the rectangle = 3 + 4 = 7cm

Verification: area of the rectangle = $3 \times 7 = 21cm^2$

Algebra

Unit 1: Non-Negative and negative Integer Powers in R and the Operations on them

lesson 1 : Non- Negative and negative integer powers in R.

lesson 2 : Rules of non-negative integer powers in R.

lesson 3 : Rules for negative integer powers in R.

lesson 4 : Operations on integer powers.

Unit TWO

Lesson One

Non - Negative and negative Integer Powers in R

Think and Discuss

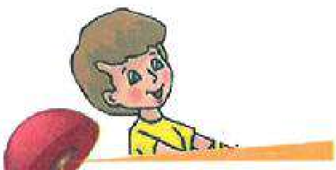
First: Non-negative intger powers:

You have previously learned the integer powers in the set of rational numbers Q :

Complete :

(1) $\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = (\text{..........})^{...}$ (2) $\frac{-2}{3} \times \frac{-2}{3} \times \frac{-2}{3} \times \frac{-2}{3} = (\text{..........})^{...}$

if $a \in R, n \in Z^+$ then $a^n = a \times a \times a \times \text{..........} \times a$
where a is repreated as a factor n times



What you'll learn

- ☆ Non-negative and negative integer powers

Example

(1) $\sqrt{2} \times \sqrt{2} \times \sqrt{2} \times \sqrt{2} \times \sqrt{2} = (\sqrt{2})^5 = 4\sqrt{2}.$

(2) $-\sqrt{2} \times -\sqrt{2} \times -\sqrt{2} \times -\sqrt{2} = (-\sqrt{2})^4 = 4$

(3) $-\sqrt{5} \times -\sqrt{5} \times -\sqrt{5} = (-\sqrt{5})^3 = -5\sqrt{5}$

If $a \in R^*$ then $a^{zero} = 1$

for example: $(\sqrt{7})^{zero} = 1$, $(\frac{-1}{\sqrt{11}})^{zero} = 1$

Key terms

- ☆ R^* the set of real numbers except zero
- ☆ Non-negative integer powers in R
- ☆ Negative intger powers in R
- ☆ Exponential equation in R

Second: Negative intgerr powers

Think and Discuss

You know that $5^3 \times 5^{-3} = 5^{3-3} = 5^0 = 1$

Complete:

$x^m \times \text{..........} = 1$ where $x \neq 0$, $m \neq 0$

If $a \in R^*$, $n \in Z^+$

then $a^{-n} = \frac{1}{a^n}$, $a^n = \frac{1}{a^{-n}}$

for example: $(\sqrt{3})^{-4} = \frac{1}{(\sqrt{3})^4} = \frac{1}{9}$, $\frac{1}{(-\sqrt{3})^{-3}} = (-\sqrt{3})^3 = -3\sqrt{3}$

Drill

If $x = 3$, $y = \sqrt{2}$, then find each of the following in the simplest form:

1. $x^{-2} y^{-4}$
2. $(x^{-2} \times y^4)^{-2}$
3. $\left(\frac{x}{y}\right)^{-3}$

Example

1. If $x = \frac{\sqrt{3}}{2}$, $y = \frac{1}{\sqrt{3}}$ and $z = \frac{\sqrt{2}}{2}$. **then find the value of:** $x^2 + (x\,z)^2 \times y^2$

Solution

The expression $= x^2 + x^2 z^2 y^2 = x^2 (1 + z^2 y^2)$

$$= \left(\frac{\sqrt{3}}{2}\right)^2 \times [1 + \left(\frac{\sqrt{2}}{2}\right)^2 \times \left(\frac{1}{\sqrt{3}}\right)^2] = \frac{3}{4} \times [1 + \frac{2}{4} \times \frac{1}{3}] = \frac{3}{4} \times \frac{7}{6} = \frac{7}{8}$$

Important Rule :

If $a^m = a^n$ then $m = n$ *where* $a \in R - \{0, 1, -1\}$

For example: If $(\sqrt{2})^x = 2\sqrt{2}$ then $(\sqrt{2})^x = (\sqrt{2})^3$ $\therefore x = 3$

If $a^n = b^n$ then $a = b$ *where* $n \in \{1, 3, 5, ...\}$

, $|a| = |b|$ *where* $n \in \{2, 4, 6, ...\}$

For example: $x^5 = \frac{1}{32}$ then $x^5 = \left(\frac{1}{2}\right)^5$ $\therefore x = \frac{1}{2}$

2. **Find the solution set for each of the following equations in R :**

A. $\left(\frac{3}{5}\right)^{x+2} = \frac{125}{27}$

B. $(3)^{x-3} = (\sqrt{3})^{x+5}$

Solution

A $(\frac{3}{5})^{x+2} = \frac{125}{27}$ $\quad \therefore (\frac{3}{5})^{x+2} = (\frac{5}{3})^3$ $\quad \therefore (\frac{3}{5})^{x+2} = (\frac{3}{5})^{-3}$

$\therefore x + 2 = -3$ $\quad \therefore x = -2 - 3$ $\quad \therefore x = -5$

The Solution Set is {-5}

B $\because [(\sqrt{3})^2]^{(x-3)} = (\sqrt{3})^{(x+5)}$ $\quad \therefore (\sqrt{3})^{2x-6} = (\sqrt{3})^{x+5}$

$\therefore 2x - 6 = x + 5$ $\quad \therefore x = 11$ $\quad$ The Solution Set is {11}

Drill *Mental math*

Solve by inspection $\frac{1}{(x+9)^4} = 0.0001$ $\quad$ What do you notice?

Unit TWO

Lesson TWO

Rules of non - negative integer powers in R

Think and Discuss

First:

Complete: $(\sqrt{3})^2 \times (\sqrt{3})^4 = (\ldots\ldots)^{\ldots\ldots}$ **What do you notice?**

> If $a \in R^*$, m, n *are two non-negative integer numbers,*
> then: $a^m \times a^n = a^{m+n}$

Generalization:

If $a \in R^*$, m, n, ..., ℓ are non-negative integer numbers

then: $a^m \times a^n \times \ldots\ldots \times a^\ell = a^{m+n+\ldots\ldots+\ell}$

From the previous rule, we find that: $(\sqrt{3})^2 \times (\sqrt{3})^4 = (\sqrt{3})^{2+4} = (\sqrt{3})^6 = 27$

Second

Complete : $(\sqrt{5})^7 \div (\sqrt{5})^3 = (\ldots\ldots)^{\ldots\ldots}$ **What do you notice?**

> If $a \in R^*$ *, and m, n are two non-negative integer numbers*
> $m \geqslant n$ then $a^m \div a^n = a^{m-n}$

From the previous rule we find that: $(\sqrt{5})^7 \div (\sqrt{5})^3 = (\sqrt{5})^{7-3} = (\sqrt{5})^4 = 25$

Third :

Complete: $(\sqrt{2} \times \sqrt{3})^2 = (\sqrt{2})^{\ldots\ldots} \times (\ldots\ldots)^{\ldots\ldots} = \ldots\ldots \times \ldots\ldots = \ldots\ldots$

> If a and b $\in R^*$, n *is a non-negative integer numbers*
> then: $(ab)^n = a^n \times b^n$

Generalization:

If a, b, c, ..., $k \in R^*$, n is a non-negative integer number then:

$(a \times b \times c \times \ldots \times k)^n = a^n \times b^n \times c^n \times \ldots \times k^n$

What you'll learn

- ☆ Rules of non-negative integer in R
- ☆ Solving problems on non-negative integer powers in R.

Key terms

- ☆ Non-negative integer powers.
- ☆ Set of real numbers.

Fourth:

Complete : $\left(\frac{\sqrt{3}}{\sqrt{5}}\right)^4 = \frac{(\ldots\ldots\ldots)^{\ldots\ldots}}{(\ldots\ldots\ldots)^{\ldots\ldots}} = \frac{\ldots\ldots\ldots}{\ldots\ldots\ldots}$

> If a, b $\in$ R*, then $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$, n *is a non-negative integer*
> *where* $b \neq 0$, $a \neq 0$

Generalization : If a, b, c, ... , k $\in$ R and n is a non-negative integer then:

$\left(\frac{a\,b\,e \times \ldots\ldots \times \ell}{c\,d\,f \times \ldots\ldots \times k}\right)^n = \frac{a^n b^n e^n \ldots\ldots \ell^n}{c^n d^n f^n \ldots\ldots k^n}$ where any of the factors of the denominator $\neq$ zero

Fifth:

Complete : $(2^2)^3 = (\ldots\ldots\ldots)^{\ldots\ldots} \times (\ldots\ldots\ldots)^{\ldots\ldots} \times (\ldots\ldots\ldots)^{\ldots\ldots} = (\ldots\ldots\ldots)^{\ldots\ldots}$. **What do you notice?**

> If *a, b* $\in$ R*, m, n are *two non-negative integers* then $(a^m)^n = a^{mn}$

Generalization: If a, b and c, ... , k $\in$ R and n is a non-negative integer, then :

$\left(\frac{b^m\, e^\ell \ldots\ldots}{f^k\, d^x \ldots\ldots}\right)^n = \frac{b^{nm}\, e^{n\ell} \ldots\ldots}{f^{nk}\, d^{nx} \ldots\ldots}$ Where any of the factors of the denominator $\neq 0$

Example

Simplify each of the following to the simplest form:

1. $\sqrt{2} \times (\sqrt{2})^2 \times (\sqrt{2})^3$
2. $\left((\sqrt{2})^3 \times (-\sqrt{2})^2\right)^2$
3. $\frac{(\sqrt{3})^5 \times (\sqrt{3})^3}{(\sqrt{3})^4}$

Solution

1. $\sqrt{2} \times (\sqrt{2})^2 \times (\sqrt{2})^3 = (\sqrt{2})^{1+2+3} = (\sqrt{2})^6 = 8$
2. $\left((\sqrt{2})^3 \times (-\sqrt{2})^2\right)^2 = (\sqrt{2})^{3\times2} \times (-\sqrt{2})^{2\times2} = (\sqrt{2})^6 \times (-\sqrt{2})^4 = 8 \times 4 = 32$
3. $\frac{(\sqrt{3})^5 \times (\sqrt{3})^3}{(\sqrt{3})^4} = (\sqrt{3})^{5+3-4} = (\sqrt{3})^4 = 9$

Unit TWO Lesson Therr

Rules for negative integer powers in R

Think and Discuss

What you'll learn

- Generalization of the laws for non-negative and negative powers in R.

Key Terms

- Negative integer powers
- Set of real number R.

Generlization the laws of exponents

If a, and b $\in$ R*, m, and n $\in$ Z then:

- $a^m \times a^n = a^{m+n}$
- $a^m \div a^n = a^{m-n}$
- $(a\,b)^n = a^n \times b^n$
- $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$
- $(a^m)^n = a^{m\,n}$

Remarks:

1. If $a \in R^*$, $n \in Z^+$ then a^n, a^{-n} each is a multiplicative inverse for the other, then , $a^n \times a^{-n} = 1$ *for example:* $(\sqrt{3})^5 \times (\sqrt{3})^{-5} = 1$

2. If a, b $\in$ R*, $n \in Z^+$ then $\left(\frac{a}{b}\right)^n = \left(\frac{b}{a}\right)^{-n}$

Example: $\left(\frac{\sqrt{2}}{\sqrt{3}}\right)^5 = \left(\frac{\sqrt{3}}{\sqrt{2}}\right)^{-5}$,where: $\left(\frac{\sqrt{2}}{\sqrt{3}}\right)^5 \times \left(\frac{\sqrt{2}}{\sqrt{3}}\right)^{-5} = 1$

Example

1. **Find in the simplest form :**

A. $5\,(\sqrt{5}\,)^{-1}$ B. $\left(\frac{\sqrt{3}}{\sqrt{2}}\right)^{-4}$ C. $\frac{2^{-1} \times 4}{3^{-1}}$

Solution

A. $5\,(\sqrt{5}\,)^{-1} = \frac{5}{\sqrt{5}} = \frac{5}{5\sqrt{}} \times \frac{\sqrt{5}}{\sqrt{5}} = \frac{5\sqrt{5}}{5} = \sqrt{5}$

B. $\left(\frac{\sqrt{3}}{\sqrt{2}}\right)^{-4} = \left(\frac{\sqrt{2}}{\sqrt{3}}\right)^{+4} = \frac{4}{9}$

C. $\frac{2^{-1} \times 4}{3^{-1}} = \frac{3 \times 4}{2} = 6$

2 Find in the simplest form $\dfrac{(15)^{-2} \times (\sqrt{5})^3 \times (3)^3}{9 \times (\sqrt{5})^{-3}}$

Solution

Expression $= \dfrac{(3)^{-2} \times (5)^{-2} \times (\sqrt{5})^3 \times (3)^3}{(3)^2 \times (\sqrt{5})^{-3}} = (3)^{-2+3-2} \times (5)^{-2} \times (\sqrt{5})^{3+3}$

