السؤال الأول: ضع دائرة حول رمز الإجابة الصحيحة فيما يلي:

1- ناتج $\left(\int \left(2^{ex} + \ln 3 - \frac{e^x+e^{-x}}{e^x}\right) dx\right)$ هو

$a)$	$(\ln(3))x - x - \frac{e^{-2x+1}}{2} + \frac{2^{ex}}{2\ln(2)} + c$	$b)$	$(\ln(3))x - x + \frac{e^{-2x+1}}{2} + \frac{2^{ex+1}}{e\ln(2)} + c$
$c)$	$(\ln(3))x - x - \frac{e^{-2x}}{2} + \frac{2^{ex}}{2\ln(2)} + c$	$d)$	$(\ln(3))x - x + \frac{e^{-2x}}{2} + \frac{2^{ex}}{e\ln(2)} + c$

2- ناتج $\left(\int e \ln 2^x . dx\right)$ هو

$a)$	$e(\ln(2^x))x + c$	$b)$	$(\ln(2^{ex}))x + c$	$c)$	$\frac{1}{2}(\ln(2))x^2 + c$	$d)$	$\frac{1}{2}(\ln(2))ex^2 + c$

3- ناتج $\left(\int \frac{e^{3x}+4e^x+5}{e^x} dx\right)$ هو

$a)$	$\frac{e^{2x}}{2} + 4x - \frac{5}{e^x} + c$	$b)$	$\frac{e^{2x}}{2} + 4x + \frac{5}{e^x} + c$	$c)$	$\frac{e^{3x}}{3} + 4x - 5e^x + c$	$d)$	$\frac{e^{3x}}{3} + 4x + 5e^x + c$

4- ناتج $\left(\int \frac{7}{x \ln 3} dx\right)$ هو

$a)$	$\ln\lvert x\rvert + c$	$b)$	$7(\ln\lvert x\rvert - 3x) + c$	$c)$	$7\ln(\lvert x\rvert - 3) + c$	$d)$	$7\ln(x-3) + c$

5- ناتج $\left(\int \left(\frac{3}{e^{-4x}} + \sqrt[3]{e^{2x}}\right) dx\right)$ هو

$a)$	$\frac{3}{2}e^{\frac{2}{3}x} - \frac{3}{4}e^{-4x} + c$	$b)$	$\frac{3}{2}e^{\frac{2}{3}x} + \frac{3}{4}e^{-4x} + c$	$c)$	$\frac{3}{2}e^{\frac{2}{3}x} + \frac{3}{4}e^{4x} + c$	$d)$	$\frac{3}{2}e^{\frac{2}{3}x} - \frac{3}{4}e^{4x} + c$

6- ناتج $\left(\int \ln\left(\frac{e.e^x}{e^{3x}}\right) dx\right)$ هو

$a)$	$x - 2x^2 + c$	$b)$	$x - x^2 + c$	$c)$	$x + x^2 + c$	$d)$	$x + 2x^2 + c$

7- ناتج $\left(\int \frac{e^x+e^{2x}-e^{-e}}{e^{3x}} dx\right)$ هو

$a)$	$\frac{-e^{-2x}}{2} - e^{-x} + \frac{e^{-e-3x}}{3} + c$	$b)$	$\frac{-e^{-2x}}{2} - e^{-x} + \frac{(e^{-e-3x})x}{3} + c$
$c)$	$\frac{-e^{-2x}}{2} - e^{-x} - \frac{e^{-e-3x}}{3} + c$	$d)$	$\frac{-e^{-2x}}{2} - e^{-x} - \frac{(e^{-e-3x})x}{3} + c$

8- ناتج $\left(\int (5^{2x} + 7^{-8x}) dx\right)$ هو

$a)$	$\frac{5^{2x}}{2\ln 5} + \frac{7^{-8x}}{8\ln 7} + c$	$b)$	$\frac{5^{2x}}{2\ln 5} - \frac{7^{-8x}}{8\ln 7} + c$	$c)$	$\frac{5^{2x}}{\ln 5} - \frac{7^{-8x}}{\ln 7} + c$	$d)$	$\frac{5^{2x}}{\ln 5} + \frac{7^{-8x}}{\ln 7} + c$

9- ناتج $\left(\int \left(\sqrt{7^{5x-1}} - \sqrt{e^{1-x}} + 3\sqrt{x}\right) dx\right)$ هو

$a)$	$\frac{2(7^{(\frac{5x-1}{2})})}{5\ln 7} - 2e^{(\frac{1-x}{2})} + \frac{4}{3}\sqrt{x^3} + c$	$b)$	$\frac{2(7^{(\frac{5x-1}{2})})}{5\ln 7} + 2e^{(\frac{1-x}{2})} + \frac{4}{3}\sqrt{x^3} + c$
$c)$	$\frac{5(7^{(\frac{5x-1}{2})})}{2\ln 7} - 2e^{(\frac{1-x}{2})} + 2\sqrt{x^3} + c$	$d)$	$\frac{5(7^{(\frac{5x-1}{2})})}{2\ln 7} + 2e^{(\frac{1-x}{2})} + 2\sqrt{x^3} + c$

10- ناتج $\left(\int \left(\frac{4^x+6^x}{2^x}\right) dx\right)$ هو

$a)$	$\frac{2^x}{\ln 2} + \frac{3^x}{\ln 3} + c$	$b)$	$\frac{2^x}{\ln 2} - \frac{3^x}{\ln 3} + c$	$c)$	$\frac{2^x \ln 4}{\ln 2} + \frac{3^x \ln 6}{\ln 3} + c$	$d)$	$\frac{2^x \ln 4}{\ln 2} + \frac{3^x \ln 6}{\ln 3} + c$

11- ناتج $\left(\int \left(\frac{2^{(x+1)}-5^{(x+1)}}{10^x}\right) dx\right)$ هو

$a)$	$\frac{2}{5^x \ln 5} + \frac{5}{2^x \ln 2} + c$	$b)$	$\frac{2}{5^x \ln 5} - \frac{5}{2^x \ln 2} + c$	$c)$	$\frac{2}{5^x \ln 2} - \frac{5}{2^x \ln 5} + c$	$d)$	$\frac{2(\frac{1}{5})^x}{\ln\frac{1}{5}} + \frac{5(\frac{1}{2})^x}{\ln\frac{1}{2}} + c$

12- ناتج $\left(\int (\pi^{2x}) dx\right)$ هو

$a)$	$\frac{\pi^{2x+1}}{2\ln\pi} + c$	$b)$	$\frac{\pi^{3x}}{3\ln\pi} + c$	$c)$	$\frac{\pi^{2x}}{2\ln\pi} + c$	$d)$	$\frac{\pi^{2x}}{\ln 2\pi} + c$

13- ناتج $\left(\int (2^x + 3^x)^2 dx\right)$ هو

$a)$	$\frac{2^{2x}}{2\ln 2} + \frac{6^x}{\ln 6} + \frac{3^{2x}}{2\ln 3} + c$	$b)$	$\frac{2^{2x}}{2\ln 2} + 2\frac{6^x}{\ln 6} + \frac{3^{2x}}{2\ln 3} + c$
$c)$	$\frac{2^{2x}}{2\ln 2} + \frac{6^x}{\ln 6} + \frac{3^{2x}}{\ln 3} + c$	$d)$	$\frac{2^{2x}}{\ln 2} + 2\frac{6^x}{\ln 6} + \frac{3^{2x}}{\ln 3} + c$

14- ناتج $(\int((2^x+3^{2x})(2^x-3^{2x}))\,dx)$ هو

$a)$	$\frac{2^x}{\ln 2}-\frac{3^{2x}}{2\ln 3}+c$	$b)$	$\frac{2^{x+1}}{2\ln 2}-\frac{3^{2x+1}}{2\ln 3}+c$	$c)$	$\frac{2^{2x}}{2\ln 2}-\frac{3^{4x}}{4\ln 3}+c$	$d)$	$\frac{2^{2x+1}}{2\ln 2}+\frac{3^{4x+1}}{4\ln 3}+c$

15- ناتج $(\int((2^x.3^{2x})+(e^x.3^{2x}))\,dx)$ هو

$a)$	$\frac{18^x}{\ln 18}+\frac{(3e)^{2x}}{2\ln(3e)}+c$	$b)$	$\frac{9^x}{\ln 9}+\frac{(9e)^x}{\ln(9e)}+c$	$c)$	$\frac{18^x}{\ln 18}+\frac{9e^x}{\ln(e)}+c$	$d)$	$\frac{18^x}{\ln 18}+\frac{(9e)^x}{\ln(9e)}+c$

16- ناتج $(\int((2+e)^x+(2+\pi)^x)\,dx)$ هو

$a)$	$\frac{(2+e)^x}{\ln(2)}+\frac{(2+\pi)^x}{\ln(2)}+c$	$b)$	$\frac{(2)^{2x}}{2\ln(2)}+\frac{(e+\pi)^x}{\ln(e+\pi)}+c$	$c)$	$\frac{(2+e)^x}{\ln(2e)}+\frac{(2+\pi)^x}{\ln(2\pi)}+c$	$d)$	$\frac{(2+e)^x}{\ln(2+e)}+\frac{(2+\pi)^x}{\ln(2+\pi)}+c$

17- ناتج $(\int(x^e+x^\pi)\,dx)$ هو

$a)$	$\frac{x^{e+1}}{e}+\frac{x^{\pi+1}}{\pi}+c$	$b)$	$\frac{x^{e+1}}{\ln(e+1)}+\frac{x^{\pi+1}}{\ln(\pi+1)}+c$	$c)$	$\frac{x^{e+1}}{e+1}+\frac{x^{\pi+1}}{\pi+1}+c$	$d)$	$x^e+\frac{x^\pi}{\ln\pi}+c$

☒ اذا كان $(I=\int(\frac{e^{2x}-1}{e^x+1})\,dx)$ ، اجب عن الفقرات (19،18)

18- ناتج $(\int(\frac{e^{2x}-1}{e^x+1})\,dx)$ هو

$a)$	e^x-x+c	$b)$	e^x+x+c	$c)$	e^x+c	$d)$	$1-e^x+c$

19- قيمة المقدار $(\int_0^1\int(\frac{e^{2x}-1}{e^x+1})\,dx)$ هي

$a)$	$1-e$	$b)$	$e-1$	$c)$	e	$d)$	$e-2$

20- ناتج $(\int(x^3.e^{\ln x})\,dx)$ هو

$a)$	$\frac{1}{4}ex^4+c$	$b)$	$\frac{1}{5e}x^5+c$	$c)$	$\frac{1}{5}x^5+c$	$d)$	$\frac{1}{4}x^4+c$

21- ناتج $(\int(e^x.\tan^{-1}(3))\,dx)$ هو

$a)$	$-\ln(\frac{e^x}{\cos(3)\ln 3})+c$	$b)$	$\tan^{-1}(3).e^x+c$	$c)$	$\ln(\frac{e^x}{\cos^{-1}(3)\ln 3})+c$	$d)$	$-\ln(\frac{e^x}{\cos^{-1}(3)})+c$

22- ناتج $(\int(e^{\ln x+1})\,dx)$ هو

$a)$	$\frac{e}{2}x^2+c$	$b)$	$x.e^{\ln x+1}+c$	$c)$	$\frac{1}{2}x^2+c$	$d)$	$\frac{1}{2}x^2+x+c$

☒ اذا كان $(w=\int(\frac{e^{3x}+1}{e^x+1})\,dx)$ ، اجب عن الفقرات (24،23)

23- قيمة (w) بدلالة $(e\,,e:$ العدد النيبيري$)$ هي

$a)$	$\frac{1}{2}e^{2x}+e^x-x+c$	$b)$	$\frac{1}{2}e^{2x}-e^x-x+c$	$c)$	$\frac{1}{2}e^{2x}-e^x+x+c$	$d)$	$\frac{1}{2}e^{2x}+e^x+x+c$

24- قيمة المقدار $(\int_0^1\int(\frac{e^{3x}+1}{e^x+1})\,dx)$ هي

$a)$	$\frac{1}{2}e^2-e+\frac{3}{2}$	$b)$	$\frac{1}{2}e^2+e-\frac{1}{2}$	$c)$	$\frac{1}{2}e^2+e+\frac{1}{2}$	$d)$	$\frac{1}{2}e^2-e+\frac{1}{2}$

25- ناتج $(\int(\frac{1}{2}e^{2\pi x-4})\,dx)$ هو

$a)$	$\frac{1}{4\pi}e^{2\pi x-3}+c$	$b)$	$\frac{1}{2\pi}e^{2\pi x-4}+c$	$c)$	$\frac{1}{\pi}e^{2\pi x-4}+c$	$d)$	$\frac{1}{4\pi}e^{2\pi x-4}+c$

26- ناتج $(\int(\frac{2}{4-3x})\,dx)$ هو

$a)$	$-\frac{2}{3}\ln\lvert 4-3x\rvert+c$	$b)$	$\frac{2}{3}\ln\lvert 4-3x\rvert+c$	$c)$	$-\frac{3}{2}\ln\lvert 4-3x\rvert+c$	$d)$	$\frac{3}{2}\ln\lvert 4-3x\rvert+c$

27- ناتج $(\int(\frac{x+5}{x^2+10x+5})\,dx)$ هو

$a)$	$-\frac{1}{2}\ln\lvert x^2+10x+5\rvert+c$	$b)$	$\frac{1}{2}\ln\lvert x^2+10x+5\rvert+c$
$c)$	$-2\ln\lvert x^2+10x+5\rvert+c$	$d)$	$2\ln\lvert x^2+10x+5\rvert+c$

28- ناتج $(\int(\frac{x^2+4x^5}{x^3+2x^6})\,dx)$ هو

$a)$	$-3\,ln\lvert x^2+4x^5\rvert+c$	$b)$	$3\,ln\lvert x^2+4x^5\rvert+c$
$c)$	$\frac{1}{3}ln\lvert x^3+2x^6\rvert+c$	$d)$	$-\frac{1}{3}ln\lvert x^3+2x^6\rvert+c$

29- ناتج $(\int(\frac{2e^x}{2+e^x})\,dx)$ هو

$a)$	$-\frac{1}{2}ln\lvert 2e^x\rvert+c$	$b)$	$\frac{1}{2}ln\lvert 2e^x\rvert+c$	$c)$	$-2\,ln\lvert 2+e^x\rvert+c$	$d)$	$2\,ln\lvert 2+e^x\rvert+c$

30- ناتج $(\int(\frac{e^x+1}{e^x+x})\,dx)$ هو

$a)$	$-\,ln\lvert e^x+x\rvert+c$	$b)$	$ln\lvert e^x+x\rvert+c$	$c)$	$-\,ln\lvert e^x+1\rvert+c$	$d)$	$ln\lvert e^x+1\rvert+c$

31- ناتج $(\int(\frac{e^x-e^{-x}}{e^x+e^{-x}})\,dx)$ هو

$a)$	$ln\lvert e^x+e^{-x}\rvert+c$	$b)$	$-ln\lvert e^x+e^{-x}\rvert+c$	$c)$	$ln\lvert e^x-e^{-x}\rvert+c$	$d)$	$-\,ln\lvert e^x-e^{-x}\rvert+c$

32- ناتج $(\int(\frac{dx}{e+x}))$ هو

$a)$	$ln\lvert x\rvert+c$	$b)$	$ln\lvert e+x\rvert+c$	$c)$	$ln\lvert e-x\rvert+c$	$d)$	$-\,ln\lvert e+x\rvert+c$

33- ناتج $(\int(\frac{dx}{\pi+x}))$ هو

$a)$	$ln\lvert\pi-x\rvert+c$	$b)$	$-ln\lvert\pi+x\rvert+c$	$c)$	$ln\lvert\pi+x\rvert+c$	$d)$	$-ln\lvert\pi-x\rvert+c$

34- ناتج $(\int(\frac{dx}{x\,ln\,x}))$ هو

$a)$	$ln\lvert ln\,x\rvert+c$	$b)$	$-ln\lvert ln\,x\rvert+c$	$c)$	$ln\lvert x\rvert+c$	$d)$	$-ln\lvert x\rvert+c$

35- ناتج $(\int(\frac{dx}{x\,ln\,x+x}))$ هو

$a)$	$ln\lvert ln\,x+1\rvert+c$	$b)$	$-ln\lvert ln\,x+1\rvert+c$	$c)$	$ln\lvert x+1\rvert+c$	$d)$	$-ln\lvert x+1\rvert+c$

36- ناتج $(\int(\frac{dx}{x^2+4x+4}))$ هو

$a)$	$\frac{1}{-x+2}+c$	$b)$	$\frac{1}{x-2}+c$	$c)$	$\frac{1}{x+2}+c$	$d)$	$\frac{1}{-x-2}+c$

37- ناتج $(\int(\frac{dx}{1+e^x}))$ هو

$a)$	$ln\lvert e^x+1\rvert+c$	$b)$	$-ln\lvert e^x+1\rvert+c$	$c)$	$ln\lvert e^{-x}+1\rvert+c$	$d)$	$-ln\lvert e^{-x}+1\rvert+c$

38- ناتج $(\int(\frac{dx}{1+e^{-x}}))$ هو

$a)$	$ln\lvert e^x+1\rvert+c$	$b)$	$-ln\lvert e^x+1\rvert+c$	$c)$	$ln\lvert e^{-x}+1\rvert+c$	$d)$	$-ln\lvert e^{-x}+1\rvert+c$

39- ناتج $(\int(\frac{dx}{1+2^{-x}}))$ هو

$a)$	$\frac{ln\lvert 2^x+1\rvert}{ln\,2}+c$	$b)$	$-\frac{ln\lvert 2^x+1\rvert}{ln\,2}+c$	$c)$	$\frac{ln\,2}{ln\lvert 2^{-x}+1\rvert}+c$	$d)$	$\frac{ln\lvert 2^{-x}+1\rvert}{ln\,2}+c$

40- ناتج $(\int(\frac{dx}{1+3^x}))$ هو

$a)$	$\frac{ln\lvert 3^{-x}+1\rvert}{ln\,3}+c$	$b)$	$-\frac{ln\lvert 3^{-x}+1\rvert}{ln\,3}+c$	$c)$	$\frac{ln\,3}{ln\lvert 3^x+1\rvert}+c$	$d)$	$-\frac{ln\lvert 3^x+1\rvert}{ln\,3}+c$

41- ناتج $(\int(\frac{e^x}{e^x+e^{-x}})dx)$ هو

$a)$	$\frac{ln\lvert e^{-2x}+1\rvert}{2}+c$	$b)$	$-\frac{ln\lvert e^{-2x}+1\rvert}{2}+c$	$c)$	$\frac{ln\lvert e^{2x}+1\rvert}{2}+c$	$d)$	$-\frac{ln\lvert e^{2x}+1\rvert}{2}+c$

42- ناتج $(\int(\frac{e^x}{e^{2x}+2e^x+1})dx)$ هو

$a)$	$-\frac{1}{e^x+1}+c$	$b)$	$-\frac{1}{e^x-1}+c$	$c)$	$\frac{1}{e^x+1}+c$	$d)$	$\frac{1}{e^x-1}+c$

43- ناتج $(\int(\frac{2^x}{4^x+2^{x+1}+1})dx)$ هو

$a)$	$-\frac{1}{(ln\,2)(2^x+1)}+c$	$b)$	$\frac{1}{(ln\,2)(2^x+1)}+c$	$c)$	$\frac{-1}{(2^x ln\,2+1)}+c$	$d)$	$\frac{-1}{(2^x ln\,2-1)}+c$

44- ناتج $(\int(\frac{3^x-1}{9^x-1})dx)$ هو

$a)$	$\frac{ln\|3^{-x}+1\|}{ln\,3}+c$	$b)$	$-\frac{ln\|3^{-x}+1\|}{ln\,3}+c$	$c)$	$-\frac{ln\|3^{x}-1\|}{ln\,3}+c$	$d)$	$-\frac{ln\|3^{x}+1\|}{ln\,3}+c$

