

أتدرّب وأحل المسائل

التكامل بالكسور الجزئية

أحد كلاًً من التكاملات الآتية:

$$(x-10x(x+5)dx \quad (1) \int$$

$$x-10x(x+5)=Ax+Bx+5 \Rightarrow x-10=A(x+5)+Bxx=0 \Rightarrow A=-2x=-5 \Rightarrow B=3 \int \\ |x+5|+C|x|+3\ln x-10x(x+5)dx=\int(-2x+3x+5)dx=-2\ln$$

$$(x^2dx \quad (2-21) \int$$

$$x^2=2(1-x)(1+x)=A1-x+B1+x \Rightarrow 2=A(1+x)+B(1-x)x=1 \Rightarrow A=1x-21 \\ |1|1+x|+C=\ln|1-x|+\ln=-1 \Rightarrow B=1 \int 21-x^2dx=\int(11-x+11+x)dx=-\ln \\ +x1-x|+C$$

$$(x-2)(x-4)dx \quad (3) \int 4$$

$$x-2)(x-4)=Ax-2+Bx-4 \Rightarrow 4=A(x-4)+B(x-2)x=2 \Rightarrow A=-2x=4 \Rightarrow B=)4 \\ |x-4|+C=2\ln|x-2|+2\ln 2 \int 4(x-2)(x-4)dx=\int(-2x-2+2x-4)dx=-2\ln \\ |x-4x-2|+C$$

$$(3x+4x^2+x)dx \quad (4) \int$$

$$3x+4x^2+x=3x+4x(x+1)=Ax+Bx+1 \Rightarrow 3x+4=A(x+1)+Bxx=0 \Rightarrow A=4x \\ |x+1|+C|x|-\ln=-1 \Rightarrow B=-13x+4x^2+x dx=\int(4x+-1x+1)dx=4\ln$$

$$(x^2x^2-4dx \quad (5) \int$$

$$x^2x^2-4dx=\int(1+4x^2-4)dx4x^2-4=4(x-2)(x+2)=Ax-2+Bx+2 \Rightarrow 4=\int \\ A(x+2)+B(x-2)x=2 \Rightarrow A=1x=-2 \Rightarrow B=-1 \int x^2x^2-4dx=\int(1+1x-2+-1x \\ |x-2x+2|+C|x+2|+C=x+\ln|x-2|-\ln+2)dx=x+\ln$$

$$(3x-6x^2+x-2dx \quad (6) \int$$

$$3x-6x^2+x-2=3x-6(x+2)(x-1)=Ax+2+Bx-1 \Rightarrow 3x-6=A(x-1)+B(x \\ +2)x=-2 \Rightarrow A=4x=1 \Rightarrow B=-1x3x-6x^2+x-2dx=\int(4x+2+-1x-1)dx=4$$

$$|x-1|+C|x+2|-\ln|\ln$$

$$(4x+104x^2-4x-3dx \ (7\int)$$

$$\begin{aligned} 4x+104x^2-4x-3 &= 4x+10(2x-3)(2x+1) = A(2x-3)+B(2x+1) \Rightarrow 4x+10=A \\ (2x+1)+B(2x-3)x &= 32 \Rightarrow A=4x=-12 \Rightarrow B=-2 \int 4x+104x^2-4x-3dx = \int \\ |2x+1|+C|2x-3|-\ln|42x-3+22x+1|dx &= 2\ln \end{aligned}$$

$$(2x^2+9x-11x^3+2x^2-5x-6dx \ (8\int)$$

$$\begin{aligned} 2x^2+9x-11x^3+2x^2-5x-6 &= 2x^2+9x-11(x-2)(x+1)(x+3) = Ax-2+B \\ x+1+Cx+3 &\Rightarrow 2x^2+9x-11 = A(x+1)(x+3)+B(x-2)(x+3)+C(x-2)(x+1) \\ x=2 \Rightarrow A=1 &x=-1 \Rightarrow B=3x=-3 \Rightarrow C=-2 \int 2x^2+9x-11x^3+2x^2-5x-6dx = \int \\ |x+3|+C|x+1|-2\ln|x-2|+3\ln(1x-2+3x+1+-2x+3)dx &= \ln \end{aligned}$$