$= (3)^{-1} \times (5)^{-2} \times (\sqrt{5})^6 \quad = \dfrac{1}{3} \times (5)^{-2} \times (5)^3 = \dfrac{1}{3} \times (5)^1 = \dfrac{5}{3}$

3 If $\dfrac{49^n \times 25^{2n} \times 3^{4n}}{7^{-n} \times 15^{4n}} = 343,$ Then calculate the value of 6^{2n}

Solution

$\because \dfrac{49^n \times 25^{2n} \times 3^{4n}}{7^{-n} \times 15^{4n}} = 343$ $\quad \therefore \dfrac{7^{2n} \times 5^{4n} \times 3^{4n}}{7^{-n} \times 5^{4n} \times 3^{4n}} = 343$

$\therefore 7^{2n+n} = 343$ $\quad \therefore 7^{3n} = 7^3$

$\therefore 3n = 3$ $\quad \therefore n = 1$ $\quad \therefore 6^{2n} = 6^{2\times1} = 36$

Unit TWO
Lesson Four

Operations on Integer Powers in R

Think and Discuss

First: Find each of the following in the simplest form:

1. $\frac{1}{(\sqrt{3})^5} \div 9\sqrt{3}$ $\frac{3\sqrt{2}}{\sqrt{3}} - \frac{(\sqrt{3})^3}{2\sqrt{2}}$

We have previously learned that :

$\frac{a}{b} \times \frac{c}{d} = \frac{ac}{bd}$ *(where* a *and* b, d $\neq 0$*)*

$\frac{a}{b} \div \frac{c}{d} = \frac{ad}{bc}$ *(where* b *and* c, d $\neq 0$*)*

$\frac{a}{b} + \frac{c}{d} = \frac{ad+bc}{bd}$ *(where* b, d $\neq 0$*)*

$\frac{a}{b} - \frac{c}{d} = \frac{ad-bc}{bd}$ *(where* b, d $\neq 0$*)*

Second: Use mental math to find: $3 \times 2^2 - 6 \div 3 \times 5 + 4$.

Check your answer using the calculator to do the operation above.

What you'll learn

- Do operation (+ , - , ×, ÷) on integer powers.

Key terms

- Non-negative integer powers.
- Negative integer powers Ordering operation.

Ordering operations :

1. Do the operations in the interior parenthesis, then the exterior parenthesis if found.
2. Calculate the powers of numbers.
3. Do multiplication or division from left to right.
4. Do addition or subtraction from left to right.

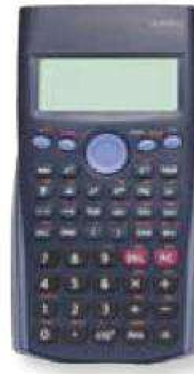
This order is followed in calculators.

Example

1 Find the result of each of the following in the simplest form:

A $2^{-3} \times 3^{-2} \div 6^{-4}$

B $(\sqrt{5})^5 \div 5\sqrt{5} + 2\sqrt{3} \times \sqrt{3}$



Solution

A $2^{-3} \times 3^{-2} \div 6^{-4} = 2^{-3} \times 3^{-2} \times 6^{4}$

$= 2^{-3} \times 3^{-2} \times 2^{4} \times 3^{4} = 2^{-3+4} \times 3^{-2+4}$

$= 2^{1} \times 3^{2} = 2 \times 9 = 18$

Calculators are used to check the previous operations as follows :

Start → 2 [$x^{\blacksquare}$] [(−)] 3 [×] 3 ▶ [$x^{\blacksquare}$] [(−)] 2 ▶ [÷] 6 [$x^{\blacksquare}$] [(−)] 4 ▶ [=]

B $(\sqrt{5})^5 \div 5\sqrt{5} + 2\sqrt{3} \times \sqrt{3} = (\sqrt{5})^5 \div (\sqrt{5})^3 + 2(\sqrt{3})^2$

$= (\sqrt{5})^{5-3} + 2 \times 3 = (\sqrt{5})^2 + 6 = 5 + 6 = 11$

2 If $\frac{3^x \times 8^x}{12^{x+1}} = \frac{1}{3}$ **Find the value of x**

Solution

$$\frac{3^x \times 2^{3x}}{(2^2 \times 3)^{x+1}} = \frac{1}{3}$$

$$\frac{3^x \times 2^{3x}}{3^{x+1} \times 2^{2x+2}} = \frac{1}{3}$$

$$3^{x-x-1} \times 2^{3x-2x-2} = \frac{1}{3}$$

$$3^{-1} \times 2^{x-2} = \frac{1}{3}$$

$$\frac{1}{3} \times 2^{x-2} = \frac{1}{3}$$

$$2^{x-2} = 1$$

$$2^{x-2} = 2^0 \rightarrow x - 2 = 0 \rightarrow x = 2$$

3 If $a = \sqrt{2}$, $b = \sqrt{3}$. Find the numerical value of:

A $\frac{b^4 - a^4}{b^2 + a^2}$ B $\frac{a^3 + b^3}{a + b}$

Solution

A $\frac{b^4 - a^4}{b^2 + a^2} = \frac{(b^2 + a^2)(b^2 - a^2)}{b^2 + a^2}$

$= b^2 - a^2 = (\sqrt{3})^2 - (\sqrt{2})^2 = 3 - 2 = 1$

B $\frac{a^3 + b^3}{a + b} = \frac{(a + b)(a^2 - ab + b^2)}{a + b} = a^2 - ab + b^2$ $(a \neq - b)$

$= (\sqrt{2})^2 - \sqrt{2} \times \sqrt{3} + (\sqrt{3})^2 = 2 - \sqrt{6} + 3 = 5 - \sqrt{6}$

Drill

1 If $\frac{6^{2n} \times 2^{2n}}{4^{2n} \times 3^{2n+4}} = 9^{-x}$ Find the value of x

2 Connecting with commercial business.

If $c = m(1 + r)^n$ where (c) is the total sum m in pounds, (r) is the profit per pound yearly and n is the number of years, then calculate (c) to the nearest pound If $m = 2.5 \times 10^4$, $r = 9.8 \times 10^{-2}$, $n = 12$.

UNIT THREE

3

Probability

Unit THREE

Lesson One

Probability

Think and Discuss

You will learn

- The meaning of inferential statistics.
- The concept of a sample.
- Random experiment.
- Sample space.
- Event.
- The concept of probability.
- Prediction.

Key-Terms

- Sample.
- Random experiment.
- Sample space.
- Event.
- Probability.
- Prediction.

You have learned before some statistical ways and procedures used in collecting and organizing data, to display data in tables and graphs and to use frequency tables or grouped frequency tables (ascending and descending). You have learned also to organize data sets by using bar graphs, line graph, histograms. frequency tables......

You have learned also how to express data in brief forms by finding mean, median, and mode used to estimate and make decisions.

Statistical inference:

Let's think

A feasibility study is always needed before starting build a factory or any investment project.

The quality assurance of production of a factory shows that 2% of the production of a certain machine is defective. What is meant by this?

A feasibility study is considered predictional way about the success of the project and achieving its objectives.

So It is necessary to start first with formulating hypotheses about the location of the project, operating supplies, employments and marketing procedures, then testing and checking these hypotheses to make a decision to start the project.

2% of the production of machines is a defective production this does not mean that each produced 100 units, you will find 2 units out of order.

Therefore, the percentage 2% means the averange of defective units when examining a large number of production sets, each set consists of 100 units. There fore the probability of producing a defective unit is 0.02.

Therefore:

Statistical inference depends on the process of producing accurate statistics and requires careful planning and selecting a representative sample of the population.

Probability is used to support conclusions made from results of a survey in many samples.

Think

What are samples and types of samples? how can a random sample be chosen? How can a regular sample be chosen? Why are samples used?

The concept of a sample

A sample is any part of a population. To obtain information about a large group, or population, smaller parts or samples are studied. A sampling method is a procedure for selecting a sample to represent the popoulation and to provide a reasonable representation of a population situations.

Probabilities are used in making decisions from a set of avilable decisions concerning with studying of a certain phenomenan in case of uncertainty or encountring the imperfect data .

Probability:

You have learned before the theoritical and experimental probability. Experimental probability depends on experiments and results of a survey.

Anyway, the probability of an event is described by the ratio:

$$\text{The probability of an event} = \frac{\text{number of outcomes in the event}}{\text{number of all possible outcomes in the sample space}}$$

As the number of trials in an experiment increases, the approximation of the experimental Probability improves and becomes closer to the theoritical probability.

Therefore, The expected number of outcomes in an event = the probability of its occurrence X number of all possible outcomes.

Theoritical probability is based on the assumption that all outcomes in the sample space occur randomly, which means all possible outcomes are equally likely. For example:

1. Tossing a regular coin. There are 2 possible ways the coin can land; heads (H) or tails (T). Each way has the same chance of happening. The chances of heads and tails are equally likely.
2. Rolling a regular die and observing the number on the upper face. Each number has the same chance of occurring. The chances of all numbers are equally likely.
3. Drawing a colored marble from a bag containing similar colored marbles with the same volume and the same number of each color. The chances of all outcomes are equally likely .
4. Drawing a card from a set of similar cards and recording what is written on it ....etc.

A random experiment is an experiment, where its all possible outcomes are known before simulating it but we can't determine the actual outcome.

Sample spaces is the set of all possible outcomes of a random experiment. The number of its elements is denoted by n (s)

An event is a subset of the sample space. If A is an event in S, then $A \subset S$, and the number of elements in A is denoted by n(A) and it is the number of outcomes in the event A.

Then: probability of occuring an event $A \subset S$, is denoted by P(A), where:

$$P(A) = \frac{\text{number of outcomes in the event A}}{\text{number of all possible outcomes in the sample space}} = \frac{n(A)}{n(S)}$$

$\because n(A) \leqslant n(S)$ $\quad\therefore \frac{n(A)}{n(S)} \leqslant 1$

$\because n(A) \in N, n(S) \in Z^+$ $\quad\therefore \frac{n(A)}{n(S)} \geqslant 0$

$\therefore 0 \leqslant \frac{n(A)}{n(S)} \leqslant 1$ $\quad$ i. e $0 \leqslant p(A) \leqslant 1$

Exemple (1) :

A numbered card is selected randomly from a set of similar cards numbered from 1 to 24 Find the probability of getting a card carries:

- **A** a multiple of 4
- **B** a multiple of 6
- **C** a multiple of 4 and 6 together.
- **D** a multiple of 4 or 6
- **E** a number divisible by 25
- **F** a positive integer less than 25

1	2	3	4
5	6	7	8
9	10	11	12
13	14	15	16
17	18	19	20
21	22	23	24

Solution

Sample Spaces = { 1, 2, 3, ..., 24}

n (S) = 24

A Let A be the event of getting a multiple of 4

$\therefore$ A = {4 , 8 , 12 , 16 , 20 , 24}

n(A) = 6

$P(A) = \frac{n(A)}{n(S)} = \frac{6}{24} = \frac{1}{4}$

B Let B be the event of getting a multiple of 6

B = {6 , 12 , 18 , 24} , n(B) = 4

$P(B) = \frac{n(B)}{n(S)} = \frac{4}{24} = \frac{1}{6}$

C Let C be the event of getting a multiple of 4 and 6 together.

C = { 12 , 24} , n (C) = 2

$P(C) = \frac{n(C)}{n(S)} = \frac{2}{24} = \frac{1}{12}$

D Let D be the event of getting a multiple of 6 or a multiple of 4 ».

D = {4 , 8 , 12 , 16 , 20 , 24 , 6 , 18}

n (D) = 8

$P(D) = \frac{n(D)}{n(S)} = \frac{8}{24} = \frac{1}{3}$

E Let E be the event of getting a number divisible by 25. This event is impossible. Why?.

E = ∅, n (s) = 0

$\therefore$ P (E) = zéro

F Let X be the event of getting a positive integer less than 25. This event is certain. Why?

X = {1 , 2 , 3 , ... , 24}

$\therefore$ n (X) = 24 = n (S)

$P(X) = \frac{n(X)}{n(S)} = \frac{n(S)}{n(S)} = 1$

Refer to the given example:

1. Impossible event (∅): an event can not be occured.
   The probabity of an impossible event = zero
2. Certain event (S): an event whose outcomes are all possible outcomes
   The probability of a certain event = 1

As illustrated in the opposite figure, where P(A) $\in$ [0, 1]

It is possible to write the probability as a fraction, decimal or percentage.

Practice

1 Selecting randomly a card out of 40 similar cards in a box numbered from1 to 40. **Find the probability** of getting a card carries:

- **A** an even number.
- **B** a number is divisible by 3.
- **C** a number is not divisible by 10.
- **D** an even number is divisible by 3.
- **E** a prime number is less than 20.