45- ناتج $(\int(\frac{1}{\ln 2^x-1})dx)$ هو

$a)$	$\frac{ln\|x\,ln\,2-1\|}{ln\,2}+c$	$b)$	$-\frac{ln\|x\,ln\,2-1\|}{ln\,2}+c$	$c)$	$\frac{ln\|ln\,2-1\|}{ln\,2}+c$	$d)$	$-\frac{ln\|ln\,2-1\|}{ln\,2}+c$

46- ناتج $(\int((1+e^x)e^x)dx)$ هو

$a)$	$e^x+e^{2x}+c$	$b)$	$e^x+2e^{2x}+c$	$c)$	$e^x+\frac{1}{2}e^{2x}+c$	$d)$	$e^x+\frac{1}{3}e^{3x}+c$

47- ناتج $(\int(\frac{8}{2x+3})dx)$ هو

$a)$	$ln\|2x+3\|+c$	$b)$	$ln\left\|(2x+3)^4\right\|+c$	$c)$	$-ln\left\|(2x+3)^4\right\|+c$	$d)$	$ln\left\|\sqrt[4]{2x+3}\right\|+c$

48- ناتج $(\int(\frac{x}{4x^2+1})dx)$ هو

$a)$	$ln\left\|(4x^2+1)^8\right\|+c$	$b)$	$ln\left\|(4x^2+1)^4\right\|+c$	$c)$	$ln\left\|\sqrt[8]{4x^2+1}\right\|+c$	$d)$	$ln\left\|\sqrt[4]{4x^2+1}\right\|+c$

49- ناتج $(\int(4\csc(4x+3)\cot(4x+3))dx)$ هو

$a)$	$-\csc(4x+3)+c$	$b)$	$\csc(4x+3)+c$
$c)$	$-\frac{1}{4}csc(4x+3)+c$	$d)$	$-16\,csc(4x+3)+c$

50- ناتج $(\int(\frac{\cos(x)}{\sin^2(x)})dx)$ هو

$a)$	$\csc(x)+c$	$b)$	$-\csc(x)+c$	$c)$	$\cot(x)+c$	$d)$	$-\sec(x)+c$

51- ناتج $(\int_0^{\frac{\pi}{3}}(\sin(4x)-\cos(2x))dx)$ هو

$a)$	$\frac{3+2\sqrt{3}}{8}$	$b)$	$\frac{3-2\sqrt{3}}{8}$	$c)$	$\frac{-1-2\sqrt{3}}{8}$	$d)$	$\frac{1-2\sqrt{3}}{8}$

52- ناتج $(\int(\cos\theta(\tan\theta+\sec\theta))\,d\theta)$ هو

$a)$	$cos\,\theta+c$	$b)$	$sec\,\theta+csc\,\theta+c$	$c)$	$cos\,\theta+\theta+c$	$d)$	$\theta-cos\,\theta+c$

53- ناتج $(\int(3\sin(\frac{\theta}{3}))\,d\theta)$ هو

$a)$	$-\cos(\theta)+c$	$b)$	$-9\cos(\theta)+c$	$c)$	$-\cos\left(\frac{\theta}{3}\right)+c$	$d)$	$-9cos\left(\frac{\theta}{3}\right)+c$

54- ناتج $(\int(4-\cot x\csc x)dx)$ هو

$a)$	$4x-csc\,x+c$	$b)$	$4x-cot\,x+c$	$c)$	$4x+csc\,x+c$	$d)$	$4x+cot\,x+c$

55- ناتج $(\int(2-2\cos(2x)+\frac{2}{x}-\sqrt{2x})dx)$ هو

$a)$	$2x-2\,sin(2x)+2\,ln\|x\|-\frac{2\sqrt{2x^3}}{3}+c$	$b)$	$2x-2sin(2x)+2\,ln\|x\|-3\sqrt{2x^3}+c$
$c)$	$2x+sin(2x)+2\,ln\|x\|-\frac{2\sqrt{2x^3}}{3}+c$	$d)$	$2x-sin(2x)+2\,ln\|x\|-\frac{2\sqrt{2x^3}}{3}+c$

56- ناتج $(\int(sec^2(x))dx)$ هو

$a)$	$-\sec x\tan x+c$	$b)$	$\sec x\tan x+c$	$c)$	$\tan x+c$	$d)$	$-\tan x+c$

57- ناتج $(\int(4\,cos(4x)+e^{2x}-\sqrt[4]{e^{x-2}})dx)$ هو

$a)$	$-\sin(4x)+\frac{e^{2x}}{2}-4e^{\frac{x-2}{4}}+c$	$b)$	$\sin(4x)+\frac{e^{2x}}{2}-4e^{\frac{x-2}{4}}+c$
$c)$	$\sin(4x)+\frac{e^{2x}}{2}+4e^{\frac{x-2}{4}}+c$	$d)$	$-\sin(4x)+\frac{e^{2x}}{2}+4e^{\frac{x-2}{4}}+c$

58- ناتج $(\int(sec(x)\,tan(x))dx)$ هو

$a)$	$-sec\,x+c$	$b)$	$sec\,x+c$	$c)$	$tan\,x+c$	$d)$	$-tan\,x+c$

59- ناتج $(\int(2\sin(1-\frac{x}{2})+(sec^2(2x)(1+\sin(2x)))dx)$ هو

a)	$8\cos(1-\frac{x}{2})+\tan(2x)+sec(2x)+c$	b)	$-8\cos(1-\frac{x}{2})+\tan(2x)+sec(2x)+c$
c)	$4\cos(1-\frac{x}{2})+\tan(2x)+sec(2x)+c$	d)	$-4\cos(1-\frac{x}{2})+\tan(2x)+sec(2x)+c$

60- ناتج $(\int(e^{3-x}-\sin(3-x)+\cos(3-x))dx)$ هو

a)	$-e^{3-x}-\sin(3-x)-\cos(3-x)+c$	b)	$-e^{3-x}+\sin(3-x)+\cos(3-x)+c$
c)	$-e^{3-x}-\sin(3-x)+\cos(3-x)+c$	d)	$-e^{3-x}+\sin(3-x)-\cos(3-x)+c$

61- ناتج $(\int(\frac{\tan(3x)-\tan(4x)}{1+\tan(3x)\tan(4x)})dx)$ هو

a)	$\ln\lvert\sin(x)\rvert+c$	b)	$-\ln\lvert\sin(x)\rvert+c$	c)	$\ln\lvert\cos(x)\rvert+c$	d)	$-\ln\lvert\cos(x)\rvert+c$

62- ناتج $(\int(4\sin(4x)\cos(4x))dx)$ هو

a)	$-\frac{1}{4}\cos(8x)+c$	b)	$-\frac{1}{2}\cos(4x)+c$	c)	$-\frac{1}{4}\cos(4x)+c$	d)	$-\frac{1}{2}\cos(8x)+c$

63- ناتج $(\int(\frac{2\tan(3x)}{1-\tan^2(3x)})dx)$ هو

a)	$\frac{1}{6}\ln\lvert\cos(6x)\rvert+c$	b)	$\frac{-1}{6}\ln\lvert\cos(6x)\rvert+c$	c)	$\ln\lvert\cos(6x)\rvert+c$	d)	$-\ln\lvert\cos(6x)\rvert+c$

64- ناتج $(\int(\frac{1+\cos(2x)}{1-\cos(2x)})dx)$ هو

a)	$-\cot(x)+x+c$	b)	$-\cot(x)-x+c$	c)	$\cot(x)+x+c$	d)	$\cot(x)-x+c$

65- ناتج $(\int(\frac{2}{1-\cos(2x)})dx)$ هو

a)	$-\cot(x)+c$	b)	$-\cot(2x)+c$	c)	$\cot(x)+c$	d)	$\cot(2x)+c$

66- ناتج $(\int(\frac{4}{2-2\cos(2x)})dx)$ هو

a)	$-\tan(x)+c$	b)	$-\cot(2x)+c$	c)	$\cot(x)+c$	d)	$\tan(x)+c$

67- ناتج $(\int(\sin^2(2x)-\cos^2(2x))dx)$ هو

a)	$2\sin(2x)+c$	b)	$-2\sin(2x)+c$	c)	$\frac{1}{2}\sin(2x)+c$	d)	$-\frac{1}{2}\sin(2x)+c$

68- ناتج $(\int(4-8\sin^2(2x))dx)$ هو

a)	$\sin(4x)+c$	b)	$-\sin(4x)+c$	c)	$4x+\sin(4x)+c$	d)	$4x-\sin(4x)+c$

69- ناتج $(\int(3-6\cos^2(3x))dx)$ هو

a)	$3x+\sin(6x)+c$	b)	$-\frac{1}{2}\sin(3x)+c$	c)	$3x-\frac{1}{2}\sin(6x)+c$	d)	$-\frac{1}{2}\sin(6x)+c$

70- ناتج $(\int((\sin(3x)\cos(5x)-\cos(3x)\sin(5x))+((\cos(3x)\cos(5x)-\sin(3x)\sin(5x)))dx)$ هو

a)	$-\frac{1}{2}\cos(2x)+\frac{1}{8}\sin(8x)+c$	b)	$\frac{1}{2}\cos(2x)+\frac{1}{8}\sin(8x)+c$
c)	$\frac{1}{2}\cos(2x)-\frac{1}{8}\sin(8x)+c$	d)	$\frac{1}{8}\cos(8x)+\frac{1}{2}\sin(2x)+c$

71- ناتج $\int(\sin(3x)\sin(4x)-\cos(3x)\sin(4x))\,dx$ هو

a)	$\frac{1}{2}(\sin(x)+\cos(x))+\frac{1}{14}(\cos(7x)+\sin(7x))+c$
b)	$\frac{1}{2}(\sin(x)-\cos(x))+\frac{1}{14}(\cos(7x)+\sin(7x))+c$
c)	$\frac{1}{2}(\sin(x)+\cos(x))+\frac{1}{14}(\cos(7x)-\sin(7x))+c$
d)	$\frac{1}{2}(\sin(x)-\cos(x))+\frac{1}{14}(\cos(7x)-\sin(7x))+c$

72- ناتج $(\int(\sqrt{\frac{1-\cos\theta}{1+\cos\theta}})d\theta)$ هو

$a)$	$\frac{1}{2}ln\lvert cos(2\theta)\rvert + c$	$b)$	$\frac{-1}{2}ln\lvert cos(2\theta)\rvert + c$	$c)$	$2ln\left\lvert cos(\frac{\theta}{2})\right\rvert + c$	$d)$	$-2ln\left\lvert cos(\frac{\theta}{2})\right\rvert + c$

73- ناتج $(\int (sin(3x)\, cos(4x) - cos(3x)\, cos(4x))\, dx)$ هو

$a)$	$\frac{1}{2}(sin(x) - cos(x)) + \frac{1}{14}(cos(7x) + sin(7x)) + c$
$b)$	$\frac{1}{2}(sin(x) - cos(x)) + \frac{1}{14}(-cos(7x) - sin(7x)) + c$
$c)$	$\frac{1}{2}(cos(x) - sin(x)) + \frac{1}{14}(-cos(7x) - sin(7x)) + c$
$d)$	$\frac{1}{2}(sin(x) - cos(x)) + \frac{1}{14}(cos(7x) - sin(7x)) + c$

74- ناتج $(\int_0^{2\pi} (sin^2(\frac{x}{4}))dx)$ هو

$a)$	0	$b)$	$\pi + 2$	$c)$	$\pi - 2$	$d)$	π

75- ناتج $(\int \frac{dx}{1-cos(x)})$ هو

$a)$	$-cot(x) + csc(x) + c$	$b)$	$cot(x) - csc(x) + c$
$c)$	$-cot(x) - csc(x) + c$	$d)$	$cot(x) + csc(x) + c$

76- ناتج $(\int \frac{dx}{1+cos(x)})$ هو

$a)$	$-cot(x) + csc(x) + c$	$b)$	$cot(x) - csc(x) + c$
$c)$	$-cot(x) - csc(x) + c$	$d)$	$cot(x) + csc(x) + c$

77- ناتج $(\int \frac{dx}{1-sin(x)})$ هو

$a)$	$-tan(x) - sec(x) + c$	$b)$	$tan(x) - sec(x) + c$
$c)$	$-tan(x) + sec(x) + c$	$d)$	$tan(x) + sec(x) + c$

78- ناتج $(\int \frac{dx}{1+sin(x)})$ هو

$a)$	$-tan(x) - sec(x) + c$	$b)$	$tan(x) - sec(x) + c$
$c)$	$-tan(x) + sec(x) + c$	$d)$	$tan(x) + sec(x) + c$

79- ناتج $(\int (cos(x)(1 + csc^2(x))\, dx)$ هو

$a)$	$-sin(x) - csc(x) + c$	$b)$	$sin(x) + csc(x) + c$
$c)$	$sin(x) - csc(x) + c$	$d)$	$-sin(x) + csc(x) + c$

80- ناتج $(\int_0^{\frac{\pi}{4}}((sin(x) - 3\, cos(x))^2)dx)$ هو

$a)$	$\pi - 1$	$b)$	$\frac{5\pi}{4} - 1$	$c)$	$\pi + 1$	$d)$	$\frac{5\pi}{4} - 1$

81- ناتج $(\int_0^{\frac{\pi}{8}}(cos^2(2x) - 4\, sin^2(x)\, cos^2(x))\, dx)$ هو

$a)$	$\frac{1}{4}$	$b)$	$\frac{1}{8}$	$c)$	0	$d)$	1

82- ناتج $(\int_0^{\pi}(4cos^2(\frac{1}{2}x))\, dx)$ هو

$a)$	$2\pi - 1$	$b)$	2π	$c)$	π	$d)$	$\pi - 1$

83- ناتج $(\int_{\frac{\pi}{4}}^{\frac{\pi}{2}}(\frac{cot^2(x)}{1+cot^2(x)})\, dx)$ هو

$a)$	$\frac{-\pi-1}{8}$	$b)$	$\frac{-\pi+1}{8}$	$c)$	$\frac{\pi-2}{8}$	$d)$	$\frac{\pi+2}{8}$

84- ناتج $\left(\int \left(\frac{sin(2x)}{1+cos(2x)}\right) dx\right)$ هو

$a)$	$-\frac{1}{2} ln\|1+cos(2x)\|+c$	$b)$	$\frac{1}{2} ln\|1+cos(2x)\|+c$
$c)$	$-\frac{1}{2} ln\|1-cos(2x)\|+c$	$d)$	$\frac{1}{2} ln\|1-cos(2x)\|+c$

85- ناتج $\left(\int (cot(x))\, dx\right)$ هو

$a)$	$-ln\|sin(x)\|+c$	$b)$	$ln\|sin(x)\|+c$	$c)$	$ln\|cos(x)\|+c$	$d)$	$-ln\|cos(x)\|+c$

86- ناتج $\left(\int (csc(x))\, dx\right)$ هو

$a)$	$-ln\|csc(x)+cot(x)\|+c$	$b)$	$-ln\|-csc(x)+cot(x)\|+c$
$c)$	$-ln\|csc(x)-cot(x)\|+$	$d)$	$-ln\|-csc(x)-cot(x)\|+c$

87- ناتج $\left(\int \left(\frac{sec(x)}{sin(x)-cos(x)}\right) dx\right)$ هو

$a)$	$-ln\|1-tan(x)\|+c$	$b)$	$-ln\|tan(x)-1\|+c$
$c)$	$ln\|1-tan(x)\|+c$	$d)$	$ln\|tan(x)-1\|+c$

88- ناتج $\left(\int \left(\frac{sin^3(x)+cos^3(x)}{sin(x)+cos(x)}\right) dx\right)$ هو

$a)$	$x-\frac{1}{2}\cos(2x)+c$	$b)$	$x+\frac{1}{2}cos(2x)+c$	$c)$	$x-\frac{1}{4}\cos(2x)+c$	$d)$	$x+\frac{1}{4}\cos(2x)+c$

89- ناتج $\left(\int \left(\frac{sin(2x)+2\cos(x)}{\sin^2(x)+2\sin(x)+1}\right) dx\right)$ هو

$a)$	$-2ln\|sin(x)+1\|+c$	$b)$	$2ln\|sin(x)+1\|+c$
$c)$	$-ln\|sin(x)+1\|+c$	$d)$	$ln\|sin(x)+1\|+c$

90- ناتج $\left(\int \left(\frac{cos^2(x)}{1-sin(x)}\right) dx\right)$ هو

$a)$	$x-cos(x)+c$	$b)$	$x+cos(x)+c$	$c)$	$-cos(x)+c$	$d)$	$cos(x)+c$

91- ناتج $\left(\int (2+tan^2(x))\, dx\right)$ هو

$a)$	$x-tan(x)+c$	$b)$	$x+tan(x)+c$	$c)$	$-tan(x)+c$	$d)$	$tan(x)+c$

92- ناتج $\left(\int (cos(\theta)(tan(\theta)+sec(\theta)))\, d\theta\right)$ هو

$a)$	$x-\cos(x)+c$	$b)$	$x+cos(x)+c$	$c)$	$-\cos(x)+c$	$d)$	$\cos(x)+c$

93- ناتج $\left(\int \left(\frac{csc(\theta)}{csc(\theta)-sin(\theta)}\right) d\theta\right)$ هو

$a)$	$-tan(\theta)+c$	$b)$	$tan(\theta)+c$	$c)$	$-cot(\theta)+c$	$d)$	$cot(\theta)+c$

94- ناتج $\left(\int (1-cot^2(x))\, dx\right)$ هو

$a)$	$x-cot(x)+c$	$b)$	$x+cot(x)+c$	$c)$	$2x-cot(x)+c$	$d)$	$2x+cot(x)+c$

95- ناتج $\left(\int \left(\frac{1}{1+cos(2x)}\right) dx\right)$ هو

$a)$	$\frac{1}{2}cot(x)+c$	$b)$	$\frac{1}{2}\tan(x)+c$	$c)$	$-\frac{1}{2}\tan(x)+c$	$d)$	$-\frac{1}{2}cot(x)+c$

96- ناتج $\left(\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} ((sin^2(x)+cos^2(x))+(tan^2(x)-sec^2(x)))\, dx\right)$ هو

$a)$	-2	$b)$	2	$c)$	-1	$d)$	0

97- ناتج $\left(\int_{-\pi}^{\pi} (cos(2x)+2sin^2(x))\, dx\right)$ هو

$a)$	2π	$b)$	-2π	$c)$	0	$d)$	1

98- ناتج $\left(\int_0^{\frac{\pi}{4}} \left(\frac{1}{1-sin(x)}\right) dx\right)$ هو

$a)$	$1+\sqrt{2}$	$b)$	$\sqrt{2}$	$c)$	$\frac{1}{\sqrt{2}}$	$d)$	$2+\sqrt{2}$

99– ناتج $\left(\int_0^{\frac{\pi}{4}}\left(\frac{1+sin(x)}{cos^2(x)}\right)dx\right)$ هو

$a)$	$1+\sqrt{2}$	$b)$	$\sqrt{2}$	$c)$	$\frac{1}{\sqrt{2}}$	$d)$	$2+\sqrt{2}$

100– ناتج $\left(\int((csc(x)-sec(x))\,(sin(x)+cos(x)))\,dx\right)$ هو

$a)$	$ln\lvert cot(x)\rvert + c$	$b)$	$ln\lvert tan(x)\rvert + c$
$c)$	$ln\lvert sin(x)\,cos(x)\rvert + c$	$d)$	$ln\lvert sec(x)\,csc(x)\rvert + c$

101– ناتج $\left(\int\left(\frac{1+tan^2(x)}{1-tan(x)}\right)\right)dx$ هو

$a)$	$-ln\lvert 1-cot(x)\rvert + c$	$b)$	$-ln\lvert 1-tan(x)\rvert + c$
$c)$	$x+ln\lvert cos(x)\rvert + c$	$d)$	$x-ln\lvert sin(x)\rvert + c$

