

## إجابات أسئلة الدرس

### التكامل بالتعويض

(١) اكتب التعويض المناسب لإيجاد قيمة كل تكامل من التكاملات الآتية:

(أ)  $\int (1-2s)(s-2)^4 ds$  (ب)  $\int 6s^2 \sqrt{(2-s)^2} ds$

(ج)  $\int (2s-3)(s^2-2s) ds$  (د)  $\int \frac{s^3-9}{(s^2-2s)^2} ds$

### الحل

(أ)  $\int (1-2s)(s-2)^4 ds$

ص =  $s-2 \Rightarrow ds = ds$   $\Rightarrow 1-2s = 1-2(v+2) = -4-2v$

$\int (-4-2v)v^4 dv = \int (-4v^4-2v^5) dv = -\frac{4v^5}{5} - \frac{2v^6}{6} + C = -\frac{4(s-2)^5}{5} - \frac{(s-2)^6}{3} + C$

(ب)  $\int 6s^2 \sqrt{(2-s)^2} ds = \int 6s^2 |2-s| ds$

(ب)  $\int 6s^2 \sqrt{(2-s)^2} ds$

ص =  $2-s \Rightarrow ds = -ds$   $\Rightarrow 6s^2 = 6(2-v)^2 = 6(4-4v+v^2) = 24-24v+6v^2$

$\int (24-24v+6v^2)(-dv) = \int (-24+24v-6v^2) dv = -24v + 12v^2 - 2v^3 + C = -24(2-s) + 12(2-s)^2 - 2(2-s)^3 + C$

$$p + \frac{u}{\sqrt{u}} = p + \frac{u^{1+\frac{1}{2}}}{1+\frac{1}{2}}$$

$$p + \frac{\sqrt{u}}{\frac{1}{2}} =$$

$$p + \frac{\sqrt{2-3x}}{\frac{1}{2}} =$$

(ج)  $\int (2-3x)^{\frac{1}{2}} dx = \frac{2-3x}{-3} \cdot \frac{2}{3} + C$

$$ص = \frac{2-3x}{-3} \Rightarrow 3x - 2 = \frac{3}{ص}$$

$$\cdot 3x = \frac{3}{ص} + 2$$

$$\frac{3x}{3x-2} = \frac{3}{ص} + 2$$

$$p + \frac{3}{3x-2} = \frac{3}{ص} + 2$$

$$p + \frac{3}{2-3x} = \frac{3}{ص} + 2$$

(د)  $\int \frac{9-x^2}{(x^2-6)^2} dx$

$$\Leftrightarrow 6-x^2 = \frac{3}{ص} \Leftrightarrow x^2 - 6 = \frac{3}{ص}$$

$$\cdot x^2 = \frac{3}{ص} + 6$$

$$= \frac{3}{ص} + \frac{9-x^2}{ص^2}$$

$$= \frac{3}{ص} + \frac{3}{ص^2} \cdot \frac{3-x}{3-x}$$

$$p + \frac{3}{1-x} = p + \frac{3}{1+x}$$

$$p + \frac{3}{(x^2-6)^2} = p + \frac{3}{3x-2}$$

(٢) جد قيمة كل من التكاملات الآتية:

(أ)  $\int \sqrt{(2-s)^2} ds$   
 (ب)  $\int (1-s)(1-2s^2-s^4+s^6) ds$   
 (ج)  $\int 2 \sqrt{2-s} ds$   
 (د)  $\int 2s^2 \sqrt{1+s^4} ds$

**الحل**

(أ)  $\int \sqrt{(2-s)^2} ds = \int (2-s) ds = 2s - \frac{s^2}{2} + C$

(ب)  $\int (1-s)(1-2s^2-s^4+s^6) ds = \int (1-s-2s^3+2s^4-s^5+s^7) ds = s - \frac{s^2}{2} - \frac{2s^4}{4} + \frac{2s^5}{5} - \frac{s^6}{6} + \frac{s^8}{8} + C$

(ج)  $\int 2 \sqrt{2-s} ds = 2 \int (2-s)^{1/2} ds = 2 \cdot \frac{2(2-s)^{3/2}}{-3/2} = -\frac{8}{3} (2-s)^{3/2} + C$