$$(4xx^2-2x-3dx \ (9\int)$$

$$\begin{aligned} 4xx^2-2x-3 &= 4x(x-3)(x+1) = Ax-3+Bx+1 \Rightarrow 4x = A(x+1)+B(x-3) \\ x|x-3|+\ln \Rightarrow A=3 &x=-1 \Rightarrow B=1 \int 4xx^2-2x-3dx = \int (3x-3+1x+1)dx = 3\ln \\ +1|+C & \end{aligned}$$

$$(8x^2-19x+1(2x+1)(x-2)2dx \ (10\int)$$

$$\begin{aligned} 8x^2-19x+1(2x+1)(x-2)2 &= A(2x+1)+B(x-2)+C(2x+1) \\ (x-2)^2+B(2x+1)(x-2)+C(2x+1)x &= -12 \Rightarrow A=2x=2 \Rightarrow C=-1x=0 \Rightarrow 1=4 \\ A-2B+C \Rightarrow B=3 &\int 8x^2-19x+1(2x+1)(x-2)2dx = \int (22x+1+3x-2+-1(x \\ |x-2|+1x-2+C|2x+1|+3\ln-2)2)dx = \ln \end{aligned}$$

$$(9x^2-3x+29x^2-4dx \ (11\int)$$

$$\begin{aligned} 9x^2-3x+29x^2-4dx &= \int (1+6-3x)9x^2-4dx \\ 6-3x9x^2-4 &= 6-3x(3x-2) \\ (3x+2)=A &3x-2+B3x+2 \Rightarrow 6-3x = A(3x+2)+B(3x-2)x=23 \Rightarrow A=1x=- \\ |323 \Rightarrow B=-2 &\int 9x^2-3x+29x^2-4dx = \int (1+13x-2+-23x+2)dx = x+13\ln \\ |3x+2|+Cx-2|-23\ln & \end{aligned}$$

$$(x^3+2x^2+2x^2+x dx \ (12\int)$$

$$x^3 + 2x^2 + 2x^2 + x dx = \int (x+1+2-xx^2+x) dx \\ 2-x = A(x+1) + Bxx = 0 \Rightarrow A=2, x=-1 \Rightarrow B=-3 \\ \int x^3 + 2x^2 + 2x^2 + x dx = Ax + Bx^2 + Cx^3 + D \\ |x+1| + C|x| - 3\ln x = \int (x+1+2x+-3x+1) dx = 12x^2 + x + 2\ln$$

$$(x^2 + x + 23 - 2x - x^2) dx \quad (13)$$

$$x^2 + x + 23 - 2x - x^2 dx = \int (-1 + 5 - x - x^2 - 2x + 3) dx \\ 5(x+3)(x-1) = Ax + 3 + Bx - 1 \Rightarrow x - 5 = A(x-1) + B(x+3) \\ x = -3 \Rightarrow A = 2, x = 1 \Rightarrow B = -1 \\ \int x^2 + x + 23 - 2x - x^2 dx = \int (-1 + 2x + 3 + -1x - 1) dx = -x + 2\ln \\ |x-1| + C - \ln$$

$$(2x - 4(x^2 + 4)(x + 2)) dx \quad (14)$$

$$2x - 4(x^2 + 4)(x + 2) = Ax + 2 + Bx + Cx^2 + 4 \Rightarrow 2x - 4 = A(x^2 + 4) + (Bx + C)(x + 2) \\ x = -2 \Rightarrow A = -1, x = 0 \Rightarrow -4 = 4A + 2C \Rightarrow C = 0, x = 1 \Rightarrow -2 = 5A + 3B + 3C \Rightarrow B = 1 \\ \int |x^2 + 4| + |x + 2| + 12\ln 2x - 4(x^2 + 4)(x + 2) dx = \int (-1x + 2 + xx^2 + 4) dx = -\ln \\ C$$