102– ناتج $\left(\int\left(\frac{1-tan^2(x)}{1+tan(x)}\right)\right)dx$ هو

$a)$	$x+ln\lvert cos(x)\rvert + c$	$b)$	$x-ln\lvert cos(x)\rvert + c$
$c)$	$x+ln\lvert sec(x\,csc(x)\rvert + c$	$d)$	$x-ln\lvert sec(x\,csc(x)\rvert + c$

103– ناتج $\left(\int\left(\frac{1}{sin^2(x)\,cos^2(x)}\right)dx\right)$ هو

$a)$	$-4\,cot(2x)+c$	$b)$	$-2\,cot(2x)+c$	$c)$	$2cot(2x)+c$	$d)$	$4cot(2x)+c$

104– ناتج $\left(\int(sin^2(x)+sin^2(x)\,tan^2(x))\,dx\right)$ هو

$a)$	$tan(x)+x+c$	$b)$	$tan^2(x)+x+c$	$c)$	$tan(x)-x+c$	$d)$	$csc^2(x)-x+c$

105– ناتج $\left(\int_{\frac{\pi}{4}}^{\frac{\pi}{2}}\left(\frac{cos(x)}{sin(x)}\right)dx\right)$ هو

$a)$	$ln\sqrt{3}$	$b)$	$ln\sqrt{2}$	$c)$	$ln\frac{1}{\sqrt{2}}$	$d)$	$-ln\sqrt{2}$

106– ناتج $\left(\int(tan^2(x))\,dx\right)$ هو

$a)$	$cot(x)-x+c$	$b)$	$cot(x)+x+c$	$c)$	$tan(x)+x+c$	$d)$	$tan(x)-x+c$

107– ناتج $\left(\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}(sin(x))\,dx\right)$ هو

$a)$	-1	$b)$	0	$c)$	2	$d)$	4

108– ناتج $\left(\int(tan^3(x)\,\cos(x))\,dx\right)$ هو

$a)$	$-\cos(x)-\sec(x)+c$	$b)$	$\cos(x)-\sec(x)+c$
$c)$	$-\cos(x)+\sec(x)+c$	$d)$	$\cos(x)+\sec(x)+c$

109– ناتج $\left(\int(cot^3(x)\,sin(x))\,dx\right)$ هو

$a)$	$-sin(x)-csc(x)+c$	$b)$	$sin(x)+csc(x)+c$
$c)$	$sin(x)-csc(x)+c$	$d)$	$-sin(x)+csc(x)+c$

110– ناتج $\left(\int\left(\frac{4}{sin^2(x)}\right)dx\right)$ هو

$a)$	$4cot(x)+c$	$b)$	$-4cot(x)+c$	$c)$	$4tan(x)+c$	$d)$	$-4tan(x)+c$

111– ناتج $\left(\int_0^{\frac{\pi}{2}}(2\,sec(\frac{1}{2}x)-tan(\frac{1}{2}x))\,dx\right)$ هو

$a)$	$4\,ln(1+\sqrt{2})$	$b)$	$4\,ln(1+\frac{1}{\sqrt{2}})$	$c)$	$2\,ln(1+\frac{1}{\sqrt{2}})$	$d)$	$2\,ln(1+\sqrt{2})$

112– ناتج $\left(\int_{\frac{\pi}{6}}^{\frac{\pi}{3}}(sec^2(x)\,csc^2(x))\,dx\right)$ هو

$a)$	$8\sqrt{3}$	$b)$	$4\sqrt{3}$	$c)$	$\sqrt{3}$	$d)$	0

113– ناتج $(\int(sin^2(x))\,dx)$ هو

$a)$	$\frac{1}{2}x+\frac{1}{4}sin(2x)+c$	$b)$	$\frac{1}{2}x-\frac{1}{4}sin(2x)+c$	$c)$	$\frac{1}{2}x+\frac{1}{4}cos(2x)+c$	$d)$	$\frac{1}{2}x-\frac{1}{4}cos(2x)+c$

114– ناتج $(\int(cos^2(x))\,dx)$ هو

$a)$	$\frac{1}{2}x+\frac{1}{4}sin(2x)+c$	$b)$	$\frac{1}{2}x-\frac{1}{4}sin(2x)+c$	$c)$	$\frac{1}{2}x+\frac{1}{4}cos(2x)+c$	$d)$	$\frac{1}{2}x-\frac{1}{4}cos(2x)+c$

115– ناتج $(\int(sin^4(x))\,dx)$ هو

$a)$	$\frac{3}{8}x-\frac{1}{4}sin(2x)-\frac{1}{32}sin(4x)+c$	$b)$	$-\frac{1}{8}x-\frac{5}{16}sin(2x)+c$
$c)$	$\frac{1}{8}x-\frac{5}{16}sin(2x)+c$	$d)$	$\frac{3}{8}x-\frac{1}{4}sin(2x)+\frac{1}{32}sin(4x)+c$

116– ناتج $(\int(cos^4(x))\,dx)$ هو

$a)$	$-\frac{1}{8}x-\frac{5}{16}sin(2x)+c$	$b)$	$-\frac{1}{8}x-\frac{5}{16}cos(2x)+c$
$c)$	$\frac{3}{8}x-\frac{1}{4}sin(2x)-\frac{1}{32}sin(4x)+c$	$d)$	$\frac{3}{8}x+\frac{1}{4}sin(2x)+\frac{1}{32}sin(4x)+c$

117– ناتج $(\int_0^{\frac{\pi}{12}}(\frac{1}{3}tan(3x))\,dx)$ هو

$a)$	$\frac{ln\sqrt{3}}{9}$	$b)$	$\frac{ln\sqrt{2}}{3}$	$c)$	$\frac{ln\sqrt{2}}{9}$	$d)$	$ln\frac{\sqrt{2}}{9}$

118– ناتج $(\int(\frac{1-tan(x)}{1+tan(x)}))\,dx)$ هو

$a)$	$-ln\lvert cos(x)-sin(x)\rvert+c$	$b)$	$ln\lvert cos(x)-sin(x)\rvert+c$
$c)$	$-ln\lvert cos(x)+sin(x)\rvert+c$	$d)$	$ln\lvert cos(x)+sin(x)\rvert+c$

119– ناتج $(\int(\frac{1+tan(x)}{1-tan(x)}))\,dx)$ هو

$a)$	$-ln\lvert cos(x)-sin(x)\rvert+c$	$b)$	$ln\lvert cos(x)-sin(x)\rvert+c$
$c)$	$-ln\lvert cos(x)+sin(x)\rvert+c$	$d)$	$ln\lvert cos(x)+sin(x)\rvert+c$

120– ناتج $(\int(cos(3x)\,cos(x)))\,dx)$ هو

$a)$	$\frac{1}{8}sin(4x)+\frac{1}{4}sin(2x)+c$	$b)$	$\frac{1}{4}sin(4x)+\frac{1}{2}sin(2x)+c$
$c)$	$\frac{1}{8}sin(2x)+\frac{1}{4}sin(4x)+c$	$d)$	$\frac{1}{2}sin(4x)+\frac{1}{4}sin(2x)+c$

121– ناتج $(\int(\frac{x^2-x-1}{x+1}))\,dx)$ هو

$a)$	$\frac{1}{2}x^2-2x-ln\lvert x+1\rvert+c$	$b)$	$\frac{1}{2}x^2+2x+ln\lvert x+1\rvert+c$
$c)$	$\frac{1}{2}x^2-2x+ln\lvert x+1\rvert+c$	$d)$	$\frac{1}{2}x^2+2x-ln\lvert x+1\rvert+c$

122– ناتج $(\int(\frac{x^2+x+1}{x^2+1}))\,dx)$ هو

$a)$	$x+ln\lvert x^2+1\rvert+c$	$b)$	$-x+ln\lvert x^2+1\rvert+c$
$c)$	$x+\frac{1}{2}ln\lvert x^2+1\rvert+c$	$d)$	$-x+\frac{1}{2}ln\lvert x^2+1\rvert+c$

123– ناتج $(\int(\frac{x^3+x}{x+1}))\,dx)$ هو

$a)$	$\frac{1}{3}x^3+\frac{1}{2}x^2+2x-2ln\lvert x+1\rvert+c$	$b)$	$\frac{1}{3}x^3+\frac{1}{2}x^2+2x+2ln\lvert x+1\rvert+c$
$c)$	$\frac{1}{3}x^3-\frac{1}{2}x^2-2x-2ln\lvert x+1\rvert+c$	$d)$	$\frac{1}{3}x^3-\frac{1}{2}x^2+2x-2ln\lvert x+1\rvert+c$

124– اذا كان $(f(x)=\lvert 4x^2-1\rvert$ ، فإن $(\int_0^1 f(x).dx)$ هو

$a)$	$\frac{3}{6}$	$b)$	$\frac{5}{6}$	$c)$	$\frac{4}{6}$	$d)$	1

125– اذا كان $(f(x) = |x^2 - 4x + 3|)$ ، فإن $(\int_0^4 f(x).dx)$ هو

$a)$	2	$b)$	3	$c)$	4	$d)$	8

126– اذا كان $(f(x) = |sin(x)|)$ ، فإن $(\int_0^{2\pi} f(x).dx)$ هو

$a)$	4	$b)$	3	$c)$	2	$d)$	1

127– اذا كان $(f(x) = \begin{cases} |1-x|, -2 \le x < 2 \\ 2x \quad , x \ge 2 \end{cases})$، فإن $(\int_{-2}^{3} f(x).dx)$ هو

$a)$	8	$b)$	9	$c)$	10	$d)$	4

128– اذا كان $(f(x) = \begin{cases} 2\cos^2(\frac{1}{2}x), 0 \le x < \frac{\pi}{2} \\ 2\sin^2(\frac{1}{2}x), \frac{\pi}{2} < x \le \pi \end{cases})$، فإن $(\int_0^{\pi} f(x).dx)$ هو

$a)$	2	$b)$	$2\pi + 2$	$c)$	$\pi + 2$	$d)$	π

129– اذا كان $(f(x) = \begin{cases} \frac{4}{x} \quad , 1 \le x < 2 \\ 2^{0.5x} - 2, 2 \le x < 4 \end{cases})$ ، فإن $(\int_1^4 f(x).dx)$ هو

$a)$	$4\,ln(2) + \frac{2}{ln(2)} - 4$	$b)$	$4\,ln(2) + \frac{4}{ln(2)} - 4$	$c)$	$4\,ln(2) + \frac{4}{ln(2)} - 2$	$d)$	$4\,ln(2) + \frac{2}{ln(2)} - 2$

130– اذا كان $(\int_a^{2a}(\frac{2x-1}{x}).dx = ln(2))$ و $(a > 0)$، فإن قيمة (a) هي

$a)$	$ln(4)$	$b)$	$\frac{1}{2}$	$c)$	2	$d)$	$ln(2)$

131– اذا كان $(\int_a^{2a}(\frac{x}{x^2+4}).dx = ln(\sqrt{2}))$ و $(a \ne 0)$، فإن قيمة (a) هي

$a)$	$\pm\sqrt{2}$	$b)$	± 1	$c)$	$\sqrt{2}$	$d)$	1

132– اذا كان $(\int_1^a(\frac{dx}{2x}) = 1)$ و $(a > 0)$، فإن قيمة (a) هي

$a)$	$2e$	$b)$	e	$c)$	e^2	$d)$	$\sqrt{e}$

133– اذا كان $(\int_1^{2a}(\frac{2}{2x-1}).dx = 1)$ و $(a > 0)$، فإن قيمة (a) هي

$a)$	$\frac{e+1}{4}$	$b)$	$\frac{e+1}{2}$	$c)$	$\frac{\sqrt{e}+1}{4}$	$d)$	$\frac{\sqrt{e}+1}{2}$

134– اذا كان $(y = \int(sin(\frac{\pi}{2} - 2x))\,dx)$ وكانت $(y = 1)$ عندما $(x = \frac{\pi}{4})$، فإن (y) تكتب على الصورة

$a)$	$\frac{1-sin(2x)}{2}$	$b)$	$\frac{1-cos(2x)}{2}$	$c)$	$\frac{1+sin(2x)}{2}$	$d)$	$\frac{1+cos(2x)}{2}$

135– اذا كان ميل المماس عند أي نقطة على منحنى الاقتران $(f(x))$ الذي يمر منحناه بنقطة الأصل هو $(cos^2(x) - sin^2(x))$ ، فإن قاعدة الاقتران $(f(x))$ هي

$a)$	$\frac{1}{2}sin(2x)$	$b)$	$\frac{1}{2}sin(2x) - \frac{1}{2}$	$c)$	$\frac{1}{2}sin(2x) + \frac{1}{2}$	$d)$	$-\frac{1}{2}sin(2x)$

136– اذا كان ميل المماس عند أي نقطة على منحنى الاقتران $(f(x))$ الذي يمر منحناه بالنقطة $(0, 4)$ هو $(e^{-x} + x^2)$ ، فإن قاعدة الاقتران $(f(x))$ هي

$a)$	$e^{-x} + \frac{1}{3}x^3 + 4$	$b)$	$e^{-x} + \frac{1}{3}x^3 + 5$	$c)$	$e^{-x+1} + \frac{1}{3}x^3 + 3$	$d)$	$e^{-x} + \frac{1}{3}x^3 + 3$

137– اذا كان ميل المماس عند أي نقطة على منحنى الاقتران $(f(x))$ الذي يمر منحناه بالنقطة $(1, -2)$ هو $(2\pi\,cos(\pi x) - 2\pi\,sin(\pi x))$ ، فإن قاعدة الاقتران $(f(x))$ هي

$a)$	$2\,sin(\pi x) - 2\,cos(\pi x)$	$b)$	$2\,sin(\pi x) + 2\,cos(\pi x) + 4$
$c)$	$2\,sin(\pi x) + 2\,cos(\pi x)$	$d)$	$2\,sin(\pi x) + 2\,cos(\pi x) - 4$

138– اذا كان ميل المماس عند أي نقطة على منحنى الاقتران $(f(x))$ الذي يمر منحناه بالنقطتين $(\frac{\pi}{4}, 5)$ و $(\frac{3\pi}{4}, 1)$ هو $(-a\,csc^2(x))$ ، فإن قاعدة الاقتران $(f(x))$ هي

$a)$	$2\,cot(x) - 3$	$b)$	$-2\,cot(x) + 3$	$c)$	$-2\,cot(x) - 3$	$d)$	$2\,cot(x) + 3$

139– اذا كان ميل المماس عند النقطة (x, y) على منحنى الاقتران $(f(x))$ هو $(2e^x - 2)$ ، اذا علمت ان $(f(ln(8)) = 2)$ فإن قاعدة الاقتران $(f(x))$ هي

$a)$	$2e^x - 2x + 2\,ln(8) + 2$	$b)$	$2e^x - 2x - 2\,ln(8) + 2$
$c)$	$2e^x - 2x + 2\,ln(8) - 14$	$d)$	$2e^x - 2x - 2\,ln(8) + 18$

140– اذا كان $(f'^{(x)} = 3^x + \frac{4}{x})$ وكان $(f(4) = 3)$ و $(f(-4) = 5)$ ، فإن قاعدة الاقتران $(f(x))$ هي

$a)$	$f(x) = \begin{cases} \frac{3^x}{ln(3)} + 4\,ln\lvert x\rvert + 3 - (\frac{81}{ln(3)} + 4\,ln(4)) & , x < 0 \\ \frac{3^x}{ln(3)} + 4\,ln\lvert x\rvert + 5 - (\frac{1}{81ln(3)} + 4\,ln(4)) & , x > 0 \end{cases}$
$b)$	$f(x) = \begin{cases} \frac{3^x}{ln(3)} + 4\,ln\lvert x\rvert + 5 - (\frac{1}{81ln(3)} + 4\,ln(4)) & , x < 0 \\ \frac{3^x}{ln(3)} + 4\,ln\lvert x\rvert + 3 - (\frac{81}{ln(3)} + 4\,ln(4)) & , x > 0 \end{cases}$
$c)$	$f(x) = \begin{cases} \frac{3^x}{ln(3)} + 4\,ln\lvert x\rvert + 5 + (\frac{1}{81ln(3)} + 4\,ln(4)) & , x < 0 \\ \frac{3^x}{ln(3)} + 4\,ln\lvert x\rvert + 3 + (\frac{81}{ln(3)} + 4\,ln(4)) & , x > 0 \end{cases}$
$d)$	$f](x) = \begin{cases} \frac{3^x}{ln(3)} + 4\,ln\lvert x\rvert + 5 - (\frac{1}{81ln(3)} - 4\,ln(4)) & , x < 0 \\ \frac{3^x}{ln(3)} + 4\,ln\lvert x\rvert + 3 - (\frac{81}{ln(3)} - 4\,ln(4)) & , x > 0 \end{cases}$

141– اذا كان $(f'(x) = -a\,sin(x))$ وكان لمنحنى الاقتران $(f(x))$ قيمة صغرى عند $(x = \pi)$ تساوي (-2)و قيمة عظمى عند $(x = 0)$ تساوي (2) ،فإن قاعدة الاقتران $(f(x))$ هي

$a)$	$2\,cos(x)$	$b)$	$-2\,cos(x)$	$c)$	$cos(x) + 2$	$d)$	$cos(x) - 2$

142– يتحرك جسيم من نقطة الأصل في مسار مستقيم وتعطى سرعته المتجهة وفق قاعدة الاقتران $(v(t) = 6\,sin(3t))$ ،حيث الزمن (t) بالثواني ، وسرعته المتجهة (v) بالمتر لكل ثانية ، فإن المسافة الكلية التي يقطعها الجسيم في الفترة $([0.2\pi])$ هي

$a)$	$0m$	$b)$	$4m$	$c)$	$6m$	$d)$	$8m$

143– يتحرك جسيم من نقطة الأصل في مسار مستقيم وتعطى سرعته المتجهة وفق قاعدة الاقتران $(v(t) = cos(3t))$ ،حيث الزمن (t) بالثواني ، وسرعته المتجهة (v) بالمتر لكل ثانية ، فإن إزاحة الجسيم في الفترة $([0.2\pi])$ هي

$a)$	$0m$	$b)$	$1m$	$c)$	$\frac{2}{3}m$	$d)$	$\frac{7}{3}m$

144– يتحرك جسيم حركة توافقية بسيطة ويعطى متجة تسارعه وفق قاعدة الاقتران $(a(t) = -sin(t) - cos(t))$ ،حيث الزمن (t) بالثواني ، وتسارعه المتجه (va) بالمتر لكل ثانية تربيع ، وكان $(v(0) = 1, s(0) = 1)$ ،فإن موقع الجسيم بعد $(\frac{\pi}{2})$ ثانية هو

$a)$	$0m$	$b)$	$1m$	$c)$	$-1m$	$d)$	$2m$

145– يتحرك جسيم في مسار مستقيم وتعطى سرعته المتجهة وفق قاعدة الاقتران $(v(t) = cos(2t))$ ،حيث الزمن (t) بالثواني ، وسرعته المتجهة (v) بالمتر لكل ثانية ، وكان الجسيم على بعد $(4m)$ من نقطة الأصل عند بدء الحركة ،فإن ازاحة الجسيم بعد $(\frac{\pi}{4})$ ثانية هي

$a)$	$3.5m$	$b)$	$4.5m$	$c)$	$0.5m$	$d)$	$2m$

146– يتحرك جسيم في مسار مستقيم وتعطى سرعته المتجهة وفق قاعدة الاقتران $(v(t) = e^{-\frac{1}{2}t})$ ،حيث الزمن (t) بالثواني ، وسرعته المتجهة (v) بالمتر لكل ثانية ، وكان موقع الجسيم الابتدائي على بعد $(2m)$ من نقطة الأصل فإن موقع الجسيم بعد (10) ثانية هي

$a)$	$4 + 2e^5m$	$b)$	$4 - 2e^5m$	$c)$	$4 - 2e^{-5}m$	$d)$	$4 + 2e^{-5}m$

147– يتحرك جسيم في مسار مستقيم وتعطى سرعته المتجهة وفق قاعدة الاقتران $(v(t) = \frac{-t}{1+t^2})$ ،حيث الزمن (t) بالثواني ، وسرعته المتجهة (v) بالمتر لكل ثانية ، فإن المسافة الكلية التي يقطعها الجسيم في الفترة $([0, 1])$ هي

