

أدرب وأحل المسائل

المماس والعمودي على المماس

أجد معادلة المماس لمنحنى كل اقتران ممّا يأتي عند النقطة المعطاة:

(1) $f(x)=x^3-6x+3$, $(2,-1)$

$$f(x)=x^3-6x+3, (2,-1), f(2)=-1, f'(x)=3x^2-6, f'(2)=12-6=6$$

معادلة المماس:

$$y-f(a)=f'(a)(x-a) \Rightarrow y-f(2)=f'(2)(x-2) \Rightarrow y-(-1)=6(x-2) \Rightarrow y+1=6x-12 \Rightarrow y=6x-13$$

(2) $f(x)=x^4-3x^3$, $(1,-2)$

$$f(x)=x^4-3x^3, (1,-2), f(1)=-2, f'(x)=4x^3-9x^2, f'(1)=-5$$

معادلة المماس:

$$y-f(a)=f'(a)(x-a) \Rightarrow y-f(1)=f'(1)(x-1) \Rightarrow y-(-2)=-5(x-1) \Rightarrow y+2=-5x+5 \Rightarrow y=-5x+3$$

(3) $f(x)=x(x^2-1)$, $(1,0)$

$$f(x)=x(x^2-1), (1,0), f(1)=0, f'(x)=(x^2-1)+2x(x-1), f'(1)=2$$

معادلة المماس:

$$y-f(a)=f'(a)(x-a) \Rightarrow y-f(1)=f'(1)(x-1) \Rightarrow y-0=2(x-1) \Rightarrow y=2x-2$$

(4) $f(x)=x+4x^2$, $(-4,-5)$

$$f(x)=x+4x^2, (-4,-5), f(-4)=-5, f'(x)=1+8x, f'(-4)=-31$$

معادلة المماس:

$$y-f(a)=f'(a)(x-a) \Rightarrow y-f(-4)=f'(-4)(x-(-4)) \Rightarrow y-(-5)=-31(x+4) \Rightarrow y+5=-31x-124 \Rightarrow y=-31x-129$$

(5) $f(x)=x+e^x$, $(0,1)$

$$f(x)=x+e^x, (0,1), f(0)=1, f'(x)=1+e^x, f'(0)=2$$

معادلة المماس:

$$y - f(a) = f'(a)(x - a) \quad y - f(0) = f'(0)(x - 0) \quad y - 1 = 2(x - 0) \quad y - 1 = 2x \quad y = 2x + 1$$

(6) $f(x) = \ln(x+e)$, $(0,1)$

$$f(x) = \ln(x+e) , (0,1) , f(0) = 1 \quad f'(x) = \frac{1}{x+e} \quad f'(0) = \frac{1}{0+e} = \frac{1}{e}$$

معادلة المماس:

$$y - f(a) = f'(a)(x - a) \quad y - f(0) = f'(0)(x - 0) \quad y - 1 = \frac{1}{e}(x - 0) \quad y - 1 = \frac{x}{e} \quad y = \frac{x}{e} + 1$$

أجد معادلة المماس لمنحنى كل اقتران ممّا يأتي عند قيمة المعطاة:

(7) $f(x) = x - 7$, $x = 16$

$$f(x) = x - 7 , x = 16 \quad f(16) = 16 - 7 = 9 \rightarrow (16, 9) \quad f'(x) = 1 \quad f'(16) = 1$$

معادلة المماس:

$$y - f(a) = f'(a)(x - a) \quad y - f(16) = f'(16)(x - 16) \quad y - 9 = 1(x - 16) \quad y - 9 = x - 16 \quad y = x - 7$$

(8) $f(x) = (x-1)e^x$, $x=1$

$$f(x) = (x-1)e^x , x=1 \quad f(1) = (1-1)e^1 = 0 \rightarrow (1, 0) \quad f'(x) = (x-1)e^x + e^x(1) = x e^x \quad f'(1) = 1e^1 = e$$

معادلة المماس:

$$y - f(a) = f'(a)(x - a) \quad y - f(1) = f'(1)(x - 1) \quad y - 0 = e(x - 1) \quad y = ex - e$$

(9) $f(x) = x + 3x - 3$, $x=4$

$$f(x) = x + 3x - 3 , x=4 \quad f(4) = 4 + 3(4) - 3 = 13 \quad f'(x) = (x-3)(1) - (x+3)(1) \quad f'(4) = (4-3) - (4+3) = -6$$

معادلة المماس:

$$y - f(a) = f'(a)(x - a) \quad y - f(4) = f'(4)(x - 4) \quad y - 13 = -6(x - 4) \quad y - 13 = -6x + 24 \quad y = -6x + 37$$

$$y = -6x + 31$$

$$(10) f(x) = (\ln x)^2, x = e$$

$$f(x) = (\ln x)^2, x = e \rightarrow f(e) = (\ln e)^2 = 1 \rightarrow (e, 1) \quad f'(x) = 2(\ln x)(1/x) \quad f'(e) = 2(\ln e)(1/e) = 2/e$$

معادلة المماس:

$$y - f(a) = f'(a)(x - a) \quad y - f(e) = f'(e)(x - e) \quad y - 1 = 2/e(x - e) \quad y - 1 = 2ex + 2 = 2ex + 3$$