$$(x^3 - 4x^2 - 2x^3 + x^2) dx \quad (15)$$

$$x^3 - 4x^2 - 2x^3 + x^2 dx = \int (1 + -5x^2 - 2x^3 + x^2) dx \\ -5x^2 - 2x^3(x+1) = Ax + Bx^2 + Cx + 1 \Rightarrow -5x^2 - 2 = Ax(x+1) + B(x+1) + Cx^2 \\ x = -1 \Rightarrow C = -7, x = 1 \Rightarrow -7 = 2A + 2B + C \Rightarrow A = 2 \\ \int x^3 - 4x^2 - 2x^3 + x^2 dx = |x+1| + C|x| + 2x - 7\ln \int (1 + 2x + -2x^2 + -7x + 1) dx = x + 2\ln$$

$$(x^2 - 5x - 12x^2) dx \quad (16-3)$$

$$x^2 - 5x - 12x^2 = x - 3 \\ 12x^2 + 5x - 2 = x - 3(4x - 1)(3x + 2) = A4x - 1 + B3x - 3 \\ + 2 \Rightarrow x - 3 = A(3x + 2) + B(4x - 1) \\ x = 14 \Rightarrow A = -1, x = -23 \Rightarrow B = 1 \\ \int 3 - x^2 - 5x - 1 \\ |3x + 2| + C|4x - 1| + 13\ln 2x^2 dx = \int (-14x - 1 + 13x + 2) dx = -14\ln$$

$$(3x^3 - x^2 + 12x - 6x^4 + 6x^2) dx \quad (17)$$

$$3x^3 - x^2 + 12x - 6x^4 + 6x^2 = 3x^3 - x^2 + 12x - 6x^2(x^2 + 6) = Ax + Bx^2 + Cx + D \\ x^2 + 6 \Rightarrow 3x^3 - x^2 + 12x - 6 = Ax(x^2 + 6) + B(x^2 + 6) + (Cx + D)(x^2) \\ x = 0 \Rightarrow B = -1 \\ x = 1 \Rightarrow 8 = 7A + 7B + C + D \dots \dots \dots \quad (1) \\ x = -1 \Rightarrow -22 = -7A + 7B - C + D \dots \dots \dots$$

$$(2)x=2 \Rightarrow 38=20A+10B+8C+4D \dots\dots (3)$$

بجمع (1)، (2) ينتج أن: $B=-14$ ، وبتعويض $B=-14$ ، نجد أن $D=0$
 وبطرح (2) من (1) ينتج أن $C=15-7A$ أي أن $14A+2C=30$ أي أن
 بالتعويض في (3) ينتج أن:

$$20A-10+8(15-7A)=38-36A=-72 \Rightarrow A=2 \\ C=15-7(2)=1 \\ \int |x^2+6| + C|x| + 1x + 12 \ln x - 6x^4 + 6x^2 dx = \int (2x^2 - 1x^2 + x^2 + 6) dx = 2 \ln (5x^2 - 2(x-2)^2) + C \quad (18)$$

$$5x-2(x-2)^2=Ax-2+B(x-2)^2 \Rightarrow 5x-2=A(x-2)+Bx=2 \Rightarrow B=8x=0 \Rightarrow -2 \\ |x-2|-8x=-2A+B \Rightarrow A=5 \int 5x-2(x-2)^2 dx = \int (5x-2+8(x-2)^2) dx = 5 \ln -2+C$$

ملاحظة: يمكن حل هذا التكامل بالتعويض $u=x-2$

كما يمكن حله بالأجزاء حيث: $u=5x-2, dv=(x-2)^2$

أجد قيمة كل من التكاملات الآتية:

$$\int (246+3x-x^2x^3+2x^2) dx \quad (19)$$

$$6+3x-x^2x^3+2x^2=6+3x-x^2x^2(x+2)=Ax+Bx^2+Cx^3+2 \Rightarrow 6+3x-x^2=A \\ x(x+2)+B(x+2)+C(x^2)x=0 \Rightarrow B=3x=-2 \Rightarrow C=-1x=1 \Rightarrow 8=3A+3B+C \Rightarrow A \\ |x+2| \mid 24=0 \int 246+3x-x^2x^3+2x^2 dx = \int 24(3x^2+1x+2) dx = (-3x-\ln 234)=34+\ln 6+32+\ln=-34-\ln$$