$a)$	$ln(4)$	$b)$	$ln(\frac{1}{\sqrt{2}})$	$c)$	$ln(2)$	$d)$	$ln(\sqrt{2})$

148- يتحرك جسيم في مسار مستقيم وتعطى سرعته المتجهة وفق قاعدة الاقتران ($v(t)=\frac{-3t}{t^2+2}$)، حيث الزمن (t) بالثواني ، وسرعته المتجهة (v) بالمتر لكل ثانية ، فإن ازاحة الجسيم في الفترة ($[0,4]$) هي

$a)$	$\frac{3}{2}ln(9)$	$b)$	$\frac{3}{2}ln(\frac{1}{9})$	$c)$	$\frac{2}{3}ln(\frac{1}{9})$	$d)$	$\frac{1}{3}ln(\frac{1}{9})$

149- يتحرك جسيم في مسار مستقيم من نقطة الاصل وتعطى سرعته المتجهة وفق قاعدة الاقتران ($v(t)=3\sqrt{2}e^{-\sqrt{2}t}$)، حيث الزمن ($t$) بالثواني ، وسرعته المتجهة (v) بالمتر لكل ثانية ، فإن موقع الجسيم بعد ($\sqrt{6}$) ثانية هي

$a)$	$3+3e^{-2\sqrt{3}}m$	$b)$	$3-3e^{-3\sqrt{2}}m$	$c)$	$3-3e^{-2\sqrt{3}}m$	$d)$	$3+3e^{-3\sqrt{2}}m$

150- يتحرك جسيم في مسار مستقيم من نقطة الاصل وتعطى سرعته المتجهة وفق قاعدة الاقتران ($v(t)=\begin{cases} t+2 & ,0\le t\le 1 \\ 4-(t-2)^2, & 1<t\le 3\end{cases}$)، حيث الزمن ($t$) بالثواني ، وسرعته المتجهة (v) بالمتر لكل ثانية ، فإن موقع الجسيم بعد (2) ثانية هي

$a)$	$4m$	$b)$	$\frac{9}{6}m$	$c)$	$\frac{37}{6}m$	$d)$	$\frac{9}{2}m$

151- ناتج ($\int(cos(x)e^{sin(x)})\,dx$) هو

$a)$	$e^{sin(x)}+c$	$b)$	$e^{cos(x)}+c$	$c)$	$-e^{sin(x)}+c$	$d)$	$-e^{cos(x)}+c$

152- قيمة ($\int_1^e(\frac{(ln(x))^2}{x})\,dx$) هو

$a)$	$\frac{1}{3}$	$b)$	$-\frac{1}{3}$	$c)$	3	$d)$	1

153- قيمة ($\int_{\frac{\pi}{4}}^{\frac{\pi}{2}}(\frac{e^{cot(x)}}{sin^2(x)})\,dx$) هو

$a)$	$1-e$	$b)$	$e+1$	$c)$	$e-1$	$d)$	$-e-1$

154- قيمة ($\int_0^1(\frac{2e^{\sqrt{x}}}{\sqrt{x}})\,dx$) هو

$a)$	$2e-2$	$b)$	$4e-4$	$c)$	$4e$	$d)$	$2e$

155- قيمة ($\int_{\frac{1}{2}}^{\frac{e}{2}}(\frac{cos(ln(4x^2))}{x})\,dx$) هو

$a)$	$\frac{1}{2}sin(2)$	$b)$	$\frac{1}{2}(sin(2)-1)$	$c)$	$-\frac{1}{2}sin(2)$	$d)$	$-\frac{1}{2}(sin(2)+1)$

156- ناتج ($\int(\sqrt{x}\,sin^2(x^{\frac{3}{2}}-1))\,dx$) هو

$a)$	$\frac{1}{3}\left(x^{\frac{3}{2}}-1\right)-\frac{1}{3}sin\left(2x^{\frac{3}{2}}-2\right)+c$	$b)$	$\frac{1}{3}\left(x^{\frac{3}{2}}-1\right)-\frac{1}{3}sin\left(x^{\frac{3}{2}}-1\right)+c$
$c)$	$\frac{1}{3}\left(x^{\frac{3}{2}}-1\right)-\frac{1}{6}sin\left(2x^{\frac{3}{2}}-2\right)+c$	$d)$	$\frac{1}{3}\left(x^{\frac{3}{2}}-1\right)+\frac{1}{6}sin\left(2x^{\frac{3}{2}}-2\right)+c$

157- ناتج ($\int(\frac{2^{\frac{1}{x^2}}}{x^3})\,dx$) هو

$a)$	$\frac{2^{\frac{1}{x^2}}}{ln(2)}+c$	$b)$	$-\frac{2^{\frac{1}{x^2}}}{ln(2)}+c$	$c)$	$\frac{2^{\frac{1}{x^2}}}{ln(4)}+c$	$d)$	$-\frac{2^{\frac{1}{x^2}}}{ln(4)}+c$

158- ناتج ($\int(csc^2(2x)\,cot(2x))\,dx$) هو

$a)$	$-\frac{1}{2}\cot^2(2x)+c$	$b)$	$\cot^2(2x)+c$	$c)$	$-\cot^2(2x)+c$	$d)$	$\frac{1}{2}\cot^2(2x)+c$

159- ناتج ($\int(\frac{e^{2x}}{1-e^x})\,dx$) هو

$a)$	$1-e^x-ln\lvert 1-e^x\rvert+c$	$b)$	$-(1-e^x)-ln\lvert 1-e^x\rvert+c$
$c)$	$1-e^x+ln\lvert 1-e^x\rvert+c$	$d)$	$-(1-e^x)+ln\lvert 1-e^x\rvert+c$

160– ناتج $\left(\int\left(\frac{2^{2x}}{2^x+1}\right)dx\right)$ هو

$a)$	$\frac{1}{ln(2)}(2^x+1+ln\lvert 2^x+1\rvert)+c$	$b)$	$ln(2)\,(2^x+1-ln\lvert 2^x+1\rvert)+c$
$c)$	$\frac{1}{ln(2)}(2^x+1-ln\lvert 2^x+1\rvert)+c$	$d)$	$ln(2)\,(2^x+1+ln\lvert 2^x+1\rvert)+c$

161– ناتج $\left(\int((x^3+x)(3x^2+1))\,dx\right)$ هو

$a)$	$\frac{3}{2}(3x^2+1)^2+c$	$b)$	$\frac{1}{2}(3x^2+1)^2+c$	$c)$	$\frac{1}{2}(x^3+x)^2+c$	$d)$	$\frac{3}{2}(x^3+x)^2+c$

162– ناتج $\left(\int(x\sqrt{2x+1})\,dx\right)$ هو

$a)$	$\frac{1}{2}\left(\frac{\sqrt[5]{(2x+1)^2}}{5}+\frac{\sqrt[3]{(2x+1)^2}}{3}\right)+c$	$b)$	$\frac{1}{2}\left(\frac{\sqrt{(2x+1)^2}}{5}+\frac{\sqrt{(2x+1)^2}}{3}\right)+c$
$c)$	$\frac{1}{2}\left(\frac{\sqrt[5]{2x+1}}{5}+\frac{\sqrt[3]{2x+1}}{3}\right)+c$	$d)$	$\frac{1}{2}\left(\frac{\sqrt{(2x+1)^5}}{5}+\frac{\sqrt{(2x+1)^3}}{3}\right)+c$

163– قيمة $\left(\int_0^4\left(\frac{1}{\sqrt{x}\,(1+\sqrt{x})^2}\right)dx\right)$ هو

$a)$	$\frac{2}{3}$	$b)$	$-\frac{2}{3}$	$c)$	$\frac{4}{3}$	$d)$	$-\frac{4}{3}$

164– قيمة $\left(\int_{-1}^{1}(x\sqrt{(x+1)^2})\,dx\right)$ هو

$a)$	$\frac{4}{3}$	$b)$	4	$c)$	$\frac{3}{4}$	$d)$	$\frac{2}{3}$

165– ناتج $\left(\int(x^2(x+2)^3)\,dx\right)$ هو

$a)$	$\frac{(x+2)^6}{6}-\frac{4(x+2)^5}{5}-(x+2)^4+c$	$b)$	$\frac{(x+2)^6}{6}+\frac{4(x+2)^5}{5}-(x+2)^4+c$
$c)$	$\frac{(x+2)^6}{6}+\frac{4(x+2)^5}{5}+(x+2)^4+c$	$d)$	$\frac{(x+2)^6}{6}-\frac{4(x+2)^5}{5}+(x+2)^4+c$

166– ناتج $\left(\int(e^{2x}\sqrt{e^x+1})\,dx\right)$ هو

$a)$	$\frac{2}{5}(e^x+1)^{\frac{5}{2}}-\frac{2}{3}(e^x+1)^{\frac{3}{2}}+c$	$b)$	$\frac{5}{2}(e^x+1)^{\frac{5}{2}}-\frac{3}{2}(e^x+1)^{\frac{3}{2}}+c$
$c)$	$\frac{2}{5}(e^x+1)^{\frac{5}{2}}+\frac{2}{3}(e^x+1)^{\frac{3}{2}}+c$	$d)$	$\frac{5}{2}(e^x+1)^{\frac{5}{2}}+\frac{3}{2}(e^x+1)^{\frac{3}{2}}+c$

167– ناتج $\left(\int(3^{2x}\sqrt{3^x+1})\,dx\right)$ هو

$a)$	$\frac{2}{5\,ln(3)}(3^x+1)^{\frac{5}{2}}+\frac{2}{3\,ln(3)}(3^x+1)^{\frac{3}{2}}+c$	$b)$	$\frac{2}{5\,ln(3)}(3^x+1)^{\frac{5}{2}}-\frac{2}{3}(3^x+1)^{\frac{3}{2}}+c$
$c)$	$\frac{2}{5\,ln(3)}(3^x+1)^{\frac{5}{2}}-\frac{2}{3\,ln(3)}(3^x+1)^{\frac{3}{2}}+c$	$d)$	$\frac{2}{5}(3^x+1)^{\frac{5}{2}}-\frac{2}{3}(3^x+1)^{\frac{3}{2}}+c$

168– ناتج $\left(\int(\sqrt{1+\sqrt{x}})\,dx\right)$ هو

$a)$	$\frac{2}{5}(1+\sqrt{x})^{\frac{5}{2}}-\frac{2}{3}(1+\sqrt{x})^{\frac{3}{2}}+c$	$b)$	$\frac{4}{5}(1+\sqrt{x})^{\frac{5}{2}}-\frac{4}{3}(1+\sqrt{x})^{\frac{3}{2}}+c$
$c)$	$2(1+\sqrt{x})^{\frac{5}{2}}-(1+\sqrt{x})^{\frac{3}{2}}+c$	$d)$	$\frac{4}{5}(1+\sqrt{x})^{\frac{5}{4}}-\frac{4}{3}(1+\sqrt{x})^{\frac{3}{4}}+c$

169– ناتج $\left(\int\left(\frac{1+x}{\sqrt{1-x}}\right)dx\right)$ هو

$a)$	$\frac{2}{3}\sqrt{(1-x)^3}+4\sqrt{1-x}+c$	$b)$	$-4\sqrt{1-x}-\frac{2}{3}\sqrt{(1-x)^3}+c$
$c)$	$\frac{2}{3}\sqrt{(1-x)^3}-4\sqrt{1-x}+c$	$d)$	$4\sqrt{1-x}-\frac{2}{3}\sqrt{(1-x)^3}+c$

170– ناتج $\left(\int\left(\frac{x}{\sqrt{1+x}}\right)dx\right)$ هو

$a)$	$\frac{2}{3}\sqrt{(x+1)^3}+2\sqrt{x+1}+c$	$b)$	$-2\sqrt{x+1}-\frac{2}{3}\sqrt{(x+1)^3}+c$
$c)$	$\frac{2}{3}\sqrt{(x+1)^3}-2\sqrt{x+1}+c$	$d)$	$2\sqrt{x+1}-\frac{2}{3}\sqrt{(x+1)^3}+c$

171– ناتج $(\int(\frac{x}{(x-1)^2})\,dx)$ هو

$a)$	$ln\lvert x-1\rvert-\frac{1}{x+1}+c$	$b)$	$ln\lvert x-1\rvert+\frac{1}{x-1}+c$
$c)$	$ln\lvert x-1\rvert-x-1+c$	$d)$	$ln\lvert x-1\rvert-\frac{1}{x-1}+c$

172– ناتج $(\int(\frac{e^{4x}}{(1+e^{2x})^{\frac{2}{3}}})\,dx)$ هو

$a)$	$\frac{3}{8}(e^{2x}+1)^{\frac{8}{3}}-\frac{3}{2}(e^{2x}+1)^{\frac{2}{3}}+c$	$b)$	$\frac{3}{4}(e^{2x}+1)^{\frac{4}{3}}-3(e^{2x}+1)^{\frac{1}{3}}+c$
$c)$	$\frac{3}{8}(e^{2x}+1)^{\frac{4}{3}}-\frac{3}{2}(e^{2x}+1)^{\frac{2}{3}}+c$	$d)$	$\frac{3}{8}(e^{2x}+1)^{\frac{4}{3}}-\frac{3}{2}(e^{2x}+1)^{\frac{1}{3}}+c$

173– ناتج $(\int(\frac{e^{2x}}{\sqrt[3]{1+e^{x}}})\,dx)$ هو

$a)$	$\frac{3}{5}\sqrt[3]{(1+e^{x})^5}-\frac{2}{3}\sqrt[3]{(1+e^{x})^2}+c$	$b)$	$\frac{3}{5}\sqrt[3]{(1+e^{x})^5}+\frac{3}{2}\sqrt[3]{(1+e^{x})^2}+c$
$c)$	$\frac{3}{5}(\sqrt[3]{1+e^{x}})^{\frac{5}{3}}+\frac{2}{3}(\sqrt[3]{1+e^{x}})^{\frac{3}{2}}+c$	$d)$	$\frac{3}{5}\sqrt[3]{(1+e^{x})^5}-\frac{3}{2}\sqrt[3]{(1+e^{x})^2}+c$

174– ناتج $(\int(\frac{2x}{\sqrt[3]{x^2+1}})\,dx)$ هو

$a)$	$\frac{2}{3}\sqrt[3]{(x^2+1)^2}+c$	$b)$	$\frac{2}{3}\sqrt{(x^2+1)^3}+c$
$c)$	$\frac{3}{2}\sqrt[3]{(x^2+1)^2}+c$	$d)$	$\frac{3}{2}\sqrt{(x^2+1)^3}+c$

175– ناتج $(\int(cos^3(x)\sqrt{sin(x)})\,dx)$ هو

$a)$	$\frac{2}{7}sin^{\frac{7}{2}}(x)-\frac{2}{3}sin^{\frac{3}{2}}(x)+c$	$b)$	$-\frac{2}{7}sin^{\frac{7}{2}}(x)-\frac{2}{3}sin^{\frac{3}{2}}(x)+c$
$c)$	$\frac{2}{7}sin^{\frac{7}{2}}(x)+\frac{2}{3}sin^{\frac{3}{2}}(x)+c$	$d)$	$-\frac{2}{7}sin^{\frac{7}{2}}(x)+\frac{2}{3}sin^{\frac{3}{2}}(x)+c$

176– ناتج $(\int(sin^3(x)\,cos^{\frac{1}{2}}(x))\,dx)$ هو

$a)$	$\frac{2}{7}cos^{\frac{7}{2}}(x)-\frac{2}{3}cos^{\frac{3}{2}}(x)+c$	$b)$	$-\frac{2}{7}cos^{\frac{7}{2}}(x)-\frac{2}{3}cos^{\frac{3}{2}}(x)+c$
$c)$	$\frac{2}{7}cos^{\frac{7}{2}}(x)+\frac{2}{3}cos^{\frac{3}{2}}(x)+c$	$d)$	$-\frac{2}{7}cos^{\frac{7}{2}}(x)+\frac{2}{3}cos^{\frac{3}{2}}(x)+c$

177– ناتج $(\int(sin^3(x)\,cos^2(x))\,dx)$ هو

$a)$	$-\frac{1}{3}cos^3(x)+\frac{1}{5}cos^5(x)+c$	$b)$	$\frac{1}{3}cos^3(x)-\frac{1}{5}cos^5(x)+c$
$c)$	$-\frac{1}{3}cos^3(x)-\frac{1}{5}cos^5(x)+c$	$d)$	$\frac{1}{3}cos^3(x)+\frac{1}{5}cos^5(x)+c$

178– ناتج $(\int(cos^5(x)\,sin^2(x))\,dx)$ هو

$a)$	$\frac{1}{3}sin^3(x)-\frac{2}{5}sin^5(x)+\frac{1}{7}sin^7(x)+c$	$b)$	$\frac{1}{3}sin^3(x)+\frac{2}{5}sin^5(x)+\frac{1}{7}sin^7(x)+c$
$c)$	$\frac{1}{3}sin^3(x)-\frac{2}{5}sin^5(x)-\frac{1}{7}sin^7(x)+c$	$d)$	$\frac{1}{3}sin^3(x)+\frac{2}{5}sin^5(x)-\frac{1}{7}sin^7(x)+c$

179– ناتج $(\int(\frac{1}{x^2}\,sin(\frac{1}{x})\,cos(\frac{1}{x}))\,dx)$ هو

$a)$	$\frac{1}{4}cos(\frac{2}{x})+c$	$b)$	$-\frac{1}{4}cos(\frac{2}{x})+c$
$c)$	$\frac{1}{4}cos(\frac{1}{x})+c$	$d)$	$-\frac{1}{4}cos(\frac{1}{x})+c$

180– ناتج $(\int(sin^3(\frac{x}{3})\,cos(\frac{x}{3}))\,dx)$ هو

$a)$	$\frac{1}{4}\sin^4(\frac{x}{3})+c$	$b)$	$\frac{3}{4}\sin^4(\frac{x}{3})+c$
$c)$	$\frac{4}{3}\sin^4(\frac{x}{3})+c$	$d)$	$\sin^4(\frac{x}{3})+c$

181- ناتج $\left(\int\left(\frac{1}{x^2}cos^2\left(\frac{1}{x}\right)\right)dx\right)$ هو

a)	$-\frac{1}{2x}-\frac{1}{4}sin\left(\frac{1}{x}\right)+c$	b)	$-\frac{1}{2x}-\frac{1}{4}sin\left(\frac{2}{x}\right)+c$
c)	$-\frac{1}{2x}-\frac{1}{2}sin\left(\frac{1}{x}\right)+c$	d)	$-\frac{1}{2x}-\frac{1}{2}sin\left(\frac{2}{x}\right)+c$

182- ناتج $\left(\int\left(\frac{cos(\sqrt{x})}{\sqrt{x}\,sin^2(\sqrt{x})}\right)dx\right)$ هو

a)	$-sec(\sqrt{x})$	b)	$-csc(\sqrt{x})$	c)	$-2csc(\sqrt{x})$	d)	$-2\,sec(\sqrt{x})$

183- ناتج $\left(\int(3\,sin^3(3x))\,dx\right)$ هو

a)	$\frac{1}{3}cos^3(3x)-\frac{1}{3}cos(3x)+c$	b)	$\frac{1}{3}cos^3(3x)+cos(3x)+c$
c)	$\frac{1}{3}cos^3(3x)+\frac{1}{3}cos(3x)+c$	d)	$\frac{1}{3}cos^3(3x)-cos(3x)+c$

184- ناتج $\left(\int(sin^5(x)\,cos^7(x))\,dx\right)$ هو

a)	$+\frac{1}{12}cos^{12}(x)+\frac{1}{5}cos^{10}(x)-\frac{1}{8}cos^8(x)+c$	b)	$-\frac{1}{12}cos^{12}(x)+\frac{1}{5}cos^{10}(x)-\frac{1}{8}cos^8(x)+c$
c)	$-\frac{1}{12}cos^{12}(x)-\frac{1}{5}cos^{10}(x)+\frac{1}{8}cos^8(x)+c$	d)	$\frac{1}{12}cos^{12}(x)-\frac{1}{5}cos^{10}(x)+\frac{1}{8}cos^8(x)+c$