2 Drawing randomly a colored marble out of a box containing 12 red marbles, 18 white marbles and 20 blue marbles.

Find the probability of selecting:

- **A** a white marble.
- **B** a red marble.
- **C** a yellow marble.
- **D** a non - red marble.
- **E** a red or blue marble.

Exemple (2)

In a survey of favorite weight of a package of wash powder. The manufacturin company asked a group of 300 Ladies using this product. The following table lists th results:

Weight (in gm)	125	250	375	500	Sum
Number of ladies	120	45	96	39	300

I: Selecting randomly a lady, what is the probability to choose:

- **A** 125 gm
- **B** 250 gm
- **C** 375 gm
- **D** 500 gm

II: What is your advice to the manager of this company according to the resul of this survey?

Solution

First:

A The probability of choosing 125 gm $= \frac{120}{300} = \frac{40}{100} = \frac{2}{5} = 0.4 = 40\%$

B The probability of choosing 250 gm $= \frac{45}{300} = \frac{15}{100} = \frac{3}{20} = 0.15 = 15\%$

C The probability of choosing 375 gm $= \frac{96}{300} = \frac{32}{100} = \frac{8}{25} = 0.32 = 32\%$

D The probability of choosing 500 gm $= \frac{39}{300} = \frac{13}{100} = 0.13 = 13\%$

Note that:

1 It is possible to write the probability in the form of a fraction, a decimal or a percent. For instance, if the probability $= \frac{3}{20}$ then the probability $= \frac{3}{20} \times (100)\,\% = 15\%$

Second: Write down your advices to the manager of the company, discuss it with your classmates and keep the report in your portfolio file.

Practice

The following table shows the results of a survey of favorite transportation means to go to school.

Transportation means	Bus	Private car	Bicycle	on foot
Number of students	3	12	24	66

Selecting randomly a student. Find the probability in percent of choosing:

A a bus user

B a bicycle user.

C a private car user.

D on foot walker.

Exemple (3)

A life insurance company has found in a sample of 10000 men, between 40 and 50 years old, 67 are dead in one year.

A What is the probability of a man to die between 40 and 50 years old in one year?

B Why are these results important for life insurance companies?

C. If the company signed life-insurance contract with 50000 men between 40 and 50 years old, then how many death-benefits should be paid in one year.

Solution

A. Death probability = $\frac{67}{10000}$ = 0.0067

B. Life-insurance companies are interested in experimental probability to find the insurance- rate (instalment).

C. The estimated number of death-cases in one year = death probability × number of insured persons = 50000 × 0.0067 = 335

Practice

In producing 300 electric lamps, 18 units found defective.

A. What is the probability of a unit to be a defective unit?

B. What is the probability of a functional unit?

C. Is it possible for a unit to be a functional unit and out of order unit in the same time?

D. Find the sum of the probability of a defective unit and the probability of a functional unit. What do you observe?

E. If a daily production of this factory was 1600 electric lamps. Find the number of the functional units in that day.

UNIT FOUR

4

Areas

Unit FOUR

Lesson One

Equality of The Areas of Two Parallelograms

Think and Discuss

You will learn

- Relation between areas of two parallelograms.
- Relation between area of a parallelogram and area of a rectangle.
- To calculate area of a parallelogram.
- Relations between a parallelogram and a triangle with common base and drawn between two parallel lines
- To calculate the area of a triangle.

Key-Terms

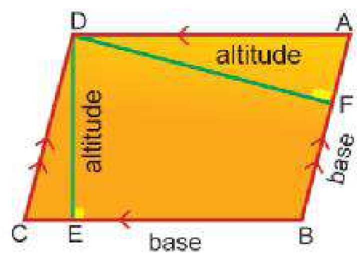
- Area .
- Parallelogram.
- Rectangle .
- Triangle .
- Base.
- Altitude .
- Two Parallel lines.

Use what you have learned before to find answers of the following questions:

- What is the definition of a parallelogram?
- What are the properties of a parallelogram?
- Is the distance between two parallel lines constant? Explain and give examples of real-life situations.
- Are rectangles, rhombuses and squares special cases of parallelograms? Why?

The altitude of a parallelogram:

In the opposite figure: A B C D is a parallelogram. If we consider $\overline{BC}$ as a base and if $\overline{DE} \perp \overline{BC}$, then the length of $\overline{DE}$ **is the corresponding altitude of the base** $\overline{BC}$.

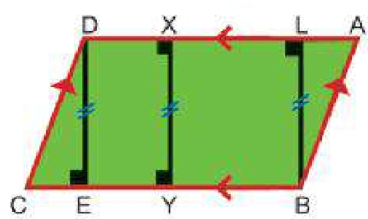


If we consider $\overline{AB}$ as a base of the parallelogram and if $\overline{DF} \perp \overline{AB}$

then the length of $\overline{DF}$ **is the corresponding altitude of the base** $\overline{AB}$.

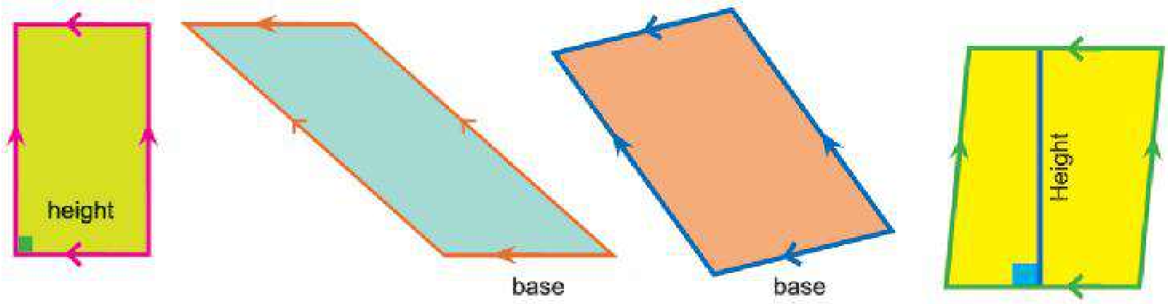
Note that: The altitude of the parallelogram corresponding to the base $\overline{BC}$ is congruent to $\overline{DE}$ where:

D E = x y = B L why?



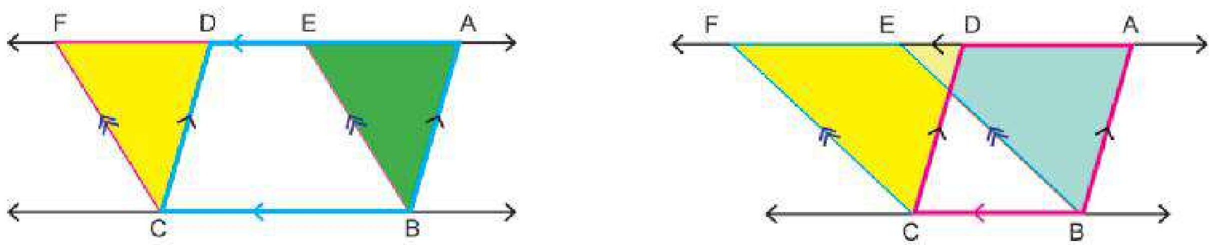
Practice

Determine a base and the corresponding altitude in each case of the following parallelograms:



Theorem 1

Surfaces of two parallelograms with common base and between two parallel straight lines, one carrying the base, are equal in area.



Given: A B C D and E B C F are two parallelograms with a common base $\overline{BC}$ and $\overleftrightarrow{BC} \parallel \overleftrightarrow{AF}$

R.T.P.: **area** ▱ A B C D = area ▱ E B C F

Proof: ∵ △ D C F is the image of △ A B E · Translation of magnitude BC in the direction of $\overrightarrow{BC}$

∴ △ D C F ≡ A B E (Translation is isometry)

∴ area of figure A B C F - area of △ D C F =

area of figure A B C F - area of △ A B E

∴ area of ▱ A B C D = area of ▱ E B C F

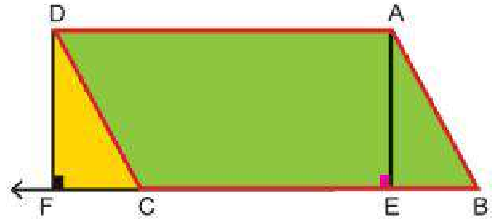
(Q.E.D.)

Let's think

In the opposite figure:

▱ A B C D , $\overline{AE} \perp \overline{BC}$

If $\overline{DF} \perp \overrightarrow{BC}$, then △ DCF is the image of △ A B E by translation with magnitude......... in the direction of ....



What is the relation between area ▱ A B C D **and area of rectangle** AEFD?

Corollaries

Corollary 1

Parallelogram and rectangle with common base and between two parallel straight lines are equal in area.

Note that:

area of rectangle = length × Width

area of rectangle A E F D = E F × A E = B C × A E why?

Thus, area of ▱ A B C D = B C × A E

Corollary 2

Area of the Parallelogram = length of the base × Corres ponding height

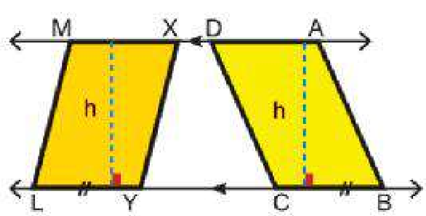
Note that:

The distance between two parallel lines is always constant.

If B C = YL, then

area ▱ A B C D = B C × ......

area ▱ X Y L M = Y L × ....



What can you conclude?

Corollary 3

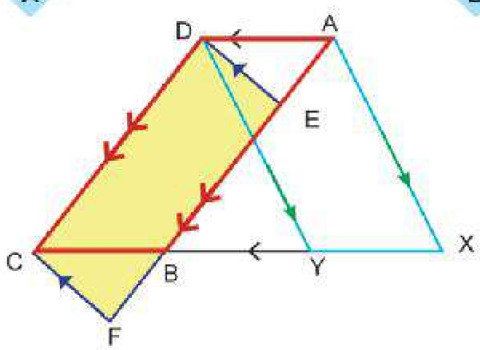
Parallelograms with bases equal in length and lying on a straight line, while the opposite sides to these bases are on another straight line are equal in area.

Practice

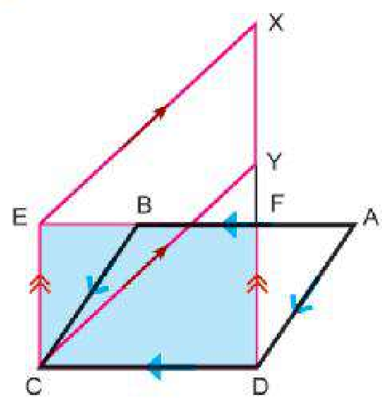
1 In the following figures:

Show that all the three parallelograms have equal areas.

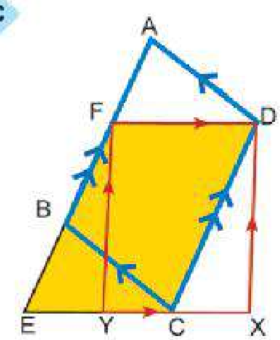
A



B

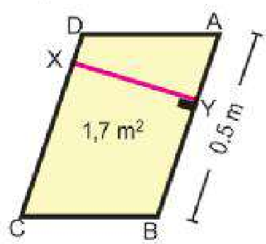


C

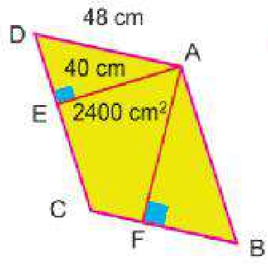


2 Complete

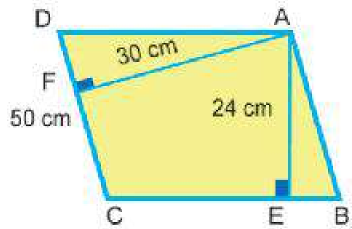
A



B



C



XY = ...

D C = ...

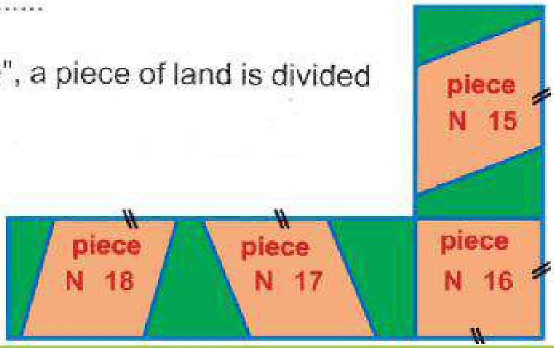
A F = ...

B C = ...