$$\int (1/31/39x^2+49x^2-4) dx \quad (20)$$

$$9x^2+49x^2-4=1+89x^2-489x^2-4=8(3x-2)(3x+2)=A3x-2+B3x+2 \\ \Rightarrow 8=A(3x+2)+B(3x-2)x=23 \Rightarrow A=2x=-23 \Rightarrow B=-2 \int -13139x^2+49x^2 \\ |3x+2| \mid 3x-2|-23 \ln -4 dx = \int -1313(1+23x-2+-23x+2) dx = (x+23 \ln 3=23-13+13-23 \ln |3x-23x+2|) \mid -1313=13+23 \ln |-1313=(x+23 \ln 343 \ln$$

$$(0117 - 5x(2x+3)(2-x)2dx \quad (21)$$

$$17 - 5x(2x+3)(2-x)2 = A2x+3 + B2-x+C(2-x)2 \Rightarrow 17 - 5x = A(2-x)2 + B(2-x)(2x+3) + C(2x+3)x = -32 \Rightarrow A=2x=2 \Rightarrow C=1x=0 \Rightarrow 17 = 4A+6B+3C \Rightarrow B=1 \int 0117 - 5x(2x+3)(2-x)2dx = \int 01(22x+3+12-x+1(2-x)2)dx = (\ln 1032 - 12 = 12 + \ln 3 + \ln 5 + 1 - \ln |2-x| + 12-x) | 01 = \ln |2x+3| - \ln$$

$$(14416x^2+8x-3dx \quad (22)$$

$$416x^2+8x-3 = 4(4x-1)(4x+3) = A4x-1 + B4x+3 \Rightarrow 4 = A(4x+3) + B(4x-1)x=14 \Rightarrow A=1x=-34 \Rightarrow B=-1 \int 14416x^2+8x-3dx = \int 14(14x-1+ -14x^2|4x-14x+3|) | 14 = 14(\ln |4x+3|) | 14 = (14\ln |4x-1| - 14\ln +3)dx = (14\ln 351937) = 14\ln 519 - \ln$$

$$(345x+5x^2+x-6dx \quad (23)$$

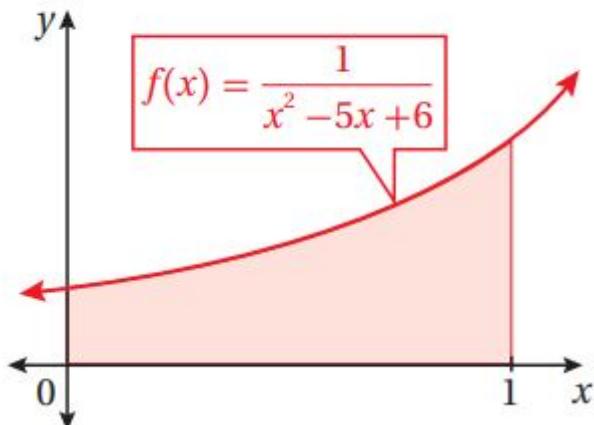
$$5x+5x^2+x-6 = 5x+5(x-2)(x+3) = Ax-2+Bx+3 \Rightarrow 5x+5 = A(x+3) + B(x-2)x=2 \Rightarrow A=3x=-3 \Rightarrow B=2 \int 345x+5x^2+x-6dx = \int 34(3x-2+2x+3)dx = 9896 = \ln 7 - 2\ln 2 + 2\ln |x+3| | 34 = 3\ln |x-2| + 2\ln = (3\ln$$

$$(344x^3-4x^2+4xdx \quad (24)$$

$$bbb4x^3-4x^2+4x = 4x(x-2)^2 = Ax+Bx-2+C(x-2)^2 \Rightarrow 4 = A(x-2)^2 + Bx(x-2) + Cx^2 = 0 \Rightarrow A=1x=2 \Rightarrow C=2x=1 \Rightarrow 4 = A-B+C \Rightarrow B=-1 A=\int 344x^3-4x^2dx = (|x|-2x-2) | 34 = (|x|-2x-2+4xdx = \int 34(1x+ -1x-2+2(x-2)^2)dx = (\ln 233 + 2 = 1 + \ln 2 - 1 - \ln |x-2| - 2x-2) | 34 = \ln n$$