185- ناتج $\left(\int(cos^6(x)\,sin^3(x))\,dx\right)$ هو

a)	$-\frac{1}{7}cos^7(x)+\frac{1}{9}sin^9(x)+c$	b)	$\frac{1}{7}cos^7(x)-\frac{1}{9}cos^9(x)+c$
c)	$-\frac{1}{7}cos^7(x)+\frac{1}{9}cos^9(x)+c$	d)	$\frac{1}{7}cos^7(x)-\frac{1}{9}sin^9(x)+c$

186- ناتج $\left(\int(cot^3(x))\,dx\right)$ هو

a)	$\frac{1}{2}cot^2(x)-ln\lvert sin(x)\rvert+c$	b)	$-\frac{1}{2}cot^2(x)-ln\lvert cos(x)\rvert+c$
c)	$-\frac{1}{2}cot^2(x)+ln\lvert sin(x)\rvert+c$	d)	$-\frac{1}{2}cot^2(x)-ln\lvert sin(x)\rvert+c$

187- ناتج $\left(\int(cot^6(x))\,dx\right)$ هو

a)	$-\frac{1}{5}cot^5(x)-\frac{1}{3}cot^3(x)-cot(x)+x+c$	b)	$-\frac{1}{5}cot^5(x)+\frac{1}{3}cot^3(x)-cot(x)-x+c$
c)	$-\frac{1}{5}cot^5(x)+\frac{1}{3}cot^3(x)+cot(x)+x+c$	d)	$-\frac{1}{5}cot^5(x)-\frac{1}{3}cot^3(x)+cot(x)+x+c$

188- ناتج $\left(\int(\csc^2(2x)\cot(2x))\,dx\right)$ هو

a)	$-\frac{1}{4}cot^2(2x)+c$	b)	$-\frac{1}{4}cot^2(x)+c$
c)	$-\frac{1}{2}cot^2(x)+c$	d)	$-\frac{1}{2}cot^2(2x)+c$

189- ناتج $\left(\int\left(\frac{\sin(\sqrt{x})}{\sqrt{x\cos^3(\sqrt{x})}}\right)dx\right)$ هو

a)	$\frac{-4}{\sqrt{cos^{\frac{1}{2}}(\sqrt{x})}}+c$	b)	$\frac{4}{\sqrt{cos^{\frac{1}{2}}(\sqrt{x})}}+c$
c)	$4cos^{-\frac{1}{2}}(\sqrt{x})+c$	d)	$-4cos^{-\frac{1}{2}}(\sqrt{x})+c$

190- ناتج $\left(\int\left(\frac{\sec(x)\tan(x)}{\sqrt{\sec(x)}}\right)dx\right)$ هو

a)	$-2\sqrt{sec(x)}+c$	b)	$\frac{1}{2}\sqrt{sec(x)}+c$
c)	$2\sqrt{sec(x)}+c$	d)	$-\frac{1}{2}\sqrt{sec(x)}+c$

191- ناتج $(\int(\sec^2(x)\tan^2(x))\,dx)$ هو

$a)$	$-3\tan^3(x)+c$	$b)$	$-\frac{1}{3}\tan^3(x)+c$
$c)$	$3\tan^3(x)+c$	$d)$	$\frac{1}{3}\tan^3(x)+c$

192- ناتج $(\int(tan^3(x)\,sec^3(x))\,dx)$ هو

$a)$	$-\frac{1}{5}sec^5(x)-\frac{1}{3}sec^3(x)+c$	$b)$	$\frac{1}{5}sec^5(x)+\frac{1}{3}sec^3(x)+c$
$c)$	$-\frac{1}{5}sec^5(x)+\frac{1}{3}sec^3(x)+c$	$d)$	$\frac{1}{5}sec^5(x)-\frac{1}{3}sec^3(x)+c$

193- ناتج $(\int(\tan^5(x)\sec^4(x))\,dx)$ هو

$a)$	$\frac{1}{6}tan^6(x)-\frac{1}{8}tan^8(x)+c$	$b)$	$-\frac{1}{6}tan^6(x)+\frac{1}{8}tan^8(x)+c$
$c)$	$\frac{1}{6}tan^6(x)+\frac{1}{8}tan^8(x)+c$	$d)$	$-\frac{1}{6}tan^6(x)-\frac{1}{8}tan^8(x)+c$

194- ناتج $(\int(\csc^5(x)\cot^3(x))\,dx)$ هو

$a)$	$\frac{1}{5}csc^5(x)+\frac{1}{7}csc^7(x)+c$	$b)$	$\frac{1}{5}csc^5(x)-\frac{1}{7}csc^7(x)+c$
$c)$	$-\frac{1}{5}csc^5(x)+\frac{1}{7}csc^7(x)+c$	$d)$	$-\frac{1}{5}csc^5(x)-\frac{1}{7}csc^7(x)+c$

195- ناتج $(\int(cos(x)\,sin^2(2x))\,dx)$ هو

$a)$	$\frac{4}{3}sin^3(x)+\frac{4}{5}sin^5(x)+c$	$b)$	$-\frac{4}{3}sin^3(x)+\frac{4}{5}sin^5(x)+c$
$c)$	$\frac{4}{3}sin^3(x)-\frac{4}{5}sin^5(x)+c$	$d)$	$-\frac{4}{3}sin^3(x)-\frac{4}{5}sin^5(x)+c$

196- ناتج $(\int(\frac{sec^4(x)}{tan^2(x)})\,dx)$ هو

$a)$	$-tan(x)+\frac{1}{tan(x)}+c$	$b)$	$tan(x)+\frac{1}{tan(x)}+c$
$c)$	$-tan(x)-\frac{1}{tan(x)}+c$	$d)$	$tan(x)-\frac{1}{tan(x)}+c$

197- ناتج $(\int(\frac{cot^3(x)}{csc^2(x)})\,dx)$ هو

$a)$	$-ln\lvert csc(x)\rvert-\frac{1}{2}sin^2(x)+c$	$b)$	$ln\lvert csc(x)\rvert-\frac{1}{2}sin^2(x)+c$
$c)$	$-ln\lvert csc(x)\rvert+\frac{1}{2}sin^2(x)+c$	$d)$	$ln\lvert csc(x)\rvert+\frac{1}{2}sin^2(x)+c$

198- ناتج $(\int(\frac{sin^3(x)}{\sqrt{cos(x)}})\,dx)$ هو

$a)$	$-2\,cos^2(x)-\frac{2}{5}cos^{\frac{5}{2}}(x)+c$	$b)$	$-2\,cos^{\frac{1}{2}}(x)+\frac{2}{5}cos^{\frac{5}{2}}(x)+c$
$c)$	$-2\,cos^{\frac{1}{2}}(x)-\frac{2}{5}cos^{\frac{5}{2}}(x)+c$	$d)$	$2\,cos^{\frac{1}{2}}(x)-\frac{2}{5}cos^{\frac{5}{2}}(x)+c$

199- ناتج $(\int(\frac{sin(2x)}{2+cos(x)})\,dx)$ هو

$a)$	$4\,ln\lvert 2+cos(x)\rvert-4+2\,cos(x)+c$	$b)$	$4\,ln\lvert 2+cos(x)\rvert+4-2\,cos(x)+c$
$c)$	$-4\,ln\lvert 2-cos(x)\rvert-4-2\,cos(x)+c$	$d)$	$4\,ln\lvert 2+cos(x)\rvert-4-2\,cos(x)+c$

200- ناتج $(\int(\frac{cos(x)}{sin^2(x)})\,dx)$ هو

$a)$	$-\frac{1}{cos(x)}+c$	$b)$	$-\frac{1}{sin(x)}+c$
$c)$	$\frac{1}{cos(x)}+c$	$d)$	$\frac{1}{sin(x)}+c$

201– ناتج $\left(\int\left(\frac{sin^3(x)}{cos^2(x)}\right) dx\right)$ هو

$a)$	$cos(x)+\frac{1}{cos(x)}+c$	$b)$	$cos(x)-\frac{1}{cos(x)}+c$
$c)$	$-cos(x)-\frac{1}{cos(x)}+c$	$d)$	$-cos(x)+\frac{1}{cos(x)}+c$

202– ناتج $\left(\int\left(\frac{tan(x)}{cos^2(x)}+\frac{sin(x)}{cos^3(x)}\right) dx\right)$ هو

$a)$	$-2\,sec^2(x)+c$	$b)$	$sec^2(x)+c$
$c)$	$2\,sec^2(x)+c$	$d)$	c

203– ناتج $\left(\int\left(\frac{cot(x)}{sin^2(x)}\right) dx\right)$ هو

$a)$	$-csc^2(x)+c$	$b)$	$csc^2(x)+c$
$c)$	$\frac{1}{2}csc^2(x)+c$	$d)$	$-\frac{1}{2}csc^2(x)+c$

204– ناتج $\left(\int(sec^6(x))\, dx\right)$ هو

$a)$	$tan(x)+\frac{1}{3}tan^3(x)+\frac{1}{5}tan^5(x)+c$	$b)$	$tan(x)+\frac{2}{3}tan^3(x)-\frac{1}{5}tan^5(x)+c$
$c)$	$tan(x)-\frac{2}{3}tan^3(x)-\frac{1}{5}tan^5(x)+c$	$d)$	$tan(x)+\frac{2}{3}tan^3(x)+\frac{1}{5}tan^5(x)+c$

205– ناتج $\left(\int(tan^5(x))\, dx\right)$ هو

$a)$	$ln\lvert sec(x)\rvert-\frac{1}{4}sec^4(x)+sec^2(x)+c$	$b)$	$ln\lvert sec(x)\rvert+\frac{1}{4}sec^4(x)+sec^2(x)+c$
$c)$	$ln\lvert sec(x)\rvert+\frac{1}{4}sec^4(x)-sec^2(x)+c$	$d)$	$ln\lvert sec(x)\rvert-\frac{1}{4}sec^4(x)-sec^2(x)+c$

206– ناتج $\left(\int(csc^6(x))\, dx\right)$ هو

$a)$	$cot(x)+\frac{2}{3}cot^3(x)+\frac{1}{5}cot^5(x)+c$	$b)$	$-cot(x)-\frac{2}{3}cot^3(x)-\frac{1}{5}cot^5(x)+c$
$c)$	$-cot(x)-\frac{1}{3}cot^3(x)-\frac{2}{5}cot^5(x)+c$	$d)$	$-cot(x)-\frac{1}{3}cot^3(x)-\frac{1}{5}cot^5(x)+c$

207– ناتج $\left(\int(cos^5(x))\, dx\right)$ هو

$a)$	$sin(x)+\frac{2}{3}sin^3(x)+\frac{1}{5}sin^5(x)+c$	$b)$	$sin(x)-\frac{2}{3}sin^3(x)+\frac{1}{5}sin^5(x)+c$
$c)$	$sin(x)+\frac{2}{3}sin^3(x)-\frac{1}{5}sin^5(x)+c$	$d)$	$sin(x)-\frac{2}{3}sin^3(x)-\frac{1}{5}sin^5(x)+c$

208– ناتج $\left(\int(sin^5(x))\, dx\right)$ هو

$a)$	$\cos(x)-\frac{2}{3}cos^3(x)+\frac{1}{5}cos^5(x)+c$	$b)$	$-\cos(x)+\frac{2}{3}cos^3(x)-\frac{1}{5}cos^5(x)+c$
$c)$	$-\cos(x)-\frac{2}{3}cos^3(x)+\frac{1}{5}cos^5(x)+c$	$d)$	$-\cos(x)-\frac{2}{3}cos^3(x)+\frac{1}{5}cos^5(x)+c$

209– ناتج $\left(\int\left(\frac{x+14}{(x-4)(x+2)}\right) dx\right)$ هو

$a)$	$3\,ln\lvert x-4\rvert+2\,ln\lvert x+2\rvert+c$	$b)$	$3\,ln\lvert x-4\rvert-2\,ln\lvert x+2\rvert+c$
$c)$	$-3\,ln\lvert x-4\rvert-2\,ln\lvert x+2\rvert+c$	$d)$	$-3\,ln\lvert x-4\rvert+2\,ln\lvert x+2\rvert+c$

210– ناتج $\left(\int\left(\frac{2x-13}{x^2-x-2}\right) dx\right)$ هو

$a)$	$3\,ln\lvert x-2\rvert+5\,ln\lvert x+1\rvert+c$	$b)$	$3\,ln\lvert x-2\rvert-5\,ln\lvert x+1\rvert+c$
$c)$	$-3\,ln\lvert x-2\rvert-5\,ln\lvert x+1\rvert+c$	$d)$	$-3\,ln\lvert x-2\rvert+5\,ln\lvert x+1\rvert+c$

211– ناتج $\left(\int\left(\frac{x}{x^2-5x+6}\right) dx\right)$ هو

$a)$	$2\,ln\lvert x-2\rvert+3\,ln\lvert x-3\rvert+c$	$b)$	$2\,ln\lvert x-2\rvert-3\,ln\lvert x-3\rvert+c$
$c)$	$-2\,ln\lvert x-2\rvert-3\,ln\lvert x-3\rvert+c$	$d)$	$-2\,ln\lvert x-2\rvert+3\,ln\lvert x-3\rvert+c$

212– ناتج $\left(\int\left(\frac{-x^2+2x+4}{x^3-4x^2+4x}\right)dx\right)$ هو

$a)$	$ln\|x\|-2\,ln\|x-2\|+\frac{4}{x-2}+c$	$b)$	$ln\|x\|+2\,ln\|x-2\|-\frac{4}{x-2}+c$
$c)$	$ln\|x\|-2\,ln\|x-2\|-\frac{4}{x-2}+c$	$d)$	$ln\|x\|+2\,ln\|x-2\|+\frac{4}{x-2}+c$

213– ناتج $\left(\int\left(\frac{x^2+8x+4}{x^3-2x^2}\right)dx\right)$ هو

$a)$	$\frac{2}{x}+6\,ln\|x-2\|+5\,ln\|x\|+c$	$b)$	$\frac{2}{x}+6\,ln\|x-2\|-5\,ln\|x\|+c$
$c)$	$\frac{2}{x}+11\,ln\|x-2\|+c$	$d)$	$\frac{2}{x}-6\,ln\|x-2\|-5\,ln\|x\|+c$

214– ناتج $\left(\int\left(\frac{5x-12}{x^2+4x}\right)dx\right)$ هو

$a)$	$3ln\|x\|+8ln\|x+4\|+c$	$b)$	$-3ln\|x\|-8ln\|x+4\|+c$
$c)$	$-3ln\|x\|+8ln\|x+4\|+c$	$d)$	$3ln\|x\|-8ln\|x+4\|+c$

215– ناتج $\left(\int\left(\frac{x^2-2}{x^3-x^2-2x}\right)dx\right)$ هو

$a)$	$ln\|x\|+\frac{1}{3}ln\|x+2\|-\frac{1}{3}ln\|x-1\|+c$	$b)$	$ln\|x\|-\frac{1}{3}ln\|x-2\|+\frac{1}{3}ln\|x+1\|+c$
$c)$	$ln\|x\|-\frac{1}{3}ln\|x+2\|+\frac{1}{3}ln\|x-1\|+c$	$d)$	$ln\|x\|+\frac{1}{3}ln\|x-2\|-\frac{1}{3}ln\|x+1\|+c$

216– ناتج $\left(\int\left(\frac{5x^2-10x-8}{x^3-4x}\right)dx\right)$ هو

$a)$	$2ln\|x\|-ln\|x-2\|+4ln\|x+2\|+c$	$b)$	$2ln\|x\|+ln\|x-2\|+4ln\|x+2\|+c$
$c)$	$2ln\|x\|-ln\|x-2\|-4ln\|x+2\|+c$	$d)$	$2ln\|x\|+ln\|x-2\|-4ln\|x+2\|+c$

217– ناتج $\left(\int\left(\frac{x^2+x}{(x^2-4)(x+4)}\right)dx\right)$ هو

$a)$	$\frac{1}{4}ln\|x-2\|+\frac{1}{4}ln\|x+2\|-ln\|x+4\|+c$	$b)$	$\frac{1}{4}ln\|x-2\|-\frac{1}{4}ln\|x+2\|+ln\|x+4\|+c$
$c)$	$\frac{1}{4}ln\|x+2\|-\frac{1}{4}ln\|x-2\|-ln\|x+4\|+c$	$d)$	$\frac{1}{4}ln\|x+2\|-\frac{1}{4}ln\|x-2\|+ln\|x+4\|+c$

218– ناتج $\left(\int\left(\frac{2x-3}{(x^2-x-2)(x+2)}\right)dx\right)$ هو

$a)$	$\frac{1}{12}ln\|x-2\|-\frac{7}{4}ln\|x+2\|+\frac{5}{3}ln\|x+1\|+c$	$b)$	$\frac{1}{12}ln\|x-2\|-\frac{7}{4}ln\|x+2\|-\frac{5}{3}ln\|x+1\|+c$
$c)$	$-\frac{1}{12}ln\|x-2\|+\frac{7}{4}ln\|x+2\|+\frac{5}{3}ln\|x+1\|+c$	$d)$	$-\frac{1}{12}ln\|x-2\|-\frac{7}{4}ln\|x+2\|+\frac{5}{3}ln\|x+1\|+c$

219– ناتج $\left(\int\left(\frac{2x^2-5x-1}{x^3-2x^2-x+2}\right)dx\right)$ هو

$a)$	$-ln\|x-2\|-2ln\|x-1\|-ln\|x+1\|+c$	$b)$	$-ln\|x-2\|+ln\|x-1\|+2ln\|x+1\|+c$
$c)$	$-ln\|x-2\|+2ln\|x-1\|+ln\|x+1\|+c$	$d)$	$-ln\|x-2\|+2ln\|x-1\|-ln\|x+1\|+c$

220– ناتج $\left(\int\left(\frac{x-2}{4x^2+4x+1}\right)dx\right)$ هو

$a)$	$\frac{1}{4}ln\|2x+1\|+2ln\|2x+1\|+\frac{1}{2x+1}+c$	$b)$	$\frac{1}{4}ln\|2x+1\|-\frac{5}{8x+4}+c$
$c)$	$-\frac{1}{4}ln\|2x+1\|+\frac{5}{8x+4}+c$	$d)$	$\frac{1}{4}ln\|2x+1\|+\frac{5}{8x+4}+c$

221– ناتج$\left(\int\left(\frac{2x+2}{(x+1)^3}\right)dx\right)$ هو

$a)$	$\frac{2}{x+1}+c$	$b)$	$2ln\|x+1\|-\frac{2}{x+1}+c$
$c)$	$-\frac{2}{x+1}+c$	$d)$	$2ln\|x+1\|+\frac{2}{x+1}+c$

222– ناتج $(\int(\frac{2x-4}{(x-1)^2})\,dx)$ هو

$a)$	$\frac{2}{x-1}+c$	$b)$	$2ln\|x-1\|-\frac{2}{x-1}+c$
$c)$	$-\frac{2}{x-1}+c$	$d)$	$2ln\|x-1\|+\frac{2}{x-1}+c$

223– ناتج $(\int(\frac{x^2}{(x+2)^3})\,dx)$ هو

$a)$	$ln\|x+2\|+\frac{4}{x-2}-\frac{2}{(x+2)^2}+c$	$b)$	$ln\|x+2\|-\frac{4}{x-2}+\frac{2}{(x+2)^2}+c$
$c)$	$ln\|x-2\|+\frac{4}{x-2}-\frac{2}{(x+2)^2}+c$	$d)$	$ln\|x-2\|+\frac{4}{x+2}-\frac{2}{(x+2)^2}+c$