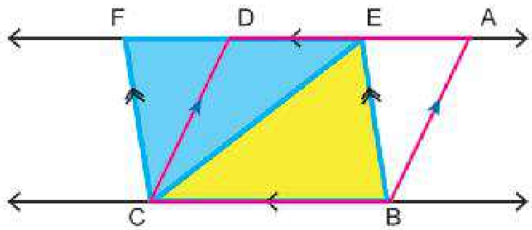
3 In the national project "Build up your home", a piece of land is divided as illustrated in the opposite figure:

Is the area of piece number 15 = the area of the pieces number 16?

State the number of piece of equal areas. Explain your answer.



Let's think



In the opposite figure: $\overleftrightarrow{BC} \parallel \overleftrightarrow{AF}$,

A B C D, and E B C F are two Parallelograms

$\overline{EC}$ is a diagonal in Parallelogram E B C F

∴ **area** △ E B C = ... area of Parallelogram E B C F

∴ **area** Parallelogram E B C F = area ....

∴ **area** △ E B C = ... area of Parallelogram A B C D

Corollary 4

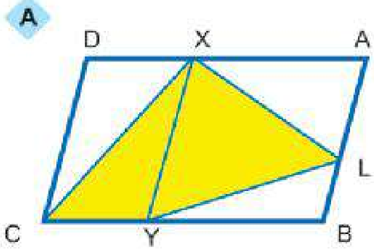
Area of a triangle is equal to half of area of a parallelogram if they have a common base lying on one of two parallel straight lines including them

Practice

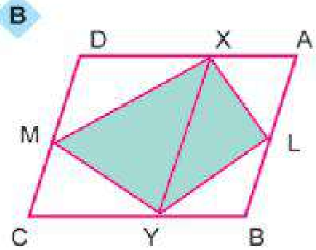
In the following figures $\overline{XY} \parallel \overleftrightarrow{AB}$ **:**

Show that the shaded area is equal to the half of the area of Parallelogram A B C D

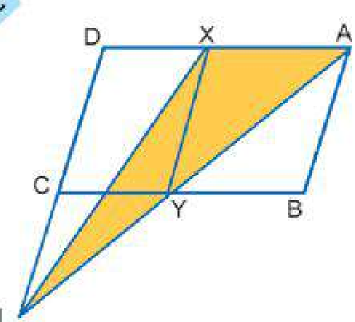
A



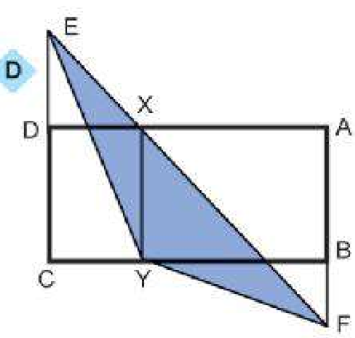
B



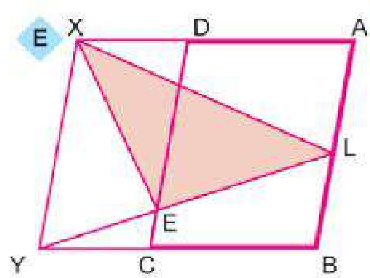
C



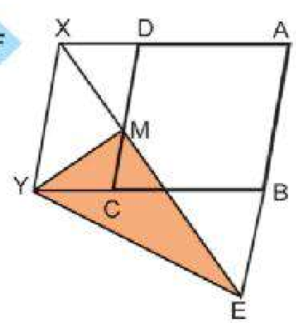
D



E

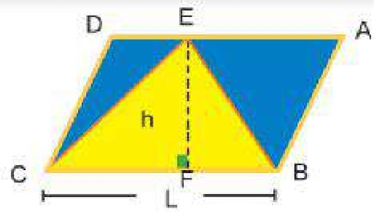


F



Unit 4 : Lesson 1

Let's think



In the opposite figure:
ABCD is a Parallelogram
area of traingle E B C = area of Parallelogram A B C D
= ....×..............×..............

Corollary 1

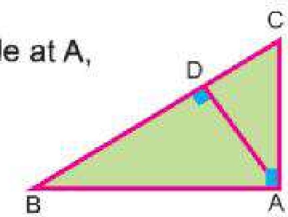
Area of the Triangle = $\frac{1}{2}$ of the length of the base × its Height

Note that:

1. The height of a triangle is the length of the perpendicular line segment drawn from a vertex to the opposite side.
2. All perpendiculur line segments of a triangle intersect in one point.

Practice

1 **In the opposite figure:** △ A B C is a right angled triangle at A, $\overline{AD} \perp \overline{BC}$



Complete:

area of the triangle A B C = $\frac{1}{2}$ A B × ....

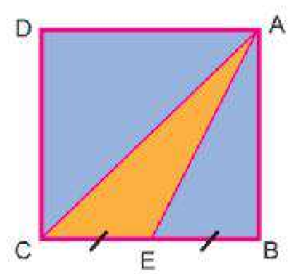
area of the triangle A B C = $\frac{1}{2}$ B C × ....

∴ A B × .... = B C × ....

Let A B = 4cm and A C = 3cm. What is the length of $\overline{AD}$?

2 **In the opposite figure:** A B C D is a square with perimeter = 24cm,

E is the midpoint of $\overline{BC}$

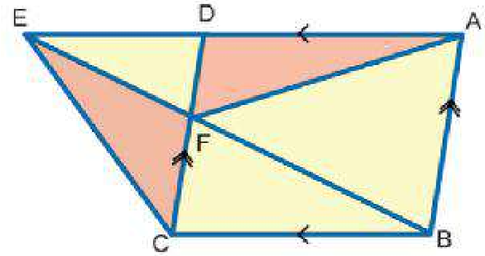


Complete:

A B = cm, C E = cm

area of the triangle A E C = cm²

Example



In the opposite figure:

A B C D is a parallelogram , E ∈ $\overrightarrow{AD}$,

$\overline{BE} \cap \overline{CD}$ = {F}

Prove that: area of the triangle A F D = area of the triangle E F C

Solution

Given: ▱ A B C D, $\overline{BE} \cap \overline{CD}$ = {F}

R.T.P : area of △ A F D = area of △ E F C

Proof : ∵ area of △ A FB = $\frac{1}{2}$ area of ▱ A B C D (corollary)

∴ area of △ A F D + area of △ B F C = $\frac{1}{2}$ area of ▱ A B C D (1)

∵ area of △ E B C = $\frac{1}{2}$ area of ▱ A B C D (corollary)

∴ area of △ E F C + area of △ B F C = $\frac{1}{2}$ area of ▱ A B C D (2)

from (1) and (2), we have:

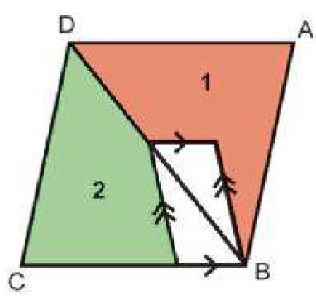
area of △ A F D = area of △ E F C (Q.E.D.)

Let's think

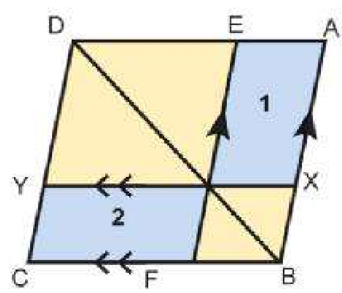
In both figures A and B :

A B C D is a parallelogram.

Why is area of Fig.(1) = area of Fig. (2) ?



(A)



(B)

Lesson TWO

Equality of the Areas of Two Triangles

Think and Discuss

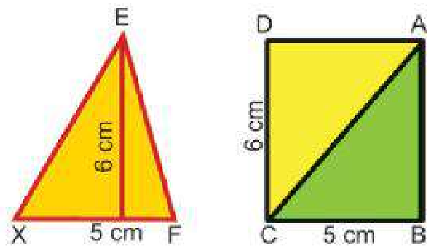
You will learn

- Relation between the areas of two triangles.

Key-Terms

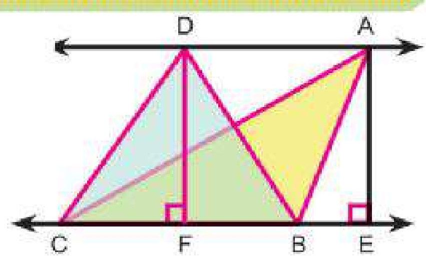
- Area of a Triangle

When two triangles are congruent, can you say that they have equal areas? When two triangles have equal areas, can you say that they are congruent?



Theorem 2

Two triangles which have the same base and the vertices opposite to this base on a straight line parallel to the base have the same area.



Given: $\overleftrightarrow{AD} \parallel \overleftrightarrow{BC}$, $\triangle ABC$ and $\triangle DBC$ have the common base $\overline{BC}$.

R.T.P : area of $\triangle ABC$ = area of $\triangle DBC$

Construction: Draw $\overline{AE} \perp \overleftrightarrow{BC}$ and $\overline{DF} \perp \overleftrightarrow{BC}$

Proof: $\because \overleftrightarrow{AD} \parallel \overleftrightarrow{BC}$, $\overleftrightarrow{AE} \perp \overline{BC}$ and $\overline{DF} \perp \overleftrightarrow{BC}$

$\therefore$ AEFD is a rectangle, AE = DF

$\because$ area of $\triangle ABC = \frac{1}{2} BC \times AE$ **(1)** ,

area of $\triangle DBC = \frac{1}{2} BC \times DF = \frac{1}{2} BC \times AE$ **(2)**

From (1) and (2), we have

$\therefore$ area of $\triangle ABC$ = area of $\triangle DBC$

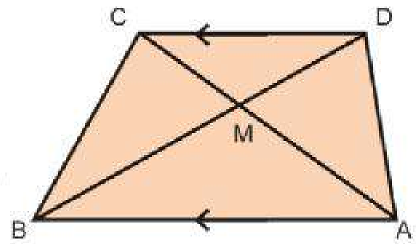
(Q.E.D.)

Practice

1 **In the opposite figure:**

$\overleftrightarrow{AB} \parallel \overleftrightarrow{DC}$, $\overline{AC} \cap \overline{BD} = \{M\}$

Complete and justify each step of your answer:



A area of $\triangle$ A D B= area .... because ....

B area of $\triangle$ D A C= area .... because ....

C area of $\triangle$ D A M .... = area .... because ....

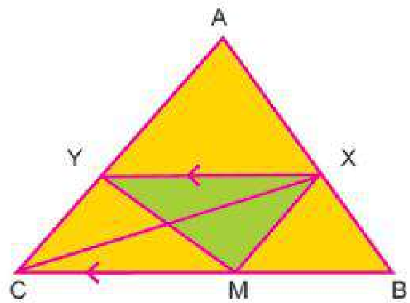
2 **In the opposite figure:**

$\triangle ABC$, $X \in \overline{AB}$, $Y \in \overline{AC}$,

$\overline{XY} \parallel \overleftrightarrow{BC}$, $M \in \overleftrightarrow{BC}$

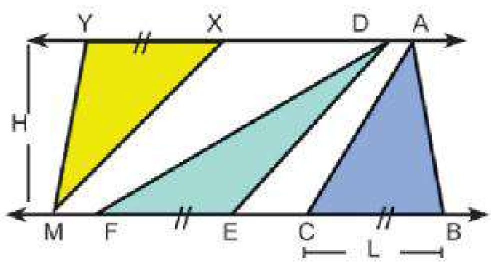
Complete: area of $\triangle$ X M Y = area ....

area of figure A X M Y = area .... Why?



Corollaries:

1 Triangles of bases equal in length and lying between two parallel straight lines are equal in area.



Note that:

$\overleftrightarrow{AY} \parallel \overleftrightarrow{BC}$, B C = E F = x y

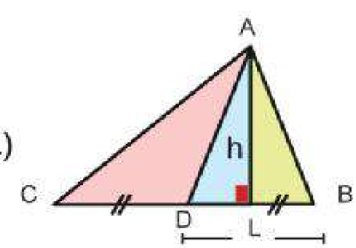
$\therefore$ area of $\triangle$ A B C = area of $\triangle$ D E F = area of $\triangle$ X Y M = $\frac{1}{2}$ L.H

2 The median of a triangle divides its surface into two triangular surfaces equal in area.

Note that:

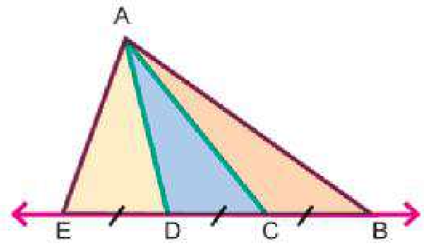
$\overline{AD}$ is a median of $\triangle$ A B C (B D = D C = L)

$\therefore$ area of $\triangle$ A B D = area of $\triangle$ A D C = $\frac{1}{2}$ L $\times$ h



3 **Triangles with congruent bases on one straight line and have a common vertex are equal in area.**

area of △ A B C = area of △ A C D = area of △ A D E



Practice

△ A B C with a median $\overline{AD}$, E ∈ $\overline{AD}$, draw $\overline{BE}$ and $\overline{CE}$

Prove that : area of △ A B E = area of △ A C E

Complete

∵ $\overline{AD}$ is a median in the triangle ....