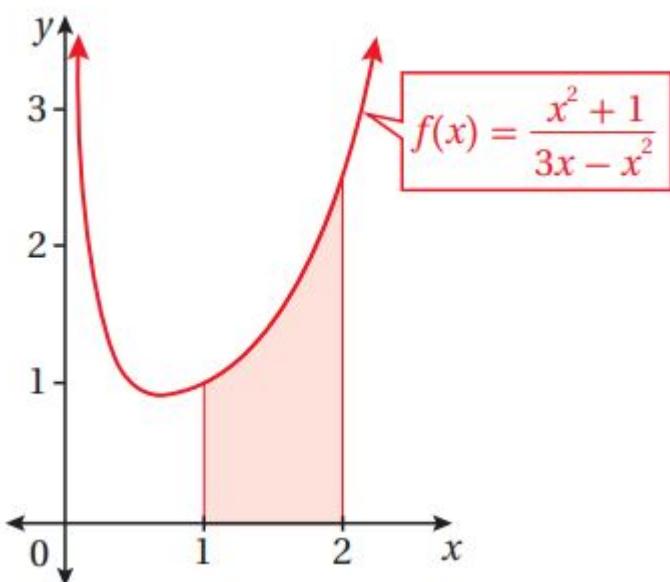
أحد مساحة المنطقة المظللة في كل من التمثيلين البيانيين الآتيين:

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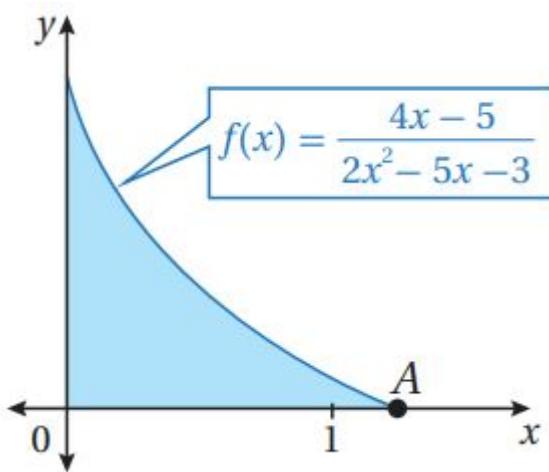


$$A = \int_0^1 x^2 - 5x + 6 dx = x^2 - 5x + 6 = 1(x-3)(x-2) = Ax - 3 + Bx - 2 \Rightarrow 1 = A(x-2) + B(x-3) \Rightarrow A = 1, x = 2 \Rightarrow B = -1 \\ A = \int_0^1 x^2 - 5x + 6 dx = \int_0^1 (1x^2 - 3 + 4) dx = \ln 2 - \ln|x-3| - \ln|x-2| \Big|_0^1 = \ln|x-3| - \ln|x-2| = (\ln$$

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$$A = \int_1^2 x^2 + 13x - x^2 dx = x^2 + 13x - x^2 = -1 + 3x + 13x - x^2 = 3x^2 + 1 + 13x = Ax + Bx^2 - x \Rightarrow 3x + 1 = A(3-x) + Bx^2 = 0 \Rightarrow A = 13 = -2 + 13 \ln 2 \\ \int_1^2 x^2 + 13x - x^2 dx = \int_1^2 (-1 + 10x + 13x - x^2) dx = 3 \Rightarrow B = -113 \ln 2 + 103 \ln x$$



يبين الشكل المجاور جزءاً من منحنى
الاقتران: $f(x) = 4x - 5 - 2x^2 - 5x - 3$

(27) أجد إحداثي النقطة A.

$$(f(x) = 0 \Rightarrow 4x - 5 = 0 \Rightarrow x = 5/4 \Rightarrow A(5/4, 0))$$

(28) أجد مساحة المنطقة المظللة.

$$49243 = \ln 498 - \ln |2x^2 - 5x - 3|_{0.5}^{5/4} = \ln A = \int_{0.5}^{5/4} 4x - 5 - 2x^2 - 5x - 3 dx = \ln$$

ملاحظة: البسط هو مشتقة المقام، فلا داعي لتجزئة الكسر.