224– ناتج $(\int(\frac{2x^2+3}{x^4-2x^2+1})\,dx)$ هو

$a)$	$\frac{1}{4}ln\|x+1\|-\frac{1}{4}ln\|x-1\|-\frac{5}{4x-4}-\frac{5}{4x+4}+c$	$b)$	$\frac{1}{4}ln\|x+1\|-\frac{1}{4}ln\|x-1\|-\frac{5}{4x-4}+\frac{5}{4x+4}+c$
$c)$	$-\frac{1}{4}ln\|x+1\|+\frac{1}{4}ln\|x-1\|-\frac{5}{4x-4}-\frac{5}{4x+4}+c$	$d)$	$\frac{1}{4}ln\|x+1\|-\frac{1}{4}ln\|x-1\|+\frac{5}{4x-4}-\frac{5}{4x+4}+c$

225– ناتج $(\int(\frac{2x^3}{x^2+1})\,dx)$ هو

$a)$	$x^2-ln\|x+1\|-ln\|x^2+1\|+c$	$b)$	$x^2+ln\|x^2+1\|+c$
$c)$	$x^2+ln\|x+1\|-ln\|x^2+1\|+c$	$d)$	$x^2-ln\|x^2+1\|+c$

226– ناتج $(\int(\frac{1-\sqrt{x}}{1+\sqrt[4]{x}})\,dx)$ هو

$a)$	$x-\frac{4}{5}\sqrt[4]{x^5}+c$	$b)$	$x+ln\|1+\sqrt[4]{x^5}\|+c$
$c)$	$x+\frac{4}{5}\sqrt[4]{x^5}+c$	$d)$	$x-ln\|1+\sqrt[4]{x^5}\|+c$

227– ناتج $(\int\frac{dx}{\sqrt[3]{x}-x})$ هو

$a)$	$\frac{3}{2}ln\|\sqrt[3]{x^2}-1\|+c$	$b)$	$\frac{3}{2}ln\|1-\sqrt[3]{x^2}\|+c$
$c)$	$-\frac{3}{2}ln\|1+\sqrt[3]{x^2}\|+c$	$d)$	$-\frac{3}{2}ln\|1-\sqrt[3]{x^2}\|+c$

228– ناتج $(\int(\frac{e^{2x}}{e^{2x}+3e^x+2})\,dx)$ هو

$a)$	$-ln(e^x+1)+2\,ln(e^{2x}+2)+c$	$b)$	$-\frac{1}{2}ln(e^x+1)+ln(e^{2x}+2)+c$
$c)$	$-2ln(e^x+1)+ln(e^{2x}+2)+c$	$d)$	$-ln(e^x+1)+\frac{1}{2}ln(e^{2x}+2)+c$

229– ناتج $(\int(\frac{csc^2(x)}{cot^2(x)-1})\,dx)$ هو

$a)$	$\frac{1}{2}ln\|cot(x)+1\|+\frac{1}{2}ln\|cot(x)-1\|+c$	$b)$	$\frac{1}{2}ln\|cot(x)+1\|-\frac{1}{2}ln\|cot(x)-1\|+c$
$c)$	$-\frac{1}{2}ln\|cot(x)+1\|-\frac{1}{2}ln\|cot(x)-1\|+c$	$d)$	$-\frac{1}{2}ln\|cot(x)+1\|-\frac{1}{2}ln\|cot(x)-1\|+c$

230– قيمة $(\int_0^{\frac{\pi}{4}}(4x\cos(4x))\,dx)$ هو

$a)$	$\frac{1}{2}$	$b)$	$-\frac{1}{2}$	$c)$	0	$d)$	$\frac{\pi-1}{4}$

231– قيمة $(\int_0^{ln\,2}(4xe^{2x})\,dx)$ هو

$a)$	$8\,ln(2)-3$	$b)$	$4ln(2)-3$	$c)$	$2ln(2)-3$	$d)$	$ln(2)-3$

232– قيمة $(\int_{\sqrt{e}}^{e}(16x^3\,ln(x))\,dx)$ هو

$a)$	$e^2(1-3e^2)$	$b)$	$3e^2(1-e^2)$	$c)$	$3e^2(e^2-1)$	$d)$	$e^2(3e^2-1)$

233- ناتج $\left(\int (x^3e^{x^2})\,dx\right)$ هو

$a)$	$-\frac{1}{2}e^{x^2}(x^2-1)+c$	$b)$	$-2e^{x^2}(x^2-1)+c$	$c)$	$2e^{x^2}(x^2-1)+c$	$d)$	$\frac{1}{2}e^{x^2}(x^2-1)+c$

234- قيمة $\left(\int_1^2 (ln(\sqrt{xe^x}))\,dx\right)$ هو

$a)$	$2ln(2)+\frac{1}{4}$	$b)$	$2\,ln(2)-\frac{1}{4}$	$c)$	$2\,ln(2)-\frac{7}{4}$	$d)$	$2\,ln(2)+\frac{7}{4}$

235- ناتج $\left(\int (e^{\sqrt{x}})\,dx\right)$ هو

$a)$	$e^{\sqrt{x}}(\sqrt{x}-1)+c$	$b)$	$2e^{\sqrt{x}}(\sqrt{x}-1)+c$	$c)$	$e^{\sqrt{x}}(2\sqrt{x}-1)+c$	$d)$	$2e^{\sqrt{x}}(2\sqrt{x}-1)+c$

236- ناتج $\left(\int (e^{\sqrt{x}})\,dx\right)$ هو

$a)$	$e^x(x^2-2x+2)+c$	$b)$	$e^x(x^2-2x-2)+c$
$c)$	$e^x(x^2+2x-2)+c$	$d)$	$e^x(x^2+2x+2)+c$

237- ناتج $\left(\int (\frac{sin(x)-x\,cos(x)}{x^2})\,dx\right)$ هو

$a)$	$\frac{1}{x^2}sin(x)+c$	$b)$	$-\frac{1}{x^2}sin(x)+c$	$c)$	$-\frac{1}{x}sin(x)+c$	$d)$	$\frac{1}{x}sin(x)+c$

238- ناتج $\left(\int (\frac{ln(x)-1}{x^2})\,dx\right)$ هو

$a)$	$-\frac{1}{x}ln(x)+\frac{1}{x}+c$	$b)$	$-\frac{1}{x}ln(x)-\frac{2}{x}+c$	$c)$	$-\frac{1}{x}ln(x)-\frac{1}{x}+c$	$d)$	$-\frac{1}{x}ln(x)+c$

239- ناتج $\left(\int (xe^{\frac{x}{2}})\,dx\right)$ هو

$a)$	$2e^{\frac{x}{2}}(x-2)+c$	$b)$	$2e^{\frac{x}{2}}(x-1)+c$	$c)$	$e^{\frac{x}{2}}(x-2)+c$	$d)$	$e^{\frac{x}{2}}(x-1)+c$

240- ناتج $\left(\int (xe^{2x})\,dx\right)$ هو

$a)$	$\frac{1}{4}e^{2x}(2x-1)+c$	$b)$	$\frac{1}{2}e^{2x}(x-1)+c$	$c)$	$\frac{1}{4}e^{2x}(x-2)+c$	$d)$	$\frac{1}{2}e^{2x}(x-2)+c$

241- قيمة $\left(\int_0^{\frac{\pi}{3}} (2\,sin(x)\,ln(sec(x)))\,dx\right)$ هو

$a)$	$-ln(2)+1$	$b)$	$-ln(2)-1$	$c)$	$-2\,ln(2)+1$	$d)$	$-2\,ln(2)-1$

242- ناتج $\left(\int (e^{-x}\,cos(2x))\,dx\right)$ هو

$a)$	$\frac{1}{5}e^{-x}(2cos(2x)-sin(2x))+c$	$b)$	$\frac{1}{5}e^{-x}(2\,sin(2x)-cos(2x))+c$
$c)$	$\frac{1}{5}e^{-x}(2cos(2x)+sin(2x))+c$	$d)$	$\frac{1}{5}e^{-x}(2\,sin(2x)+cos(2x))+c$

243- اذا كان $\left(m=\int (\frac{e^x}{sec(x)})\,dx\right)$ وكان $\left(n=\int (\frac{e^x}{csc(x)})\,dx\right)$ فإن $(m-n)$هو

$a)$	$-e^x\,sin(x)$	$b)$	$-e^x\,cos(x)$	$c)$	$e^x\,sin(x)$	$d)$	$e^x\,cos(x)$

244- اذا كان $\left(m=\int_0^{\pi} (e^x\,cos(x))\,dx\right)$ وكان $\left(n=\int_0^{\pi} (e^x\,sin(x))\,dx\right)$ ، فإن قيمة كل من (m,n)هي

$a)$	$(-\frac{1+e^{\pi}}{2},\frac{1+e^{\pi}}{2})$	$b)$	$(\frac{1+e^{\pi}}{2},-\frac{1+e^{\pi}}{2})$	$c)$	$(-\frac{1-e^{\pi}}{2},\frac{1+e^{\pi}}{2})$	$d)$	$(-\frac{1+e^{\pi}}{2},-\frac{1-e^{\pi}}{2})$

245- اذا كان $\left(m=\int_0^{\pi} (x\,sin(x))\,dx\right)$ وكان $\left(n=\int_0^{\pi} (x\,cos(x))\,dx\right)$ فإن $(m-n)$هو

$a)$	$-\pi+2$	$b)$	$\pi-2$	$c)$	$\pi+2$	$d)$	π

246- قيمة $\left(\int_0^3 (ln(2x+3))\,dx\right)$ هو

$a)$	$\frac{15}{2}ln(3)+6$	$b)$	$\frac{15}{2}ln(3)+3$	$c)$	$\frac{15}{2}ln(3)-3$	$d)$	$\frac{15}{2}ln(3)-6$

247- قيمة $\left(\int_0^2 (xe^{-x})\,dx\right)$ هو

$a)$	$-e^{-2}-1$	$b)$	$-e^{-2}+1$	$c)$	$-3e^{-2}-1$	$d)$	$-3e^{-2}+1$

248– ناتج $(\int (x\,sec^2(x))\,dx)$ هو

$a)$	$x\,tan(x) + ln\lvert sin(x)\rvert + c$	$b)$	$x\,tan(x) + ln\lvert cos(x)\rvert + c$
$c)$	$x\,tan(x) - ln\lvert sin(x)\rvert + c$	$d)$	$x\,tan(x) - ln\lvert cos(x)\rvert + c$

249– ناتج $(\int (3x\,ln(x))\,dx)$ هو

$a)$	$\frac{3}{4}x^2(2\,ln(x) - 1) + c$	$b)$	$x\,ln(x) - \frac{3}{4}x^2 + c$
$c)$	$\frac{3}{4}x^2(2\,ln(x) + 1) + c$	$d)$	$3\,ln(x)(x - 1) + c$

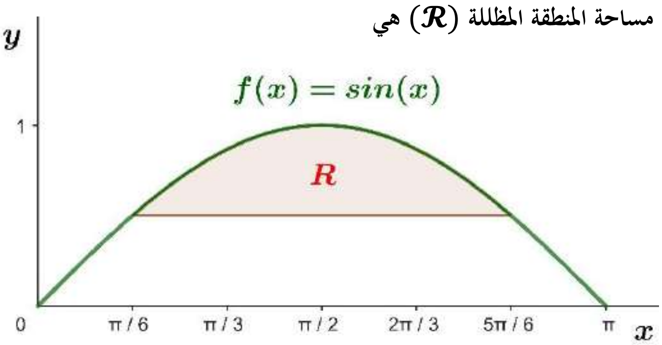
250– ناتج $(\int (e^{cos(x)}\,sin(2x))\,dx)$ هو

$a)$	$2\cos(x)e^{cos(x)} + 2e^{cos(x)} + c$	$b)$	$-2\cos(x)e^{cos(x)} - 2e^{cos(x)} + c$
$c)$	$-2\cos(x)e^{cos(x)} + 2e^{cos(x)} + c$	$d)$	$2\cos(x)e^{cos(x)} - 2e^{cos(x)} + c$

251– ناتج $(\int ((ln(x))^2)\,dx)$ هو

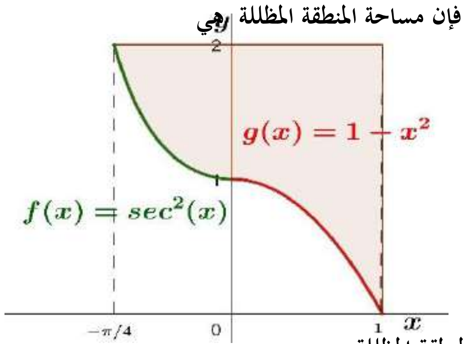
$a)$	$x(ln(x))^2 - 2x\,ln(x) + 2x + c$	$b)$	$-x(ln(x))^2 - 2x\,ln(x) + 2x + c$
$c)$	$x(ln(x))^2 - 2x\,ln(x) - 2x + c$	$d)$	$-x(ln(x))^2 + 2x\,ln(x) + 2x + c$

252– يبين الشكل المجاور التمثيل البياني لمنحنى الاقتران $(f(x) = sin(x))$ ، فإن مساحة المنطقة المظللة $(\mathcal{R})$ هي



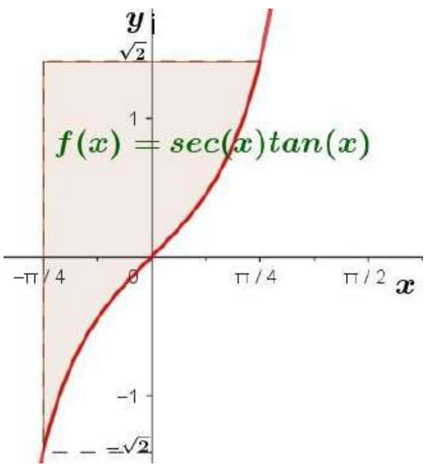
$a)$	$\sqrt{3} - \frac{\pi}{3}u^2$
$b)$	$2\sqrt{3} - \frac{\pi}{3}u^2$
$c)$	$\sqrt{3}u^2$
$d)$	$2\sqrt{3}u^2$

253– يبين الشكل المجاور التمثيل البياني لمنحنى الاقتران $(f(x) = sec^2(x), g(x) = 1 - x^2)$ ، فإن مساحة المنطقة المظللة هي



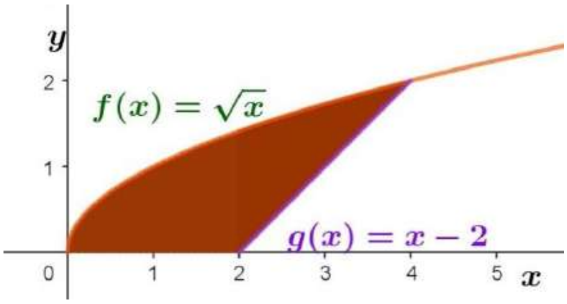
$a)$	$\frac{\pi}{2} - \frac{1}{3}u^2$
$b)$	$\frac{\pi}{2} + \frac{1}{3}u^2$
$c)$	$\frac{\pi}{2} + \frac{7}{3}u^2$
$d)$	$\frac{\pi}{2} + \frac{5}{3}u^2$

254– يبين الشكل المجاور التمثيل البياني لمنحنى الاقتران $(f(x) = sec(x)\,tan(x))$ ، فإن مساحة المنطقة المظللة هي



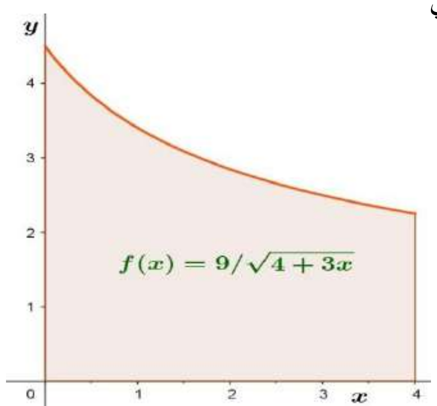
$a)$	$\frac{\pi}{\sqrt{2}}u^2$
$b)$	$\sqrt{2}u^2$
$c)$	$\frac{\pi}{\sqrt{2}} - 2\sqrt{2}u^2$
$d)$	$\frac{\pi}{\sqrt{2}} + 2\sqrt{2}u^2$

255– يبين الشكل المجاور التمثيل البياني لمنحنى الاقتران $(f(x)=\sqrt{x}, g(x)=x-2)$ ، فإن مساحة المنطقة المظللة هي



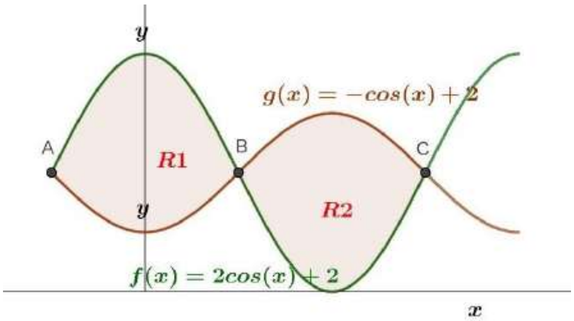
$a)$	$7u^2$
$b)$	$\frac{2\sqrt{8}}{3}+\frac{16}{3}u^2$
$c)$	$\frac{10}{3}u^2$
$d)$	$\frac{16}{3}u^2$

256– يبين الشكل المجاور التمثيل البياني لمنحنى الاقتران $(f(x)=\frac{9}{\sqrt{4+3x}})$ ، فإن مساحة المنطقة المظللة هي



$a)$	$12u^2$
$b)$	$32u^2$
$c)$	$24u^2$
$d)$	$54u^2$

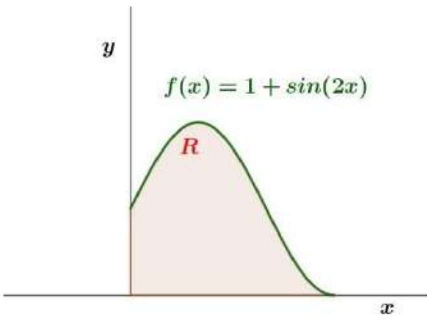
257– يبين الشكل المجاور التمثيل البياني لمنحنى الاقتران $(f(x)=2\cos(x)+2, g(x)=-\cos(x)+2)$ ، فإن مساحة المنطقة المظللة $(\mathcal{R}1,\mathcal{R}2)$ هي



$a)$	$12u^2$
$b)$	$6u^2$
$c)$	$16u^2$
$d)$	$8u^2$

☒ يبين الشكل المجاور التمثيل البياني لمنحنى الاقتران $(f(x)=1+\sin(2x))$ ، اجب عن الفقرتين (259،258)

258– مساحة المنطقة المظللة هي



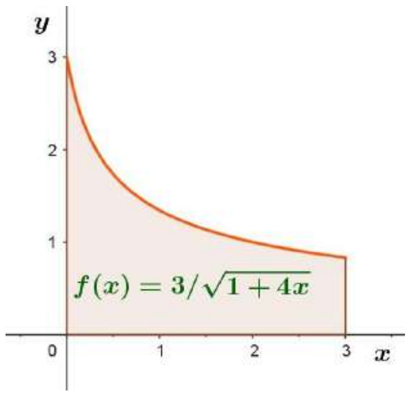
$a)$	$\frac{3\pi}{4}+\frac{1}{2}u^2$	$b)$	$\frac{3\pi}{4}u^2$
$c)$	$\frac{3\pi}{4}-\frac{1}{2}u^2$	$d)$	$\frac{3\pi}{4}-1u^2$

259– حجم المجسم الناتج عن دوران المنطقة المظللة حول محور (x) هي

$a)$	$\frac{9\pi^2}{12}+2\pi u^3$	$b)$	$\frac{9\pi^2}{8}+2\pi u^3$
$c)$	$\frac{9\pi^2}{8}-2\pi u^3$	$d)$	$\frac{9\pi^2}{12}-2\pi u^3$

☒ يبين الشكل المجاور التمثيل البياني لمنحنى الاقتران $(f(x)=\frac{3}{\sqrt{1+4x}})$ ، اجب عن الفقرتين (261،260)