(د)  $\int 2s^2 \sqrt{1+s^4} ds = \int 2s^2 (1+s^4)^{1/2} ds$   
 Let  $u = 1+s^4$ , then  $du = 4s^3 ds$   
 $\int 2s^2 \sqrt{1+s^4} ds = \frac{1}{2} \int \frac{1}{s} \sqrt{u} du = \frac{1}{2} \int u^{-1/2} du = \frac{1}{2} \cdot 2u^{1/2} = \sqrt{1+s^4} + C$

(ج)  $\int 2 \sqrt{2-s} ds = 2 \int (2-s)^{1/2} ds = 2 \cdot \frac{2(2-s)^{3/2}}{-3/2} = -\frac{8}{3} (2-s)^{3/2} + C$

(د)  $\int 2s^2 \sqrt{1+s^4} ds = \int 2s^2 (1+s^4)^{1/2} ds$

Let  $u = 1+s^4$ , then  $du = 4s^3 ds$   
 $\int 2s^2 \sqrt{1+s^4} ds = \frac{1}{2} \int \frac{1}{s} \sqrt{u} du = \frac{1}{2} \int u^{-1/2} du = \frac{1}{2} \cdot 2u^{1/2} = \sqrt{1+s^4} + C$

(ج)  $\int 2 \sqrt{2-s} ds = 2 \int (2-s)^{1/2} ds = 2 \cdot \frac{2(2-s)^{3/2}}{-3/2} = -\frac{8}{3} (2-s)^{3/2} + C$

٣) احسب قيمة كل من التكاملات الآتية:

أ)  $\int \sqrt{4s+1} ds$

ب)  $\int \frac{3s^2(1-s)^2}{s^2} ds$

ج)  $\int \frac{2s^2}{\sqrt{s^2-1}} ds$

د)  $\int \frac{s^2-3}{s^2(s^3-2)} ds$

**الحل**

أ)  $\int \sqrt{4s+1} ds = \int \sqrt{4(s+\frac{1}{4})} ds$

$$\int \sqrt{4(s+\frac{1}{4})} ds = \int \frac{1+\frac{1}{4}}{4 \times (1+\frac{1}{4})} ds$$

$$\int \frac{\sqrt{4(s+\frac{1}{4})}}{4} ds$$

$$\frac{1}{4} \left[ \sqrt{4(s+\frac{1}{4})} - \frac{1}{2} \ln |2\sqrt{s+\frac{1}{4}} - 1| \right] + C$$

$$\frac{1}{4} (2\sqrt{s+\frac{1}{4}} - \ln |2\sqrt{s+\frac{1}{4}} - 1|) + C$$

$$\frac{1}{x} (1 - 2x) = \frac{1}{3} = \frac{1}{3} \times 2x + \frac{1}{3}$$

$$(ب) \int_{-1}^1 x^2 (1 - x^2) dx = \text{مساحة}$$

$$(ج) \int_{-1}^1 x^2 \sqrt{1 - x^2} dx =$$

$$\int_{-1}^1 x^2 (1 - x^2)^{\frac{1}{2}} dx$$

$$\text{هنا } 1 - x^2 = \frac{1 - x^2}{1 - x^2} \Leftrightarrow x^2 = \frac{1 - x^2}{1 - x^2} \Leftrightarrow x^2 = 1 - x^2$$

$$\int_{-1}^1 x^2 \sqrt{1 - x^2} dx = \int_{-1}^1 \frac{1 - x^2}{\sqrt{1 - x^2}} dx$$

$$\int_{-1}^1 \frac{1 - x^2}{\sqrt{1 - x^2}} dx = \int_{-1}^1 \frac{1 - x^2}{1 - x^2} dx$$

$$\frac{2}{3} \left[ \sqrt{1 - x^2} \right]_{-1}^1 = \frac{2}{3} \left( \sqrt{1 - 1} - \sqrt{1 - 1} \right)$$

$$\left( \sqrt[3]{-1} - \sqrt[3]{1} \right) \frac{x}{2}$$

$$\left( -1 - 1 \right) \frac{x}{2}$$

$$\frac{x}{2} = 1 \times \frac{x}{2}$$

$$\int_1^2 \frac{x^2 - 2}{(x^3 - 6)^2} dx = \int_1^2 \frac{u^2 - 2}{(u^3 - 6)^2} \cdot \frac{1}{3} du$$