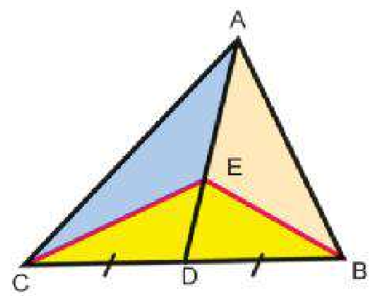
∴ area of △ A B D = area .... **(1)**

∵ .... is a median in △ E B C

∴ area of △ E B D = area .... **(2)**

subtracting **(2)** from **(1)** , then

area of △ A B E = ...



Exemple :

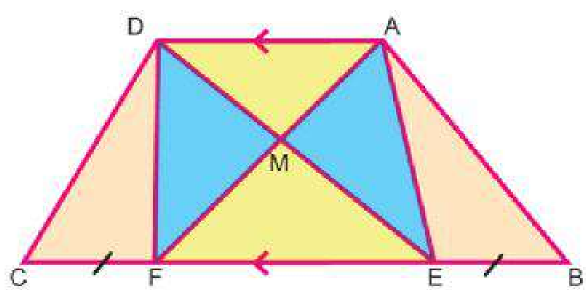
In the opposite figure:

$\overline{AD}$ // $\overline{BC}$, E ∈ $\overline{BC}$, F ∈ $\overline{BC}$ where B E = C F , $\overline{AF}$ ∩ $\overline{ED}$ = {M}

prove that :

first: area of △ A ME = area of △ D M F

Second: area of the figure A B E M = area of the figure D C F M



Proof:

∵ $\overline{AD}$ // $\overline{EF}$, and △ A E F and △DEF have a common base $\overline{EF}$

∴ area of △ A E F = area of △ D E F

subtracting area of △ ME F from both sides, then

area of △ A E M = area of △ D F M **(1) (I.Q.E.D)**

$\because BE = CF$, $\overline{AD} \parallel \overleftrightarrow{BC}$

$\therefore$ area of $\triangle ABE$ = area of $\triangle DCF$ **(2)**

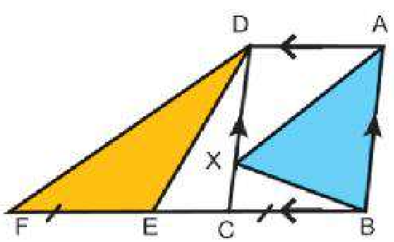
Adding **(1)** and **(2)** **we have:**

area of the figure ABEM = area of figure DCFM **(Q.E.D)**

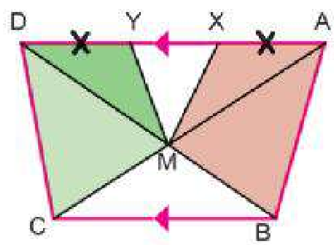
Practice

Show that all the shaded figures have equal areas (Use given information):

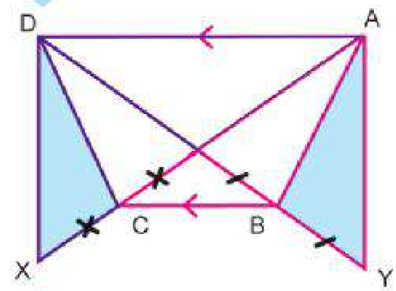
A



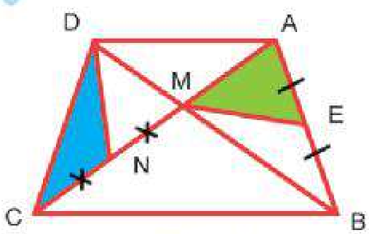
B



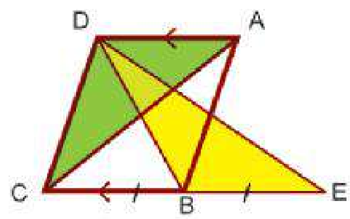
C



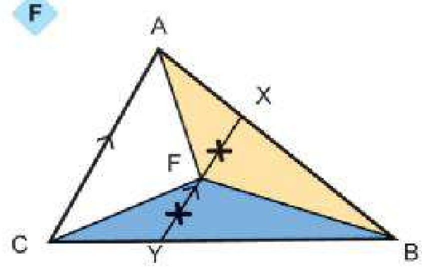
D



E



F



Theorem 3

If two triangles are equal in area and drawn on the same base and in one side of it, then their vertices lie on a straight line parallel to this base.

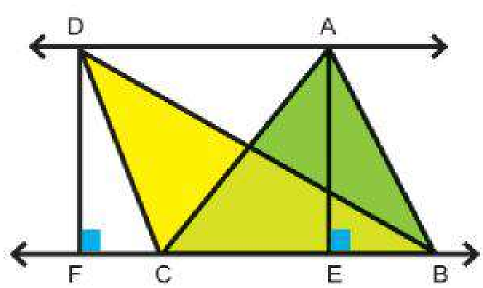
Given: area of $\triangle ABC$ = area of $\triangle DBC$.

$\overline{BC}$ is a common base

R.T.P: $\overleftrightarrow{AD} \parallel \overleftrightarrow{BC}$

Construction:

Draw $\overline{AE} \perp \overleftrightarrow{BC}$, $\overline{DF} \perp \overleftrightarrow{BC}$



Proof: $\because$ area of $\triangle ABC$ = area of $\triangle DBC$

$\therefore \frac{1}{2} BC \times AE = \frac{1}{2} BC \times DF$

$\therefore AE = DF$

$\because \overline{AE} \perp \overleftrightarrow{BC}$, $\overline{DF} \perp \overleftrightarrow{BC}$

$\therefore \overline{AE} \parallel \overline{DF}$

$\therefore$ Figure AEFD is a rectangle Thus : $\overleftrightarrow{AD} \parallel \overleftrightarrow{BC}$

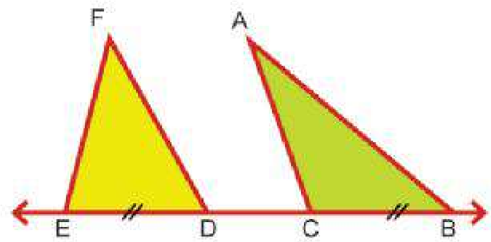
Let's think

1 **In the opposite figure:**

B, C, D, and E are collinear , where

$BC = DE$

If area of $\triangle ABC$ = area of $\triangle FDE$. What can you conclude ? Explain your answer.



2 **In the opposite figure:** $D \in \overleftrightarrow{BC}$,$A \in \overleftrightarrow{FE}$

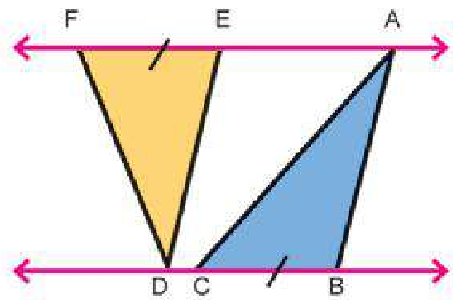
$BC = EF$

If:

area of $\triangle ABC$ = area of $\triangle DEF$

What can you conclude? Explain your answer.

Note that: $\overleftrightarrow{AF} \parallel \overleftrightarrow{BC}$. Why?



Exemple

ABCD is a parallelogram , $\overline{AC} \cap \overline{BD} = \{M\}$

$E \in \overrightarrow{AB}$ where area of $\triangle AME$ = area of $\triangle ABC$

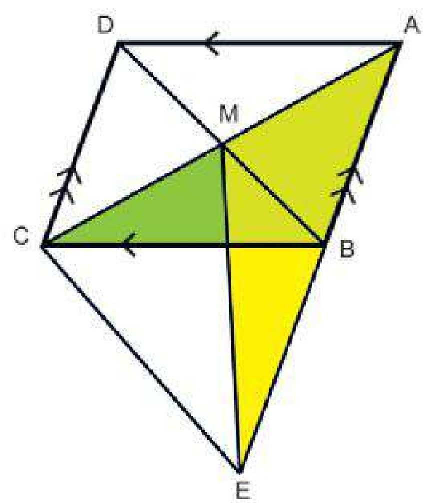
Prove that : The figure BECD is a parallelogram.

Proof: $\because$ area of $\triangle AME$ = area of $\triangle ABC$

Subtracting area of $\triangle ABM$ from both sides

$\therefore$ area of $\triangle BME$ = area of $\triangle BMC$

and both triangles have the common base $\overline{BM}$ and in one side of the base $\overline{BM}$.



∴ $\overleftrightarrow{CE} \parallel \overleftrightarrow{BM}$ **(1)**

∵ The figure A B C D is a parallelogram

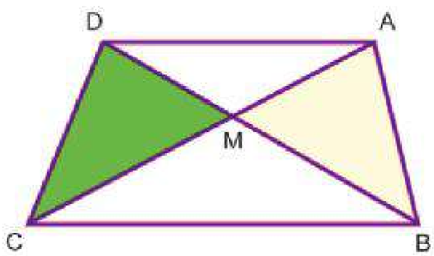
∴ $\overleftrightarrow{BE} \parallel \overleftrightarrow{DC}$ **(2)**

from **(1)** and **(2)** the figure DBEC is parallelograr

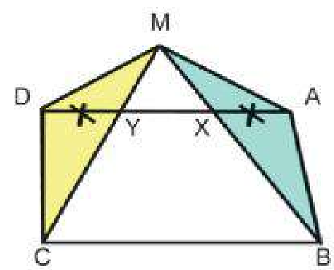
Practice

1 **In the following figures all the colored triangles have the same area . Explain why** $\overline{AD} \parallel \overline{BC}$.

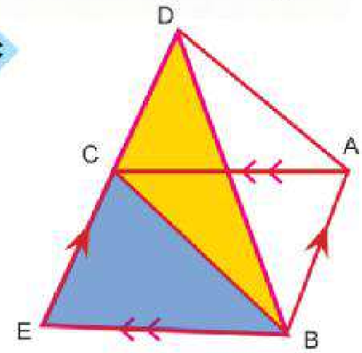
A



B



C



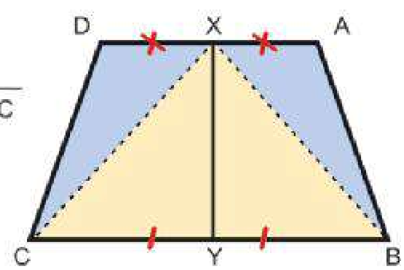
2 **In the opposite figure:**

A B C D is a quadrilateral, where

X is the midpoint of $\overline{AD}$ and Y is the midpoint of $\overline{BC}$

area of the figure ABYX = area of the figure DCYX

Prove that: $\overline{AD} \parallel \overline{BC}$



Problem solving Tip

Draw $\overline{BX}$, $\overline{CX}$

In △ X B C, $\overline{XY}$ is a median, what can you conclude?

area of △ AXB = area .... why?

$\overline{AD} \parallel \overline{BC}$ Why?

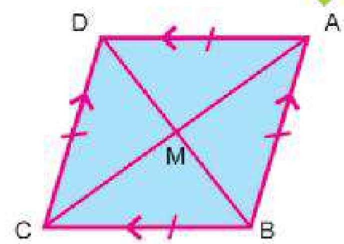
Aeras of Some geometric Figures

Lesson Three

Think and Discuss

You have learned before that the rhombus is a parallelogram whose sides are equal in length.

- What is the Relation between the diagonals of the rhombus?
- How can you calculate the area of the rhombus?



You will learn

- To find the area of a rhombus.
- To find the area of a square in terms of its diagonal.
- To find the area of a Trapezium.

Key - terms

- Square.
- Rhombus.
- Trapezium
- Area.

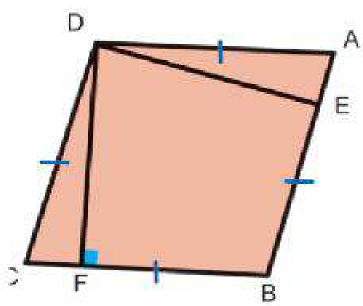
Area of the rhombus:

1. If the side length of a rhombus is b and its height is h, then Area of Rhombus = $b \times h$

i.e:

area of the rhombus

= base length × height



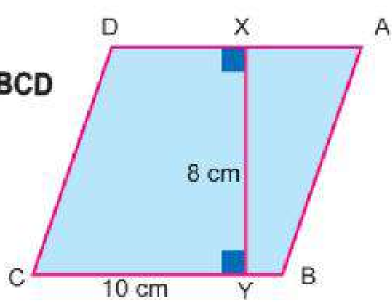
Let's think

is D E = D F? Explain.