260– مساحة المنطقة المظللة هي

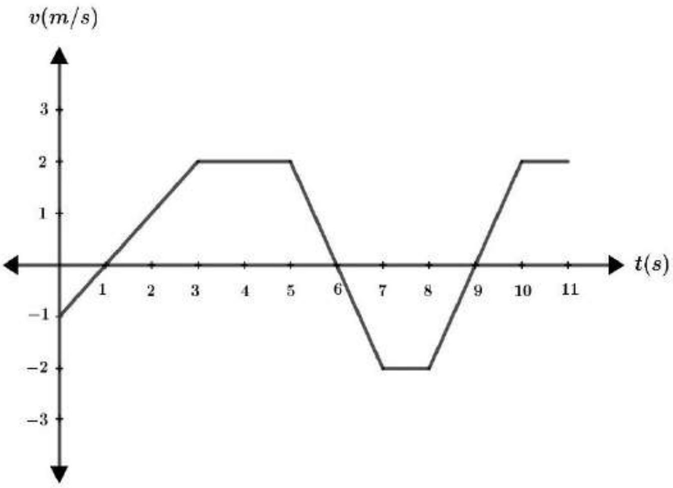


$a)$	$\frac{3}{2}\sqrt{13}u^2$	$b)$	$\frac{3}{2}\sqrt{13}+\frac{3}{2}u^2$
$c)$	$\frac{2}{3}\sqrt{13}u^2$	$d)$	$\frac{3}{2}\sqrt{13}-\frac{3}{2}u^2$

261– حجم المجسم الناتج عن دوران المنطقة المظللة حول محور (x) هي

$a)$	$9\pi\ln 13\,u^3$	$b)$	$\frac{9\pi}{4}\ln 12\,u^3$
$c)$	$\frac{9\pi}{4}\ln 13\,u^3$	$d)$	$\frac{9\pi}{4}\ln 3\,u^3$

☒ يبين الشكل المجاور منحنى السرعة المتجهة($v(m/s)$) – الزمن ($t(s)$) لجسيم يتحرك على المحور (x) في الفترة الزمنية ($[0,11]$)، اذا بدأ الجسيم الحركة من ($x=9$) عندما ($t=0$)، اجب عن الفقرات (262،263،264)



262– إزاحة الجسيم في الفترة الزمنية المعطاه هي

$a)$	$5.5m$ نحو اليمين	$b)$	$6.5m$ نحو اليمين
$c)$	$5.5m$ نحو اليسار	$d)$	$6.5m$ نحو اليسار

263– المسافة التي قطعها الجسيم في الفترة الزمنية المعطاه هي

$a)$	$15.5m$	$b)$	$16.5m$
$c)$	$14.5m$	$d)$	$13.5m$

264– الموقع النهائي للجسيم هو

$a)$	$13m$	$b)$	$15.5m$
$c)$	$16m$	$d)$	$14.5m$

265–يتحرك جسيم في مسار مستقيم ويعطى تسارعه بالمعادلة التفاضلية ($\frac{dv}{dt}=-\frac{v^2}{100}, t\geq 0$) حيث الزمن ($t(s)$) وسرعتها المتجهة ($v(m/s)$)، اذا بدأ الجسيم الحركة بسرعة متجهة ابتدائية مقدارها ($20m/s$)، فإن سرعته المتجهة بعد ($20s$) هي

$a)$	$4m/s$	$b)$	$5m/s$	$c)$	$0.25m/s$	$d)$	$0.2m/s$

266– يمكن نمذجة معدل تحلل مادة مشعة بالمعادلة التفاضلية ($\frac{dx}{dt}=-\lambda x, \lambda>0$) حيث ($x$) تمثل كمية المادة المتبقية من المادة المشعة بالمليغرام بعد (t) يوماً وكان عمر النصف للمادة المشعة هو الوقت اللازم لتحلل نصفها و (a) كتلة المادة الابتدائية ، فإن عمر النصف للمادة المشعة هو

$a)$	$\frac{ln(0.5)}{\lambda}$	$b)$	$\frac{ln(2)}{\lambda}$	$c)$	$\frac{\lambda}{ln(2)}$	$d)$	$\frac{\lambda}{ln(0.5)}$

267– يتحرك جسيم في مسار مستقيم وتعطى سرعته المتجهة بالاقتران ($v(t)=\frac{t}{9}-\frac{1}{\sqrt{t+6}}$) حيث الزمن ($t(s)$) والسرعة المتجهة ($v(m/s)$)، فإن المسافة الكلية التي قطعها الجسيم في الفترة ($[1,10]$) هي

$a)$	$\frac{45}{18}-2\sqrt{7}\,m$	$b)$	$2\sqrt{7}-\frac{45}{18}m$	$c)$	$2\sqrt{7}-\frac{5}{2}m$	$d)$	$\frac{155}{18}-2\sqrt{7}\,m$

268– الاقتران الذي يعد حلا للمعادلة التفاضلية ($y`=3y$) هو

$a)$	$f(x)=ke^{-3x}, k\in\mathcal{R}$	$b)$	$f(x)=ke^{3x}, k\in\mathcal{R}$
$c)$	$f(x)=e^{-3x}+k, k\in\mathcal{R}$	$d)$	$f(x)=0, k\in\mathcal{R}$

269– الاقتران الذي يعد حلا للمعادلة التفاضلية ($y`+y=x+1$) هو

$a)$	$f(x)=e^{-x}+1$	$b)$	$f(x)=e^{-x}+x$
$c)$	$f(x)=e^{-x}+x+1$	$d)$	$f(x)=x+1$

270– الاقتران الذي يعد حلا للمعادلة التفاضلية ($y``+4y=0$) هو

$a)$	$f(x)=ke^{4x}, k\in\mathcal{R}$	$b)$	$f(x)=a\cos(2x)+b\sin(2x)\ , a, b\in\mathcal{R}$
$c)$	$f(x)=ke^{-4x}, k\in\mathcal{R}$	$d)$	$f(x)=a\cos(\sqrt{2}x)+b\sin(\sqrt{2}x)\ , a, b\in\mathcal{R}$

271– الحل العام للمعادلة التفاضلية ($y`=y$) التي تحقق ($f(1)=2$) هو

$a)$	$f(x)=2^x$	$b)$	$f(x)=2x$
$c)$	$f(x)=2e^{x-1}$	$d)$	$f(x)=2e^x-1$

272– الحل العام للمعادلة التفاضلية ($y`+2y=6$) التي تحقق ($f(0)=2$) هو

$a)$	$f(x)=-e^{-2x}+3$	$b)$	$f(x)=2e^{2x}+3$
$c)$	$f(x)=-2e^{-2x}+3$	$d)$	$f(x)=-e^{2x}+6$

273– الحل الخاص للمعادلة التفاضلية $((y+1)y` = 3x)$ عند النقطة $(0,1)$ هو

$a)$	$y^2+2y=3x^2-3$	$b)$	$y^2+y=3x^2+3$
$c)$	$y^2+2y=3x^2+3$	$d)$	$2y^2+y=3x^2+3$

274– الحل للمعادلة التفاضلية $(y` = \frac{2x}{y}e^{y-x})$

$a)$	$e^{-y}(-y+1)=e^{-x}(2x-2)+c$	$b)$	$e^{y}(y-1)=e^{x}(2x-2)+c$
$c)$	$e^{-y}(-y-1)=e^{-x}(-2x-2)+c$	$d)$	$e^{-y}(y-1)=e^{-x}(2x-2)+c$

275– الحل للمعادلة التفاضلية $(y` = \frac{cos(x)}{sin(y)})$

$a)$	$cos(y)=-sin(x)+c$	$b)$	$cos(x)=-sin(y)+c$
$c)$	$cos(y)=sin(x)+c$	$d)$	$cos(x)=sin(y)+c$

276– الحل الخاص للمعادلة التفاضلية $(y` = x\cos^2(y))$ عند النقطة $(1,\frac{\pi}{4})$ هو

$a)$	$2\,tan(y)=x^2+3$	$b)$	$2\,tan(y)=x^2+1$
$c)$	$2\,tan(y)=x^2-3$	$d)$	$2\,tan(y)=x^2-1$

277– الحل الخاص للمعادلة التفاضلية $(\frac{dy}{dx}=\frac{3}{y\,cos^2(x)})$ عندما $(y=2)$ عند $(x=\frac{\pi}{4})$ هو

$a)$	$\frac{1}{2}y^2=3\,tan(x)-1$	$b)$	$\frac{1}{2}y^2=3\,tan(x)-2$
$c)$	$\frac{1}{2}y^2=3\,tan(x)+1$	$d)$	$\frac{1}{2}y^2=3\,tan(x)$

278– تمثل المعادلة التفاضلية $(\frac{dy}{dx}=y\,cos(x))$ ميل المماس لمنحنى علاقة ما ، فإن قاعدة هذه العلاقة اذا كان منحناها يمر بالنقطة $(0,1)$ هي

$a)$	$ln\lvert y\rvert=-sin(x)$	$b)$	$ln\lvert y\rvert=sin(x)+1$
$c)$	$ln\lvert y\rvert=sin(x)$	$d)$	$ln\lvert y\rvert=sin(x)-1$

279– تمثل المعادلة التفاضلية $(\frac{dy}{dx}=-\frac{2x}{3y})$ ميل المماس لمنحنى علاقة ما ، فإن قاعدة هذه العلاقة اذا كان منحناها يمر بالنقطة $(5,4)$ هي

$a)$	$\frac{3}{2}y^2=-x^2+49$	$b)$	$\frac{3}{2}y^2=-x^2+1$
$c)$	$\frac{3}{2}y^2=x^2+49$	$d)$	$\frac{3}{2}y^2=-x^2-1$

280– الحل الخاص للمعادلة التفاضلية $(\frac{dy}{dx}=\frac{\sqrt{y}}{x})$ عند النقطة $(1,4)$ هو

$a)$	$2\sqrt{y}=ln\lvert x\rvert+3$	$b)$	$2\sqrt{y}=ln\lvert x\rvert+4$
$c)$	$\sqrt{y}=ln\lvert x\rvert+4$	$d)$	$2\sqrt{y}=ln\lvert x\rvert-4$

السؤال الثاني:

1– جد قيمة التكامل الآتي:

$$\int_0^1 5x(1-x^2)^{\frac{3}{2}}.dx$$

2– جد ناتج كلا من التكاملات الآتية:

$$1-\int cos(x)\sqrt{4-sin(x)}\,.dx$$

$$2-\int \frac{4x^3-7x}{x^4-3x^2+4}.dx$$

$$3-\int \frac{x^4+3x^2-3x+2}{x^3-x^2-2x}.dx$$

$$4-\int \frac{e^{2x}+1}{e^x+1}.dx$$

$$5-\int \frac{dx}{e^{2x}+2e^{x}+1}$$

$$6-\int \frac{dx}{1+\sqrt[3]{x}}$$

$$7-\int \frac{2x-3}{(x^2-3x+2)(x-1)}dx$$

3- جد ناتج كلا من التكاملات الآتية:

$$1-\int x^2 \sin(2x).dx$$

$$2-\int x^2 \cos(x).dx$$

$$3-\int x^5 \cos(x).dx$$

$$4-\int e^{-x}\sin(2x).dx$$

$$5-\int \cos(\ln(x)).dx$$

السؤال الثالث:

1- اوجد الحل الخاص للمعادلة التفاضلية عندما $(y=4)$ عند $(x=1)$

$$\frac{dy}{dx}=\frac{3(y-1)}{(2x+1)(x+2)}$$

2- اوجد الحل العام للمعادلات التفاضلية الاتية:

$$1-(1+x^2)\frac{dy}{dx}=x\tan(y)$$

$$2-\frac{dy}{dx}+e^{x}y=e^{x}y^2$$

$$3-\frac{dy}{dx}=xy+y$$

3- اوجد المساحة للمنطقة المحصورة و حجم المجسم الناتج من دورانها في الربع الاول حول محور (x) في الحالتين الأتيتين:

$1-f(x)=e^{x}, g(x)=x, x=0, x=2$

$2-f(x)=x^3, g(x)=x, x=0, x=2$

4- اوجد حجم المجسم الناتج من دوران المنطقة المحصورة في الربع الأول بين منحنى الاقتران $(f(x)=\sqrt{xe^{-x}})$ والمستقيمين $(x=1, x=2)$ حول محور (x).

$$1-\int(2^{ex}+ln\,3-\frac{e^x+e^{-x}}{e^x})\,dx=\int 2^{ex}.dx+\int ln\,3\,.dx-\int(1+e^{-2x}).dx$$

$$=\int 2^{ex}.dx+\int ln\,3\,.dx-\int 1.dx-\int e^{-2x}.dx=\frac{2^{ex}}{e\,ln\,2}+ln(3)x-x-\frac{e^{-2x}}{-2}$$

$$=\frac{2^{ex}}{e\,ln\,2}+\frac{e^{-2x}}{2}+ln(3)x-x+c$$

الإجابة الصحيحة هي (d)

$$2-\int e\,ln\,2^x.dx=\int e\ln(2)\,x.dx=e\ln(2)\frac{x^2}{2}+c=\frac{1}{2}\ln(2)ex^2+c$$

الإجابة الصحيحة هي (d)

$$3-\int\frac{e^{3x}+4e^x+5}{e^x}dx=\int\frac{e^{3x}}{e^x}.dx+\int\frac{4e^x}{e^x}.dx+\int\frac{5}{e^x}.dx=\int e^{2x}.dx+\int 4.dx+\int 5e^{-x}.dx$$

$$=\frac{e^{2x}}{2}+4x-\frac{5}{e^x}+c$$

الإجابة الصحيحة هي (a)

$$4-\int\frac{7}{x\,ln\,3}dx=\frac{7}{ln\,3}\int\frac{1}{x}.dx=\frac{7}{ln\,3}ln|x|=7\,ln(|x|-3)+c$$

الإجابة الصحيحة هي (c)

$$5-\int(\frac{3}{e^{-4x}}+\sqrt[3]{e^{2x}})\,dx=3\int e^{4x}.dx+\int(e^{2x})^{\frac{1}{3}}.dx=3\int e^{4x}.dx+\int e^{\frac{2x}{3}}.dx$$

$$=\frac{3e^{4x}}{4}+\frac{3e^{\frac{2x}{3}}}{2}+c$$

الإجابة الصحيحة هي (c)

$$6-\int ln\left(\frac{e.e^x}{e^{3x}}\right)dx=\left(\int ln\,e\,.dx+x\int ln\,e\,.dx\right)-3x\int ln\,e\,.dx=\int 1.dx+x\int 1.dx-3x\int 1\,.dx$$

$$=x+x^2-3x^2=x-2x^2+c$$

الإجابة الصحيحة هي (a)

$$7-\int\frac{e^x+e^{2x}-e^{-e}}{e^{3x}}dx=\int\frac{e^x}{e^{3x}}.dx+\int\frac{e^{2x}}{e^{3x}}.dx-\int\frac{e^{-e}}{e^{3x}}.dx=\int e^{-2x}.dx+\int e^{-x}.dx-\int e^{-e-3x}.dx$$

$$=-\frac{e^{-2x}}{2}-e^{-x}+\frac{e^{-e-3x}}{3}+c$$

الإجابة الصحيحة هي (a)

$$8-\int(5^{2x}+7^{-8x})\,dx=\int 5^{2x}.dx+\int 7^{-8x}.dx=\frac{5^{2x}}{2\,ln\,5}-\frac{7^{-8x}}{8\,ln\,7}+c$$

الإجابة الصحيحة هي (b)

$$9-\int\left(\sqrt{7^{5x-1}}-\sqrt{e^{1-x}}+3\sqrt{x}\right)dx=\int(7^{5x-1})^{\frac{1}{2}}.dx-\int(e^{1-x})^{\frac{1}{2}}.dx+3\int x^{\frac{1}{2}}.dx$$

$$=\int 7^{\frac{5}{2}x-\frac{1}{2}}.dx-\int e^{\frac{1}{2}-\frac{1}{2}x}.dx+3\int x^{\frac{1}{2}}.dx=\frac{2\left(7^{\frac{5}{2}x-\frac{1}{2}}\right)}{5\ln 7}+2\,e^{\frac{1}{2}-\frac{1}{2}x}+\frac{6x^{\frac{3}{2}}}{3}$$

$$\frac{2(7^{(\frac{5x-1}{2})})}{5\,ln\,7}+2e^{(\frac{1-x}{2})}+2\sqrt{x^3}+c$$

الإجابة الصحيحة هي (b)

$$10-\int\left(\frac{4^x+6^x}{2^x}\right)dx=\int\frac{4^x}{2^x}.dx+\int\frac{6^x}{2^x}.dx=\int\frac{(2^2)^x}{2^x}.dx+\int\frac{(2\times3)^x}{2^x}.dx$$

$$\Rightarrow\int\frac{2^{2x}}{2^x}.dx+\int\frac{2^x.3^x}{2^x}.dx=\int 2^x.dx+\int 3^x.dx=\frac{2^x}{ln\,2}+\frac{3^x}{ln\,3}$$

الإجابة الصحيحة هي (a)

$$11-\int\left(\frac{2^{(x+1)}-5^{(x+1)}}{10^x}\right)dx=\int\frac{2^x.2}{2^x.5^x}.dx-\int\frac{5^x.5}{2^x.5^x}.dx=2\int 5^{-x}.dx-5\int 2^{-x}.dx$$

$$=2\frac{5^{-x}}{ln\,5}+5\frac{2^{-x}}{ln\,2}=\frac{2}{5^x ln\,5}+\frac{5}{2^x ln\,2}+c$$

الإجابة الصحيحة هي (a)

$$12-\int(\pi^{2x})\,dx=\frac{\pi^{2x}}{2\,ln\,\pi}+c$$

الإجابة الصحيحة هي (c)

$$13-\int(2^x+3^x)^2\,dx=\int((2^x)^2+(3^x)^2+(2\times2^x\times3^x)).dx=\int\left(2^{2x}+3^{2x}+2(6^x)\right).dx$$

$$=\int 2^{2x}.dx+\int 3^{2x}.dx+2\int 6^x.dx=\frac{2^{2x}}{2\,ln\,2}+\frac{3^{2x}}{2\,ln\,3}+2\frac{6^x}{ln\,6}+c$$

الإجابة الصحيحة هي (b)

$$14-\int((2^x+3^{2x})(2^x-3^{2x}))\,dx=\int((2^x)^2-(3^{2x})^2).dx=\int(2^{2x}-3^{4x}).dx$$

$$\int(2^{2x}).dx-\int(3^{4x}).dx=\frac{2^{2x}}{2\,ln\,2}-\frac{3^{4x}}{4\,ln\,3}+c$$

الإجابة الصحيحة هي (c)

$$15-\int((2^x.3^{2x})+(e^x.3^{2x}))\,dx=\int((2\times3^2)^x+(e\times3^2)^x).dx=\int(18^x+(9e)^x).dx$$

$$\int 18^x.dx+\int(9e)^x.dx=\frac{18^x}{ln\,18}+\frac{(9e)^x}{ln\,9e}+c$$

الإجابة الصحيحة هي (d)

$$16-\int((2+e)^x+(2+\pi)^x)\,dx=\int(2+e)^x\,dx+\int(2+\pi)^x\,dx=\frac{(2+e)^x}{ln(2+e)}+\frac{(2+\pi)^x}{ln(2+\pi)}+c$$

الإجابة الصحيحة هي (d)

$$17-\int(x^e+x^\pi)\,dx=\int x^e.dx+\int x^\pi.dx=\frac{x^{e+1}}{e+1}+\frac{x^{\pi+1}}{\pi+1}+c$$

الإجابة الصحيحة هي (c)

$$18-\int(\frac{e^{2x}-1}{e^x+1}).dx=\int\frac{(e^x-1)(e^x+1)}{e^x+1}.dx=\int(e^x-1).dx=\int e^x.dx-\int 1.dx=e^x-x+c$$

الإجابة الصحيحة هي (a)

$$19-\int_0^1\frac{e^{2x}-1}{e^x+1}.dx=(e^x-x)\Big|_0^1=(e^1-1)-(e^0-0)=e-1-1+0=e-2$$