أجد كلاً من التكاملات الآتية:

$$\int x dx (29x + \cos 2x \cos \sin x)$$

$$\begin{aligned} xu + u^2 x dx &= \int \sin x + \cos 2x \cos x \int \sin x dx dx = du - \sin x \Rightarrow du dx = -\sin u = \cos \\ x &= \int -1u + u^2 du - 1u + u^2 = -1u(1+u) = Au + B \\ 1 &\Rightarrow -1 = A(1+u) \times du - \sin \\ x \cos + Buu &= 0 \Rightarrow A = -1u = -1 \Rightarrow B = 1 \int -1u + u^2 du = \int (-1u + 1 + u) du \Rightarrow \int \sin \\ x | + C &= \ln |\cos|1+u| + C_1 + \cos x | + C = \ln |1 + \cos x| + \ln |\cos x| dx = -\ln x + \cos 2 \\ x | + C | 1 + \sec n & \end{aligned}$$

$$\int (1x^2 + xx dx) (30)$$

$$\begin{aligned} u = x \Rightarrow u^2 = x \Rightarrow dx = 2udu &\int 1x^2 + xx dx = \int 1u^4 + u^3 2udu = \int 2u^3 + u^2 du = 2u^3 + \\ u^2 &= 2u^2(u+1) = Au + Bu^2 + Cu + 1 \Rightarrow 2 = Au(u+1) + B(u+1) + Cu^2u = 0 \Rightarrow B = 2 \\ u = -1 \Rightarrow C = 2u &= 1 \Rightarrow 2 = 2A + 2B + C \Rightarrow A = -2 \int 2u^3 + u^2 du = \int (-2u + 2u^2 + 2u \\ |u+1| - 2u + C|u+1| + C &\Rightarrow \int 1x^2 + xx dx = 2\ln|u| - 2u + 2\ln + 1) du = -2\ln \end{aligned}$$

$$\int (e^{2x} e^{2x} + 3ex + 2dx) (31)$$

$$u = ex \Rightarrow du/dx = ex = u \Rightarrow dx = du/u \int e^{2x} e^{2x+3ex+2} dx = \int u^2 u^2 + 3u + 2 \times du$$

$$u = \int uu^2 + 3u + 2 du = u(u+1)(u+2) = Au+1+Bu+2 \Rightarrow u = A(u+2) + B(u+1)$$

$$u = -1 \Rightarrow A = -1, u = -2 \Rightarrow B = 2 \int uu^2 + 3u + 2 du = \int (-1u+1+2u+2)$$

$$(ex(ex+1)+2\ln|u+2|+C) \Rightarrow \int e^{2x} e^{2x+3ex+2} dx = -\ln|u+1| + 2\ln u = -\ln|u+2| + C$$

$$(x-4)dx (32x(\sin 2x \sin x \cos x))$$

$$\int xu(u^2-x-4)dx = \int \cos x (\sin 2x \sin x \cos x) dx = ducosx \Rightarrow du/dx = \cos u = \sin x = \int 1/(u^2-4) du = 1/(u-2)(u+2) = Au + Bu - 2 + Cu + 4 \times du \cos 2 \Rightarrow 1 = A(u-2)(u+2) + B(u+2) + Cu(u-2)$$

$$u=0 \Rightarrow A=-14, u=2 \Rightarrow B=18, u=-2 \Rightarrow C=18 \int 1/(u^2-4) du = \int (-14u+18u-2+18u+2) du = -14\ln|x| + 18\ln|\sin x - 4| dx = -14\ln x (\sin 2x \sin |u+2| + C) \Rightarrow \int \cos u - 2 + 18\ln|x+2| + C |\sin 2| + 18\ln$$