Practice

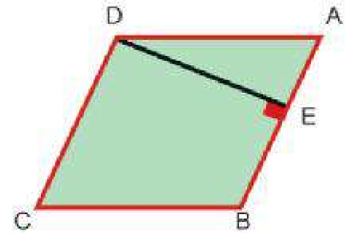
1. find the area of the rhombus ABCD

   A. area = ...



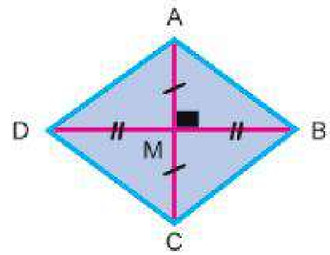
B Perimeter of rhombus A B C D = 24cm, D E = 5cm

area =



2 **You know:**

The diagonals of the rhombus are perpendicular and bisecting each other. Refres to the opposite figure, complete:



area of rhombus A B C D = 2 area △ A B D

$= 2 \quad \times \frac{1}{2}$ B D × ...

$= \frac{1}{2} \quad \times$ B D $\times 2$

$= \frac{1}{2}$ B D ×

Area of the rhombus = half of the product of the lengths of its diagonals.

a **The Square is a rhombus whose diagonals are equal in length.**

Area of the square = $\frac{1}{2}$ of the square of the length of its diagonal

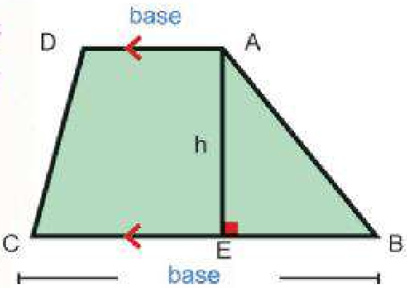
Practice

Find the area of the following figures:

1. A rhombus whose side length is 12cm and whose height is 8cm.
2. A rhombus whose diagonals length are 8cm and 10cm.
3. A square whose diagonal length is 8cm.
4. A rhombus whose perimeter is 52cm and the length of one of its diagonal is 10cm.
5. A rhombus whose perimeter is 60cm and the measure of one of its angles is 60°.

Trapezium

A Trapezium is a quadrilateral whose two opposite sides are parallel. The two opposite sides are called bases and the other two sides are called legs.



In the opposite figure:

$\overline{AD}$, $\overline{BC}$ are bases of the Trapezium A B C D

$\overline{AB}$ and $\overline{DC}$ are legs of the Trapezium A B C D

A Trapezium has only one height which is the perpendicular distance between its bases

Let's think

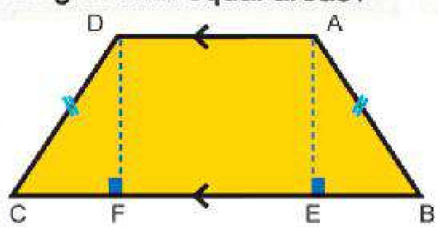
Does the diagonal of a trapezium divide it into two triangles with equal areas?

If ABCD is an isosceles Trapezium, in $\overline{AB}$, $\overline{DC}$

: is m (∠ B) = m (∠ C)?

Draw $\overline{AE} \perp \overline{BC}$ and $\overline{DF} \perp \overline{BC}$

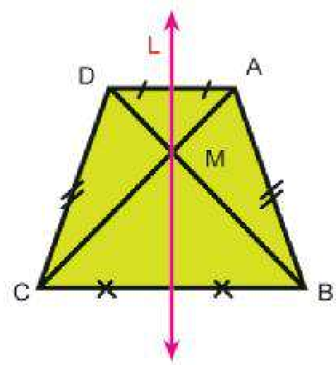
Explain your answer.



Isosceles Trapezium:

If: A B C D is a Trapezium with A B = C D, then

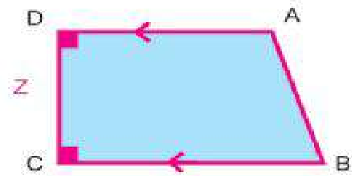
1. The base angles are equal in measure.
   m (∠ B) = m (∠ C), m (∠ A) = m (∠ D)
2. The diagonals are equal in length A C = B D
   $\overline{AC} \cap \overline{BD}$ = {M}
   ∴ A M = D M, B M = C M



3. The isosceles trapezium has only one axis of symmetry (L) which is the perpendicular bisector of its bases.

Right Trapezium

A right Trapezium is a Trapezium whose one of its legs is perpendicular to its two parallel bases

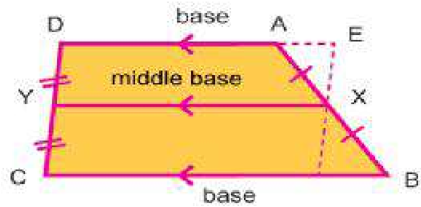


In the opposite figure: $\overline{DC} \perp \overline{BC}$ and $\overline{CD} \perp \overline{AD}$,

∴ The height of Trapezium = The length of $\overline{CD}$

Middle base of Trapezium

A middle base of a trapezium is a segment $\overline{XY}$ whose endpoints are the midpoints of the non-parallel sides of Trapezium A B C D



Note that:

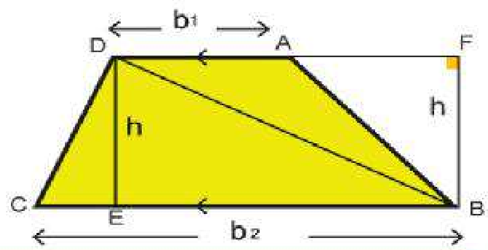
$\overline{XY}$ // $\overline{BC}$ // $\overline{AD}$

The length of $\overline{XY} = \frac{1}{2}(AD + BC)$

Practice

Find the length of the middle base of a trapezium whose two bases lengths are 7cm and 13cm.

Area of trapezium:



Area of trapezium A B C D = area △ A B D + area △ D B C

$= \frac{1}{2} AD \times BF \quad + \quad \frac{1}{2} BC \times DE$

$= \frac{1}{2} b_1 h \quad + \quad \frac{1}{2} b_2 \times h$

$= \frac{1}{2}(b_1 + b_2)\, h$

Area of a Trapezium = half of the sum of lengths of the two parallel bases × height.

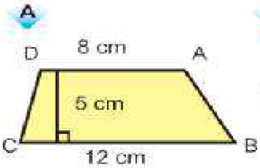
Note that: **The middle base of the trapezium is parallel to the two bases and its length is equal to half of the sum of their lengths.**

Area of a Trapezium = the length of the middle base × its Height.

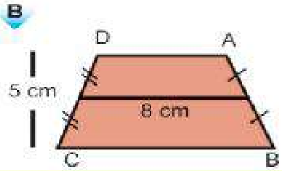
Practice

Find the area of each of the following figurers by using the given data :

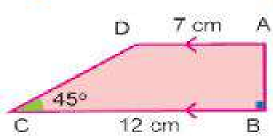
A



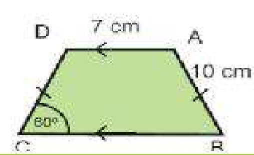
B



C



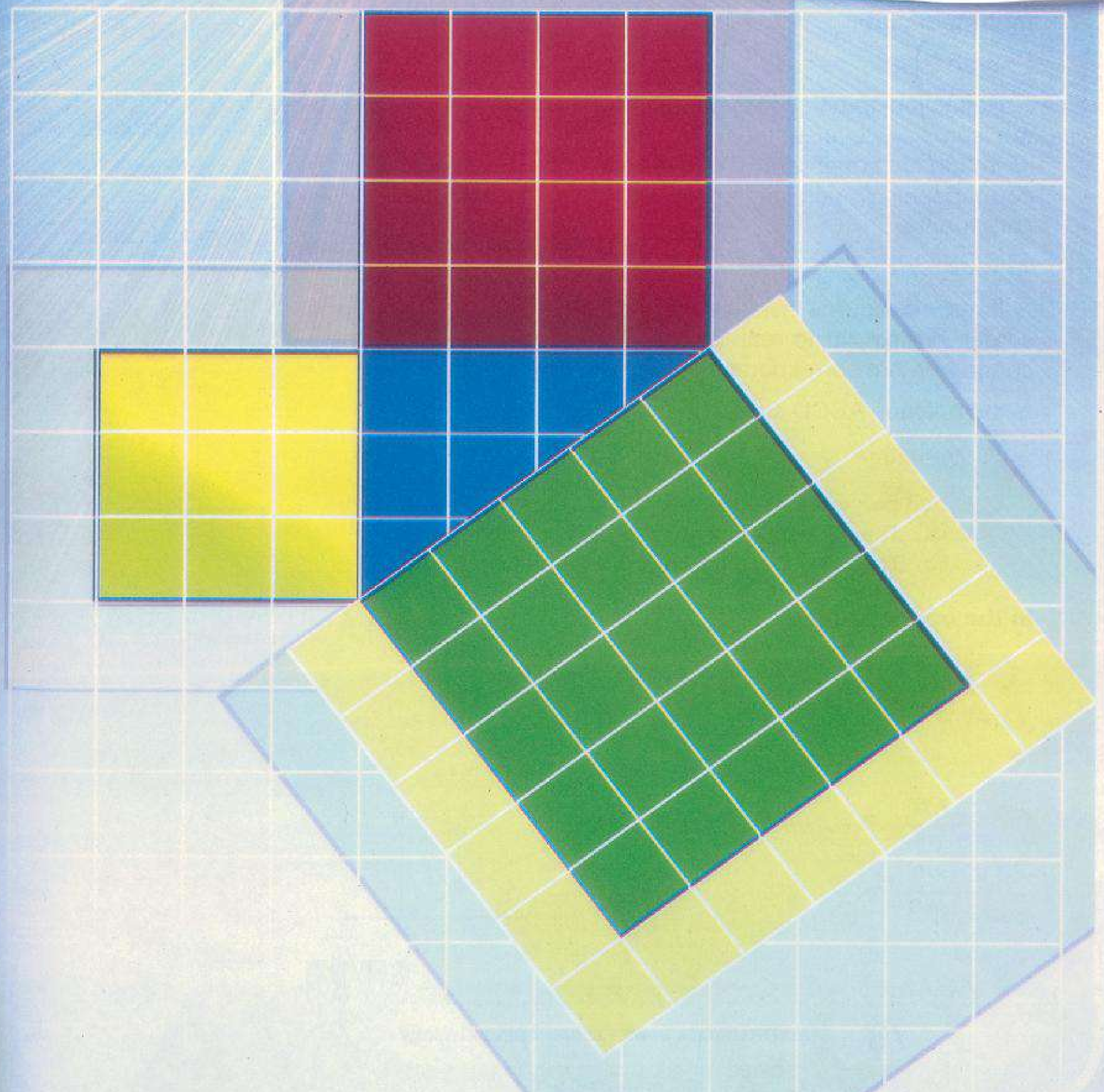
D



UNIT FIVE

5

Similarity and Projections



Similarity

Lesson One

Think and Discuss

You will learn

- The concept of similarity.
- Similar of Polygons.
- Similar of triangles.

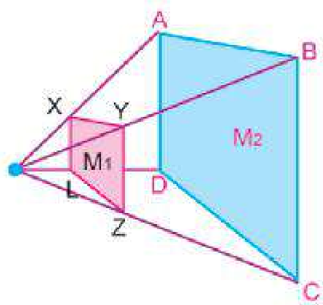
Key-Terms

- Similar.
- Proportional sides.
- corresponding angles.

During displaying examples and applications in the multmedia lab.

Usama Said:

Reflection, translation and rotation are isometry, because the figure and its image are congruent. This means corresponding sides and angles are congruent .



Ahmed Said:

Exercises figures displayed on the screen are similar to real figures. Corresponding angles are congruent, but corresponding sides are proportional.

Is the Polygon ABCD similar to the polygon XYZL? Why?

Definition

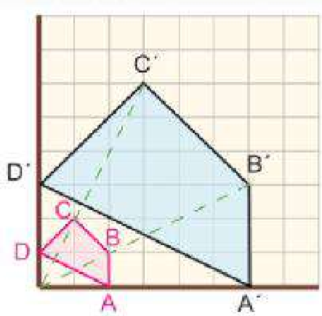
Two polygons are similar if:

- The corresponding angles are congruent
- The corresponding sides are proportional.

In the opposite figure

Exemple :

$$\frac{A'B'}{AB} = \frac{B'C'}{BC} = \frac{C'D'}{CD} = \frac{A'D'}{AD} = \frac{3}{1}$$



m (∠ A′) = m (∠ A), m (∠ B′) = m (∠ B),
m (∠ C′) = m (∠ C), m (∠D′) = m (∠ D)

The polygon ABCD is similar to the polygon A`B`C`D`.