$$v = \frac{u^2 - 2}{u^3 - 6} \Rightarrow 3 - u^3 = \frac{2v}{u} \Rightarrow u^3 - 3 = \frac{2v}{u}$$

$$= \int_1^2 \frac{2v}{u} - \int_1^2 \frac{2v}{u^3} = \frac{2v}{u} - \frac{2v}{2u^2} = \frac{2v}{u} - \frac{v}{u^2}$$

$$\int_1^2 \frac{1}{u} = \int_1^2 \frac{1}{u} = \int_1^2 \frac{1}{1+u} = \int_1^2 \frac{1}{1+u} = \ln|1+u|$$

$$\frac{1}{2} - \frac{1}{2} = \frac{1}{1 \times 3 - 6} - \frac{1}{2 \times 3 - 6} = \int_1^2 \frac{1}{u^3 - 6}$$

$$\text{مفر} = \frac{1}{2} + \frac{1}{2} =$$

٤) إذا علمت أن ق(٨) = ٥، ق(٢٧) = ٦، فجد قيمة التكامل الآتي:  $\int_2^3 \frac{3^x - 2^x}{3^x - 2^x} dx$

**الحل**

$$v = \frac{3^x - 2^x}{3^x - 2^x} \Rightarrow \frac{dv}{dx} = \frac{3^x \ln 3 - 2^x \ln 2}{3^x - 2^x}$$

$$= \int_2^3 \frac{3^x \ln 3 - 2^x \ln 2}{3^x - 2^x} dx = \int_2^3 \frac{3^x \ln 3 - 2^x \ln 2}{3^x - 2^x} dx$$

$$\int_2^3 \frac{3^x \ln 3 - 2^x \ln 2}{3^x - 2^x} dx = \int_2^3 \frac{3^x \ln 3 - 2^x \ln 2}{3^x - 2^x} dx$$

$$0 - 6 - = (8 -) 5 - (27) 6 = (5 -) 5 - (6) 6$$

$$11 - =$$

(٥) إذا علمت أن  $\int_0^2 (س) دس = ٣$ ، فجد قيمة التكامل الآتي:  $\int_{-1}^2 ٨س ق(س٢ + ١) دس$

**الحل**

$$٥س = س٢ + ١ \Leftrightarrow س٢ = ٥س - ١ \Leftrightarrow دس = \frac{٥س}{٢س} = \frac{٥}{٢}$$

$$\int_{-1}^2 ٨س ق(س٢ + ١) دس = \int_{-1}^2 ٨س ق(٥س - ١) دس$$

$$\text{عند } س = -١ \Rightarrow س٢ = ٥(-١) - ١ = -٦ \Rightarrow ٢ = ١ + (-٦)$$

$$\text{عند } س = ٢ \Rightarrow س٢ = ٥(٢) - ١ = ٩ \Rightarrow ٥ = ١ + ٩$$

$$\int_{-1}^2 ٨س ق(س٢ + ١) دس = \int_{-٦}^9 ٤ دس = ٤(٩ - (-٦)) = ٤(١٥) = ٦٠$$

(٦) حل المسألة الواردة في بداية الدرس.  
جد قيمة التكامل الآتي:

$$\int_0^2 ٢س \sqrt{٩ + س٢} دس$$

**الحل**

$$\int_0^2 ٢س \sqrt{٩ + س٢} دس = \int_0^2 (٩ + س٢) دس$$

$$\Leftrightarrow ٥س = ٩ + س٢ \Leftrightarrow دس = \frac{٥س}{٢س} = \frac{٥}{٢}$$

$$\int_0^2 ٢س \sqrt{٩ + س٢} دس = \int_0^2 \frac{٥س}{٢} دس$$

$$\int_0^2 \frac{٥س}{٢} دس = \frac{٥}{٢} \int_0^2 س دس = \frac{٥}{٢} \left[ \frac{س٢}{٢} \right]_0^2 = \frac{٥}{٢} \left( \frac{٢٢}{٢} - 0 \right) = \frac{٥}{٢} (٢) = ٥$$

$$\left( \sqrt[٣]{٩ + ٢} - \sqrt[٣]{٩ + ٤} \right) \frac{٥}{٢} = \left( \sqrt[٣]{١١} - \sqrt[٣]{١٣} \right) \frac{٥}{٢}$$

$$\frac{١٩٦}{٣} = ٩٨ \times \frac{٥}{٢} =$$