Note that:

1. The order of corresponding vertices should be kept in giving names of similar polygons, Similarity is denoted by the sign (~). Fig. A`B`C`D` (~) Fig. ABCD means two similar figures.
2. The proportional ratio between corresponding sides is called the ratio of enlargement or drawing scale.

   Notice: If the proportional ratio = 1, then the two polygons are congruent.
3. All the regular polygon that have the same number of sides are similar. why?
4. If two polygons are similar, then the corresponding angles are congruent and the corresponding sides are proportional as well.

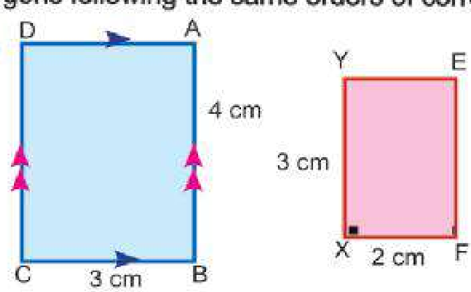
Think: **The square and the rectangle are not similar although the corresponding angles are congruent . Why?**

The corresponding sides of a square and a rhombus are proportional but they are not similar.

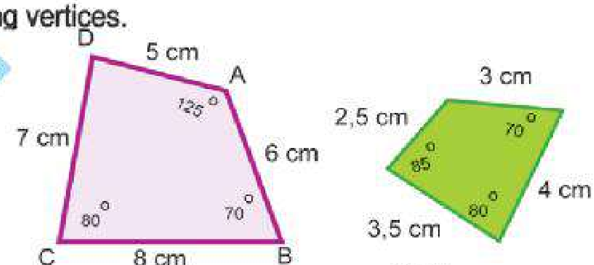
Practice

1 Which of the following pairs of polygons are similar and why? write the similar polygons following the same orders of corresponding vertices.

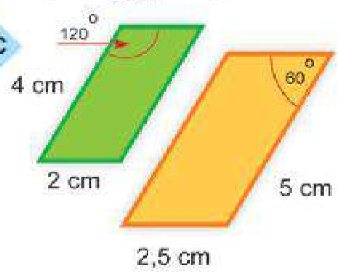
A



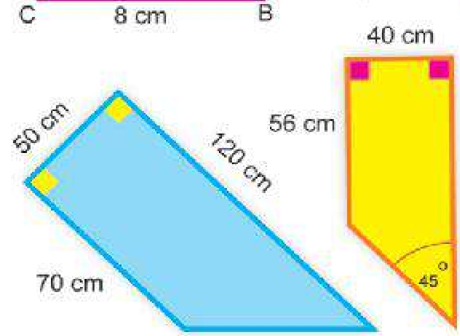
B



C



D



Similarity of two triangles

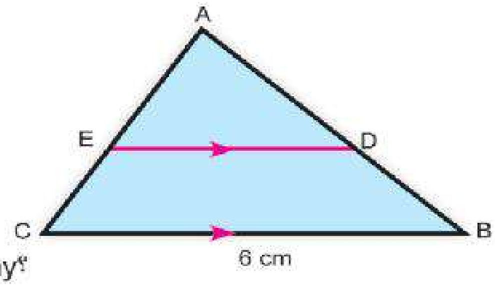
definition

Two triangles are similar if there exists one of the following conditions :

- The corresponding angles are congruent.
- the corresponding sides are proportional.

Exemple :

In the opposite figure: ABC is a triangle in which A B = 5cm, BC = 6cm, AC = 4cm, and D ∈ $\overline{AB}$ where AD = 3cm, $\overline{DE}$ \\ $\overline{BC}$ ، $\overline{DE}$ ∩ $\overline{AC}$ = {E}.



A **Prove that** △ ADE ~ △ ABC.

B **Find the length of** $\overline{DE}$ and $\overline{AE}$

Solution

∵ $\overline{DE}$ \\ $\overline{BC}$

∴ m (∠ ADE) = m (∠ B), m (∠ AED) = m (∠ C) Why?

∵ ∠A is common in △ ADE and △ ABC .

∴ △ ADE ~ △ ABC corresponding angles are congruent, So:

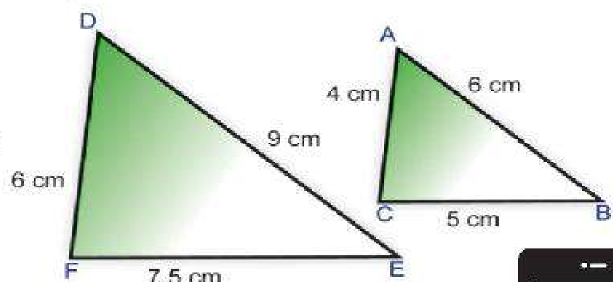
$$\frac{AD}{AB} = \frac{DE}{BC} = \frac{AE}{AC} \quad \therefore \frac{3}{5} = \frac{DE}{6} = \frac{AE}{4}$$

$$\therefore DE = \frac{3 \times 6}{5} = 3.6\text{cm and } AE = \frac{3 \times 4}{5} = 2.4\text{cm}$$

Practice

Using the given in the opposite figures.

Prove that



A △ DEF ~ △ ABC

B $\frac{\text{Perimeter of } \triangle DEF}{\text{Perimeter of } \triangle ABC}$ = the ratio of the similarity

Note that :

the ratio between the perimeters of two similar triangles = the ratio between any two of corresponding sides

Converse of Pythagoras Theorem

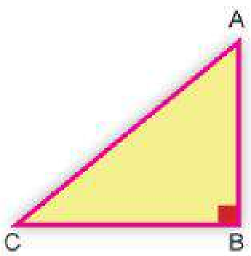
Lesson Two

Think and Discuss

You know from Pythagoras Theorem that if $\triangle$ ABC is a right angled triangle at B, then

$$(AC)^2 = (AB)^2 + (BC)^2$$

Now, we will learn the converse of the pythagorean Theorem.



You will learn

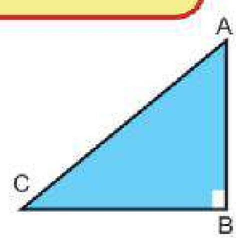
- Converse of pythagoras theorem.
- Using pythagoras theorem on solving problems.

Converse of Pythagoras Theorem:

In a triangle if the sum of the areas of two squares on two sides is equal to the area of the square on the third side, then the angle opposite to this side is a right angle.

i.e. in $\triangle$ ABC, if : $(AB)^2 + (BC)^2 = (AC)^2$

then : m ($\angle$B) = 90° and $\triangle$ ABC is a right angled triangle at B.



The converse of Pythagoras Theorem can be rewritten as:

In a triangle if the square of the length of a side is equal to the sum of the squares of the lengths of the other two sides, then the angle opposite to this side is a right angle.

Corollary:

In the triangle ABC, if $\overline{AC}$ is the longest side and $(AB)^2 + (BC)^2 \neq (AC)^2$, then $\triangle$ ABC is not a right triangle

Lesson Three Projections

Think and Discuss

You will learn

- To find the projection of a point on a line.
- To find the projection of a line segment on a line.
- To find the prjection of a ray on a line.
- To find the projection of a line on a line.

Key-Terms

- Projection.
- Point.
- Line segment.
- Ray.
- Straight line.

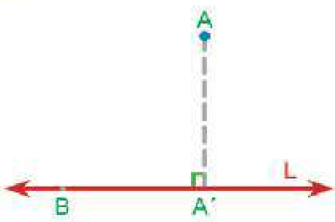
A piece of chalk falls down on the earth:

Does it fall down vertically (perpendicular to the earth)?

Does it leave a mark on the earth?

Projection of a point on a straight line

In the opposite figure:



L is a straight line, A and B are two points, where $A \notin L$ and $B \in L$.

Draw $\overline{AA`} \perp L$, where $A` \in L$.

The point A` (the point of intersection of $\overline{AA`}$ and L) **is called the projection of A on L.**

$\because B \in L$ $\therefore$ The projection of B on L is itself.

Note that:

- **Projection of a point on a straight line is that the point of intersection of the perpendicular segment from this point and the straight line.**
- **If the point lies on the straight line, its projection on it is the same point.**

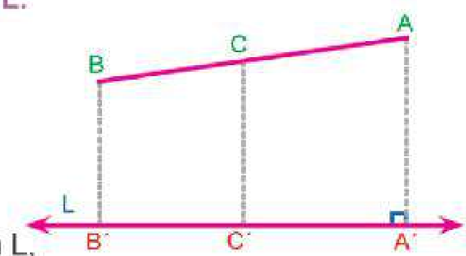
The Projection of a line Segment on a given straight line

Finding the projection of line segment $\overline{AB}$ on a line L.

If : A` is the projection of A on the straight line L

and B` is the projection of B on the straight line L,

then $\overline{A`B`}$ is the projection of $\overline{AB}$ on L.

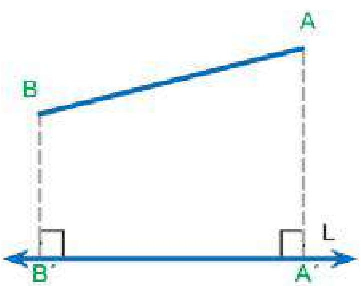


Note that If C ∈ $\overline{AB}$ and C` is its projection on L,

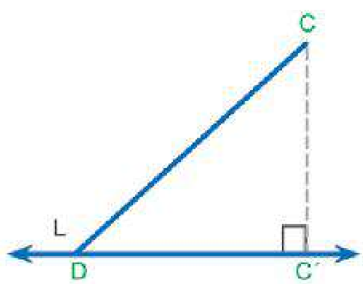
then C` ∈ $\overline{A`B`}$

Practice

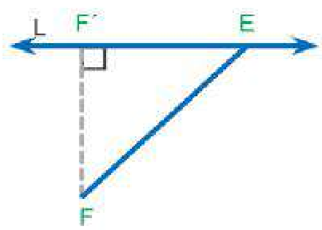
The following figures illustrate segments in different locations. Complete by writing down the projection of each one as shown in the first example:



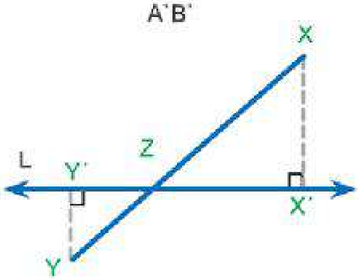
The projection of $\overline{AB}$ on L is $\overline{A`B`}$



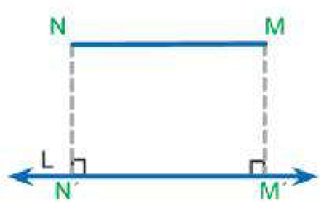
The projection of $\overline{CD}$ on L is ....



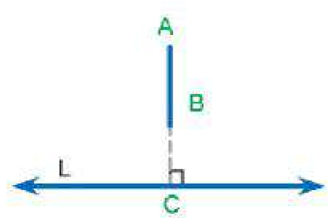
The projection of $\overline{EF}$ on L is ....



The projection of $\overline{XY}$ on L is ....



The projection of $\overline{MN}$ on L is ....



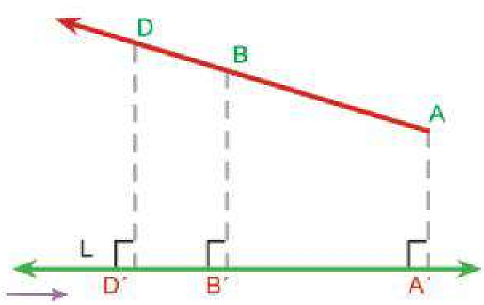
The projection of $\overline{AB}$ on L is ....

Note and Discuss:

- **A** The length of the projection of a line segment on a given line is less than or equal to the length of the segment itself.
- **B** When is the length of the projection of a line segment on a given line equal to the length of the segment itself?
- **C** When is the length of the projection of a segment on a given line equal to zero?

The Projection of a Ray on a straight line

Finding the projection of $\overrightarrow{AB}$ on L.



Note that A` is the projection of A on L

B` is the projection of B on L.

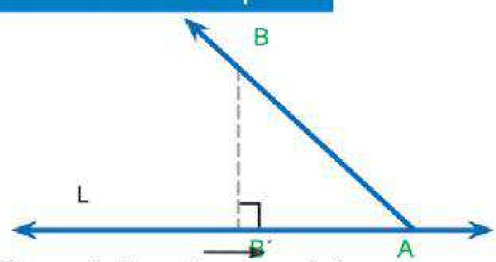
If $D \in \overrightarrow{AB}$, $D \notin \overline{AB}$

and D` is the projection of D on L,

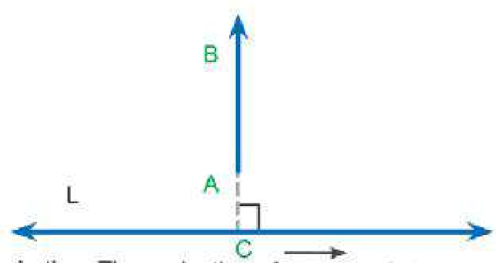
then $D` \in \overrightarrow{A`B`}$

∴ The projection of $\overrightarrow{AB}$ on L is $\overrightarrow{A`B`}$:

Observe and complete:



The projection of $\overrightarrow{AB}$ on L is...............

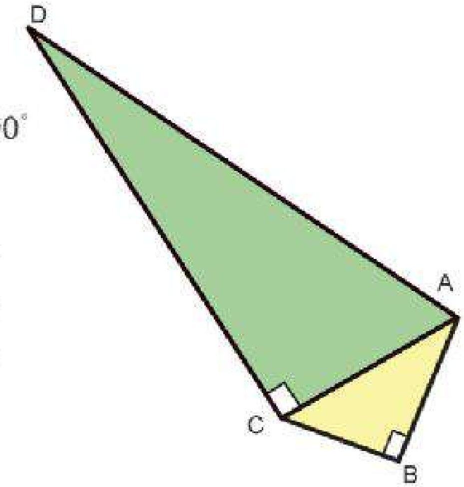


If $\overrightarrow{AB} \perp$ L, then The projection of $\overrightarrow{AB}$ on L is ....

Let's think

- **A** What is the projection of a line on a given line?
- **B** Can the projection of a line on a given line be a point?
- **C** Explain your answers by drawing different figures of a projection of a line on a given line and keep it in your portfolio file.

Practice:(1)

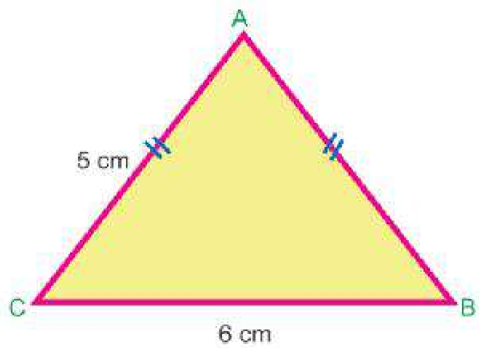


In the opposite figure: m (∠ B) = m (∠ ACD) = 90°

Complete:

- **A** The projection of $\overline{AD}$ on $\overleftrightarrow{CD}$ is ....
- **B** The projection of $\overline{AC}$ on $\overleftrightarrow{CD}$ is ....
- **C** The projection of $\overline{AC}$ on $\overleftrightarrow{AB}$ is ....

Practice:(2)



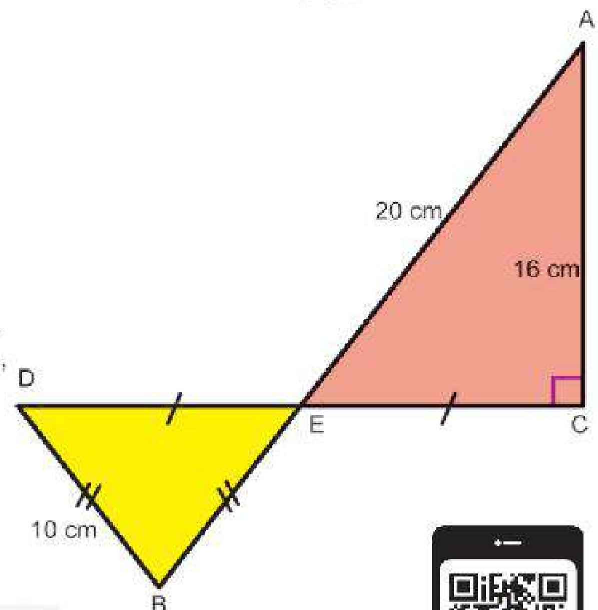
In the opposite figure:

ABC is a triangle, with AB = AC = 5cm, and BC = 6cm.

Find:

- **A** The length of the projection of $\overline{AB}$ on $\overleftrightarrow{BC}$.
- **B** The area of the triangle ABC.

Practice:(3)



In the opposite figure:

$\overline{AB} \cap \overline{CD}$ = {E}, E is the midpoint of $\overline{CD}$,

AC = 16cm, AE = 20cm,

BD = B E = 10cm.

Find:

- **A** The length of the projection of $\overline{BD}$ on $\overleftrightarrow{CD}$
- **B** The length of the projection of $\overline{AB}$ on $\overleftrightarrow{CD}$.

Euclidean Theorem

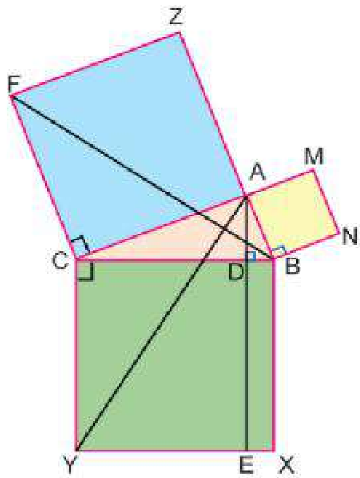
Lesson Four

Think and Discuss

In the opposite figure:

1. ABC is a right angled triangle at A. ABNM, ACFZ and BXYC are squares drawn on the sides of the triangle ABC.

2. Draw $\overrightarrow{AD} \perp \overline{BC}$ and intersects it at D and intersects $\overline{XY}$ at E. Draw $\overline{BF}$ and $\overline{AY}$ as shown in the opposite figure.



You will learn

- Euclidean Theorem.
- Applications on Euclidean theorem.

Note that:

$m(\angle BCF) = m(\angle YCA)$

$\triangle BCF \equiv \triangle YCA$ **Why?** ?

area of $\triangle BCF = \frac{1}{2}$ the area of the square ACFZ **Why?**

area of $\triangle YCA = \frac{1}{2}$ the area of the rectangle EYCD **Why?**

Thus: the **area** of the square ACFZ = the **area** of the rectangle EYCD

$AC^2 = CD \times CY$ **Why?**

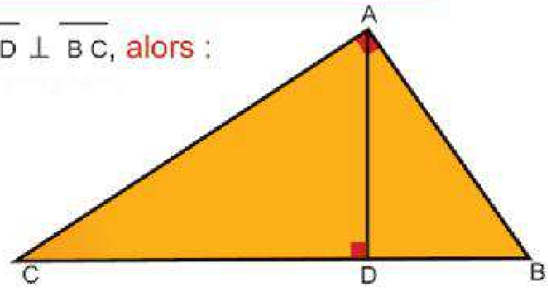
$\therefore AC^2 = CD \times CB$

= The length of the projection of $\overline{AC}$ × The length of the hypotenuse $\overline{BC}$

Euclidean Theory:

In the right-angled triangle, the area of the square on a side of the right angle is equal to the area of the rectangle which its dimensions are the length of the projection of this side on the hypotenuse and the length of the hypotenuse

i.e. ABC is a right angled triangle at A and $\overline{AD} \perp \overline{BC}$, alors :



$BA^2 = BD \times BC$

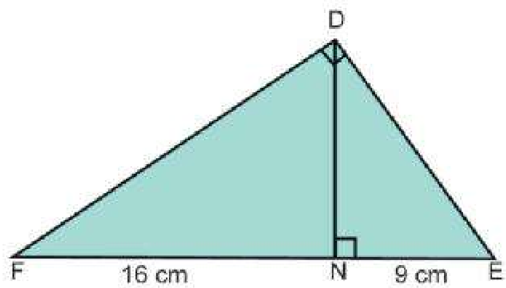
$CA^2 = CD \times CB$

Corollary:

$$(AD)^2 = DB \times DC$$

Practice

In the opposite figure:



$\triangle$ DEF is a right angled triangle at D, $\overline{DN} \perp \overline{EF}$,

EN = 9cm and NF = 16cm

Complete:

$(DE)^2$ = EN × EF (Euclidean Theorem)

= .... × ....   ∴ DE = ...cm

$(DF)^2$ = FN × ......   (Euclidean Theorem)

= .... × ....   ∴ DF = ...cm

$(DN)^2$ = NE × NF.   (...........)

= .... × ....   ∴ DN = ...cm

Let's think

Is DN × EF = DE × DF ?   Why?

Lesson Five

Classifying of Triangles according to their Angles

Think and Discuss

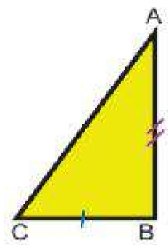


Fig. (1)

∠B is a right angle

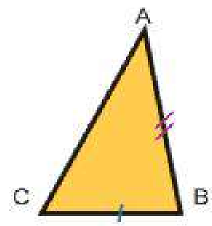


Fig. (2)

∠ B is an acute angle

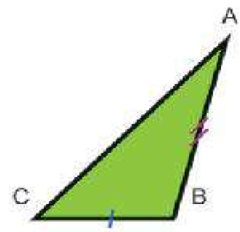


Fig. (3)

∠ B is an obtuse angle

You will learn

- to determine the type of a triangle according to its angles

Key-Terms

- A right angled triangle.
- An acute angled triangle
- An obtuse angled triangle.

Note that: **The length of $\overline{AB}$, does not change in all figures.**

The length of $\overline{BC}$, **does not change in all figures, Does it mean the length of** $\overline{AC}$ changes according to the opposite angle?.

Complete by writing **> , or = or < :**

in Fig. (1) ∴ m (∠B) ≐ 90˚ ∴ $(AB)^2 + (BC)^2$ .... $(AC)^2$

in Fig. (2) ∴ m (∠B) < 90˚ ∴ $(AB)^2 + (BC)^2$ .... $(AC)^2$

in Fig. (3) ∴ m (∠B) > 90˚ ∴ $(AB)^2 + (BC)^2$ .... $(AC)^2$

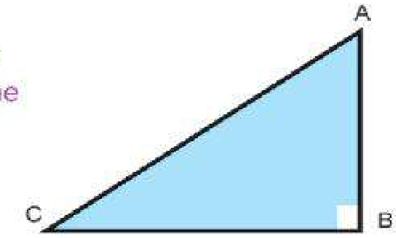
when is m (∠ B)= 90°?:

Determining the type of triangle according to its angles, in case of knowing the lengths of its three sides.

We compare the square length of the longest side of the triangle and the sum of squares of the other two sides

I: If the

square length of the longest side is equal to the sum of the squares lengths of the other two sides, then the triangle is a right angled triangle.

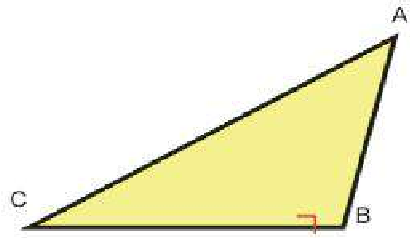


In △ ABC: $(AC)^2 = (AB)^2 + (BC)^2$

∴ ∠ **B is a right angle.**

II: If the

square length of the longest side > sum of squares lengths of the other two sides, then the triangle is an obtuse angled triangle.

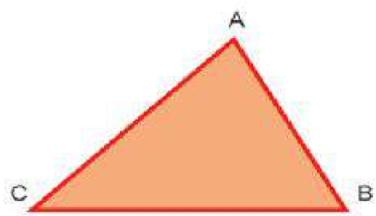


In △ ABC: $(AC)^2 > (AB)^2 + (BC)^2$

∴ ∠ **B is an obtuse angle.**

III: If the

square length of the longest side < the sum of squares lengths of the two other sides, then the triangle is an acute triangle.



In △ ABC: $(AC)^2 < (AB)^2 + (BC)^2$

∴ ∠B is an acute angle why?

Exemple :

Determine the type of the angle which has the greatest measure in △ ABC, where

AB = 8cm , BC = 10cm and CA = 7cm

What is the type of the triangle according to its angles?

Solution

∵ The greatest angle is opposite to the longest side.

∴ ∠A is the greatest angle in △ ABC, since $\overline{BC}$ is the longest side. $(BC)^2 = (10)^2 = 100$

$AB^2 + AC^2 = (8)^2 + (7)^2$

$= 64 + 49 = 113$

∵ $(BC)^2 < (AB)^2 + (AC)^2$ ∵ ∠ A is an acute angle

∵ ∠ A is the greatest angle

∵ △ ABC is an acute angled triangle

المواصفات الفنيــة:

رقــم الكتـــاب:	٢٠/٢/٣٣/١٥/٧/١١٠٩
مقـــاس الكتـــاب:	$\frac{١}{٨}$ (٥٧ × ٨٢) سم
طبـــع المتـــن:	٤ ألوان
طبـــع الغـــلاف:	٤ ألوان
ورق المتـــن:	٨٠ جم أبيض
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