الإجابة الصحيحة هي (d)

$$20-\int(x^3.e^{ln\,x})\,dx=\int x^3\times x.dx=\int x^4.dx=\frac{x^5}{5}=\frac{1}{5}x^5+c$$

الإجابة الصحيحة هي (c)

$$21-\int(e^x.tan^{-1}(3))\,dx=tan^{-1}(3)\int e^x.dx=tan^{-1}(3).e^x+c$$

الإجابة الصحيحة هي (b)

$$22-\int (e^{\ln x+1})\,dx = \int e.e^{\ln x}.dx = e\int x.dx = \frac{ex^2}{2} = \frac{e}{2}x^2 + c$$

الإجابة الصحيحة هي (a)

$$23-\int (\frac{e^{3x}+1}{e^x+1})\,dx = \int \frac{(e^x+1)(e^{2x}-e^x+1)}{e^x+1}.dx = \int (e^{2x}-e^x+1).dx$$
$$= \int e^{2x}.dx - \int e^x.dx + \int 1.dx = \frac{e^{2x}}{2} - e^x + x + c$$

الإجابة الصحيحة هي (c)

$$24-\int_0^1 \frac{e^{3x}+1}{e^x+1}.dx = \left(\frac{e^{2x}}{2} - e^x + x\right)\Big|_0^1 = (\frac{1}{2}e^2 - e + 1) - (\frac{1}{2}e^0 - e^0 + 0) = \frac{1}{2}e^2 - e + 1 - \frac{1}{2} + 1$$
$$= \frac{1}{2}e^2 - e + \frac{3}{2}$$

الإجابة الصحيحة هي (a)

$$25-\int (\frac{1}{2}e^{2\pi x-4} = \frac{1}{2}\int e^{2\pi x-4}.dx = \frac{e^{2\pi x-4}}{2\times 2\pi} = \frac{e^{2\pi x-4}}{4\pi} + c$$

الإجابة الصحيحة هي (d)

$$26-\int (\frac{2}{4-3x})\,dx = 2\int (\frac{1}{4-3x}).dx = \frac{2}{-3}\int -3\times(\frac{1}{4-3x}).dx = \frac{2}{-3}\int \left(\frac{-3}{4-3x}\right).dx$$
$$= -\frac{2}{3}\ln|4-3x| + c\,, \int \frac{g`(x)}{g(x)}.dx = \ln|g(x)| + c$$

الإجابة الصحيحة هي (c)

$$27-\int (\frac{x+5}{x^2+10x+5})\,dx = \frac{1}{2}\int 2(\frac{x+5}{x^2+10x+5})\,dx = \frac{1}{2}\int (\frac{2x+10}{x^2+10x+5})\,dx$$
$$= \frac{1}{2}\ln|x^2+10x+5| + c, \int \frac{g`(x)}{g(x)}.dx = \ln|g(x)| + c$$

الإجابة الصحيحة هي (b)

$$28-\int (\frac{x^2+4x^5}{x^3+2x^6})\,dx = \frac{1}{3}\int 3(\frac{x^2+4x^5}{x^3+2x^6})\,dx = \frac{1}{3}\int (\frac{3x^2+12x^5}{x^3+2x^6})\,dx$$
$$= \frac{1}{3}\ln|x^3+2x^6| + c\,, \int \frac{g`(x)}{g(x)}.dx = \ln|g(x)| + c$$

الإجابة الصحيحة هي (c)

$$29-\int (\frac{2e^x}{2+e^x})\,dx = 2\int (\frac{e^x}{2+e^x})\,dx = 2\ln|2+e^x| + c\,, \int \frac{g`(x)}{g(x)}.dx = \ln|g(x)| + c$$

الإجابة الصحيحة هي (d)

$$30-\int (\frac{e^x+1}{e^x+x})\,dx = \ln|e^x+x| + c\,, \int \frac{g`(x)}{g(x)}.dx = \ln|g(x)| + c$$

الإجابة الصحيحة هي (d)

$$31-\int (\frac{e^x-e^{-x}}{e^x+e^{-x}})\,dx = \ln|e^x+e^{-x}| + c\,, \int \frac{g`(x)}{g(x)}.dx = \ln|g(x)| + c$$

الإجابة الصحيحة هي (a)

$$32-\int (\frac{dx}{e+x} = \ln|e+x| + c\,, \int \frac{g`(x)}{g(x)}.dx = \ln|g(x)| + c$$

الإجابة الصحيحة هي (b)

$$33-\int (\frac{dx}{\pi+x}) = \ln|\pi+x| + c\,, \int \frac{g`(x)}{g(x)}.dx = \ln|g(x)| + c$$

الإجابة الصحيحة هي (c)

$$34-\int(\frac{dx}{x\ln x}=\int(\frac{\frac{1}{x}.dx}{\ln x})=\ln|\ln x|+c\,,\int\frac{g`(x)}{g(x)}.dx=\ln|g(x)|+c$$

الإجابة الصحيحة هي (a)

$$35-\int\frac{dx}{x\ln x+x}=\int\frac{dx}{x\,(\ln x+1)}=\int(\frac{\frac{1}{x}.dx}{\ln x+1})=\ln|\ln x+1|+c\,,\int\frac{g`(x)}{g(x)}.dx=\ln|g(x)|+c$$

الإجابة الصحيحة هي (a)

$$36-\int\frac{dx}{x^2+4x+4}=\int\frac{dx}{(x+2)^2}=\int(x+2)^{-2}.dx=\frac{(x+2)^{-1}}{-1}=\frac{1}{-(x+2)}=\frac{1}{-x-2}+c$$

الإجابة الصحيحة هي (d)

$$37-\int\frac{dx}{1+e^x}=\int\frac{e^{-x}}{e^{-x}(1+e^x)}.dx=\int\frac{e^{-x}}{e^{-x}+e^{-x}e^x}.dx=\int\frac{e^{-x}}{e^{-x}+1}.dx=-\int\frac{-e^{-x}}{e^{-x}+1}.dx$$
$$=-\ln|e^{-x}+1|+c\,,\int\frac{g`(x)}{g(x)}.dx=\ln|g(x)|+c$$

الإجابة الصحيحة هي (d)

$$38-\int\frac{dx}{1+e^{-x}}=\int\frac{e^x}{e^x(1+e^{-x})}.dx=\int\frac{e^x}{e^x+e^{-x}e^x}.dx=\int\frac{e^x}{e^x+1}.dx$$
$$=\ln|e^x+1|+c\,,\int\frac{g`(x)}{g(x)}.dx=\ln|g(x)|+c$$

الإجابة الصحيحة هي (a)

$$39-\int\frac{dx}{1+2^{-x}}=\int\frac{2^x}{2^x(1+2^{-x})}.dx=\int\frac{2^x}{2^x+2^{-x}2^x}.dx=\int\frac{2^x}{2^x+1}.dx=\frac{1}{\ln(2)}\int\ln(2)\frac{2^x}{2^x+1}.dx$$
$$=\frac{1}{\ln(2)}\int\frac{2^x\ln(2)}{2^x+1}.dx=\frac{1}{\ln(2)}\ln|2^x+1|+c\,,\int\frac{g`(x)}{g(x)}.dx=\ln|g(x)|+c$$

الإجابة الصحيحة هي (a)

$$40-\int\frac{dx}{1+3^x}=\int\frac{3^{-x}}{3^{-x}(1+3^x)}.dx=\int\frac{3^{-x}}{3^{-x}+3^{-x}3^x}.dx=\int\frac{3^{-x}}{3^{-x}+1}.dx=\frac{1}{\ln(3)}\int\ln(3)\frac{3^{-x}}{3^{-x}+1}.dx$$
$$=\frac{1}{\ln(3)}\int\frac{3^{-x}\ln(3)}{3^{-x}+1}.dx=\frac{1}{\ln(3)}\ln|3^{-x}+1|+c\,,\int\frac{g`(x)}{g(x)}.dx=\ln|g(x)|+c$$

الإجابة الصحيحة هي (a)

$$41-\int\frac{e^x}{e^x+e^{-x}}.dx=\int\frac{e^x\,e^x}{e^x(e^x+e^{-x})}.dx=\int\frac{e^{2x}}{e^{2x}+1}.dx=\frac{1}{2}\int\frac{2\,e^{2x}}{e^{2x}+1}.dx=\frac{1}{2}\ln|e^{2x}+1|+c$$

الإجابة الصحيحة هي (c)

$$41-\int\frac{e^x}{e^x+e^{-x}}.dx=\int\frac{e^x\,e^x}{e^x(e^x+e^{-x})}.dx=\int\frac{e^{2x}}{e^{2x}+1}.dx=\frac{1}{2}\int\frac{2\,e^{2x}}{e^{2x}+1}.dx$$
$$=\frac{1}{2}\ln|e^{2x}+1|+c\,,\int\frac{g`(x)}{g(x)}.dx=\ln|g(x)|+c$$

الإجابة الصحيحة هي (c)

$$42-\int\frac{e^x}{e^{2x}+2e^x+1}.dx=\int\frac{e^x}{(e^x+1)^2}.dx=\int e^x(e^x+1)^{-2}.dx=\frac{(e^x+1)^{-1}}{-1}=\frac{1}{-1(e^x+1)}$$
$$=\frac{1}{-e^x-1}+c\,,\int g`(x).(g(x))^n.dx=\frac{(g(x))^{n+1}}{n+1}+c$$

او بالتكامل بالتعويض

$$\int\frac{e^x}{e^{2x}+2e^x+1}.dx=\int\frac{e^x}{(e^x+1)^2}.dx\,,\quad \boxed{u=e^x+1\Rightarrow du=e^x\,dx\Rightarrow dx=\frac{du}{e^x}}$$
$$=\int\frac{e^x}{(e^x+1)^2}.dx=\int\frac{e^x}{(u)^2}.\frac{du}{e^x}=\int(u^{-2}).du=\frac{u^{-1}}{-1}=-\frac{1}{e^x+1}+c$$

الإجابة الصحيحة هي (a)

$$43-\int \frac{2^x}{4^x+2^{x+1}+1}.dx=\int \frac{2^x}{4^x+2.2^x+1}.dx=\int \frac{2^x}{(2^x+1)(2^x+1)}.dx=\int \frac{2^x}{(2^x+1)^2}.dx$$

$$=\int 2^x(2^x+1)^{-2}.dx=\frac{1}{ln(2)}\int ln(2).2^x(2^x+1)^{-2}.dx=\frac{1}{ln(2)}.\frac{(2^x+1)^{-1}}{-1}=\frac{1}{ln(2)}.\frac{1}{-1(2^x+1)}$$

$$=\frac{-1}{(2^x+1)(ln(2))}+c\,,\int g`(x).(g(x))^n.dx=\frac{(g(x))^{n+1}}{n+1}+c$$

او بالتكامل بالتعويض

$$\int \frac{2^x}{4^x+2^{x+1}+1}.dx==\int \frac{2^x}{4^x+2.2^x+1}.dx=\int \frac{2^x}{(2^x+1)(2^x+1)}.dx=\int \frac{2^x}{(2^x+1)^2}.dx$$

$$=\int 2^x(2^x+1)^{-2}.dx,\quad \boxed{u=2^x+1\Rightarrow du=2^x\,ln(2)\,dx\Rightarrow dx=\frac{du}{2^x\,ln(2)}}$$

$$=\int 2^x(u^{-2}).\frac{du}{2^x\,ln(2)}=\frac{1}{ln(2)}.\frac{u^{-1}}{-1}=-\frac{1}{(2^x+1)\,ln(2)}+c$$

الإجابة الصحيحة هي (a)

$$44-\int \frac{3^x-1}{9^x-1}.dx=\int \frac{3^x-1}{(3^x+1)(3^x-1)}.dx=\int \frac{1}{3^x+1}.dx=\int \frac{3^{-x}}{3^{-x}(3^x+1)}.dx=\int \frac{3^{-x}}{3^{-x}3^x+3^{-x}}.dx$$

$$=\int \frac{3^{-x}}{1+3^{-x}}.dx=-\frac{1}{ln(3)}\int \frac{-ln(3).3^{-x}}{1+3^{-x}}.dx=-\frac{1}{ln(3)}\ln|1+3^{-x}|+c,\int \frac{g`(x)}{g(x)}.dx=\ ln|g(x)|+c$$

الإجابة الصحيحة هي (a)

$$45-\int \frac{1}{\ln 2^x-1}.dx=\int \frac{1}{x\ln(2)-1}.dx=\frac{1}{ln(2)}\int \frac{\ln(2)}{x\ln(2)-1}.dx$$

$$=\frac{1}{ln(2)}\ln|x\ln(2)-1|+c,\int \frac{g`(x)}{g(x)}.dx=\ ln|g(x)|+c$$

الإجابة الصحيحة هي (a)

$$46-\int (1+e^x)e^x.dx=\int (e^x+e^{2x}).dx=\int e^x.dx+\int e^{2x}.dx=e^x+\frac{1}{2}e^{2x}+c$$

الإجابة الصحيحة هي (c)

$$47-\int \frac{8}{2x+3}.dx=4\int \frac{1}{4}.\frac{8}{2x+3}.dx=4\int \frac{2}{2x+3}.dx=4\ln|2x+3|$$

$$=\ln\left|(2x+3)^4\right|+c,\int \frac{g`(x)}{g(x)}.dx=\ ln|g(x)|+c$$

الإجابة الصحيحة هي (b)

$$48-\int \frac{x}{4x^2+1}.dx=\frac{1}{8}\int \frac{8x}{4x^2+1}.dx=\frac{1}{8}ln\left|4x^2+1\right|=ln\left|(4x^2+1)^{\frac{1}{8}}\right|$$

$$=ln\left|\sqrt[8]{4x^2+1}\right|+c,\int \frac{g`(x)}{g(x)}.dx=\ ln|g(x)|+c$$

الإجابة الصحيحة هي (c)

$$49-\int (4\csc(4x+3)\cot(4x+3)\,.dx=4\int (\csc(4x+3)\cot(4x+3)\,.dx=4(-\frac{1}{4}\csc(4x+3))$$

$$=-\csc(4x+3)+c$$

الإجابة الصحيحة هي (a)

$$50-\int \frac{cos\,(x)}{sin^2(x)}.dx=\int (csc(x)\,cot(x)\,.dx=-csc(x)+c\,,csc(x)=\frac{1}{sin(x)},cot(x)=\frac{cos(x)}{sin(x)}$$

الإجابة الصحيحة هي (b)

$$51-\int_0^{\frac{\pi}{3}}(\sin(4x)-\cos(2x)).dx=\int_0^{\frac{\pi}{3}}\sin(4x).dx-\int_0^{\frac{\pi}{3}}\cos(2x).dx=-\frac{1}{4}\cos(4x)-\frac{1}{2}\sin(2x)\Big|_0^{\frac{\pi}{3}}$$

$$=(-\frac{1}{4}\cos(\frac{4\pi}{3})-\frac{1}{2}\sin(\frac{2\pi}{3}))-(-\frac{1}{4}\cos(0)-\frac{1}{2}\sin(0))$$

$$=((-\frac{1}{4}\times-\frac{1}{2})-(\frac{1}{2}\times\frac{\sqrt{3}}{2}))-((-\frac{1}{4}\times 1)-(\frac{1}{2}\times 0))=(\frac{1}{8}-\frac{\sqrt{3}}{4})-(-\frac{1}{4}-0)=\frac{1}{8}-\frac{\sqrt{3}}{4}+\frac{1}{4}$$

$$=\frac{1-2\sqrt{3}+2}{8}=\frac{3-2\sqrt{3}}{8}$$

الإجابة الصحيحة هي (b)

$$52-\int(\cos\theta(\tan\theta+\sec\theta))\,d\theta=\int(\cos\theta\tan\theta+\cos\theta\sec\theta)\,d\theta$$

$$=\int(\sin\theta+1)\,d\theta,\ \sec(x)=\frac{1}{\cos(x)},\tan(x)=\frac{\sin(x)}{\cos(x)}$$

$$=\int\sin\theta.d\theta+\int 1.d\theta=-\cos\theta+\theta=\theta-\cos\theta+c$$

الإجابة الصحيحة هي (d)

$$53-\int(3\sin(\frac{\theta}{3}))\,d\theta=3\int(\sin(\frac{\theta}{3}))\,d\theta=3(-3\cos(\frac{\theta}{3}))=-9\cos(\frac{\theta}{3})+c$$

الإجابة الصحيحة هي (d)

$$54-\int(4-\cot x\csc x)dx=\int 4.dx-\int\cot x\csc x.dx=4x-(-\csc x)=4x+\csc x+c$$

الإجابة الصحيحة هي (c)

$$55-\int(2-2\cos(2x)+\frac{2}{x}-\sqrt{2x})dx=\int 2.dx-2\int\cos(2x).dx+2\int\frac{1}{x}.dx-\sqrt{2}\int x^{\frac{1}{2}}.dx$$

$$=2x-2(\frac{1}{2}\sin(2x))+2(\ln|x|)-\sqrt{2}(2\frac{x^{\frac{3}{2}}}{3})=2x-\sin(2x)+2\ln|x|-\frac{2\sqrt{2x^3}}{3}+c$$

الإجابة الصحيحة هي (d)

$$56-\int(\sec^2(x))dx=\tan(x)+c$$

الإجابة الصحيحة هي (c)

$$57-\int(4\cos(4x)+e^{2x}-\sqrt[4]{e^{x-2}})dx=4\int\cos(4x).dx+\int e^{2x}.dx-\int e^{\frac{x-2}{4}}.dx$$

$$=4(\frac{1}{4}\sin(4x))+\frac{1}{2}e^{2x}-4e^{\frac{x-2}{4}}=\sin(4x)+\frac{1}{2}e^{2x}-4e^{\frac{x-2}{4}}+c$$

الإجابة الصحيحة هي (b)

$$58-\int(\sec(x)\tan(x))dx=\sec(x)+c$$

الإجابة الصحيحة هي (b)

$$59-\int(2\sin(1-\frac{x}{2})+(\sec^2(2x)(1+\sin(2x)))dx$$

$$=\int(2\sin(1-\frac{x}{2})+\sec^2(2x)+(\sec^2(2x)\sin(2x)))dx$$

$$=\int(2\sin(1-\frac{x}{2}).dx+\int\sec^2(2x).dx+\int(\sec(2x)\tan(2x)).dx,\ \sec(x)=\frac{1}{\cos(x)},\tan(x)=\frac{\sin(x)}{\cos(x)}$$

$$=2\left(-(-2)\cos(1-\frac{x}{2})\right)+\frac{1}{2}\tan(2x)+\frac{1}{2}\sec(2x)=8\cos(1-\frac{x}{2})+\tan(2x)+\sec(2x)+c$$

الإجابة الصحيحة هي (a)

$$60-\int (e^{3-x}-sin(3-x)+cos(3-x))dx=\int e^{3-x}.dx-\int sin(3-x).dx+\int cos(3-x).dx$$
$$=-e^{3-x}-(-(-)\,cos(3-x))+(-\,sin(3-x))=-e^{3-x}-sin(3-x)-cos(3-x)+c$$

الإجابة الصحيحة هي (a)

$$61-\int \frac{tan(3x)-tan(4x)}{1+tan(3x)\,tan(4x)}.dx=\int tan(3x-4x).dx,\; tan(x-y)=\frac{tan(x)-tan(y)}{1+tan(x)\,tan(y)}$$
$$=\int tan(-x).dx=-\int tan(x).dx,\; tan(-x)=-\,tan(x)$$
$$=-\int tan(x).dx=-\int \frac{sin(x)}{cos(x)}.dx=\int \frac{-\,sin(x)}{cos(x)}.dx=ln|cos(x)|+c,\;\int \frac{g`(x)}{g(x)}.dx=ln|g(x)|+c$$

الإجابة الصحيحة هي (c)

$$62-\int 4\,sin(4x)\,cos(4x).dx=\int 2(2\,sin(4x)\,cos(4x)).dx$$
$$=2\int sin(8x).dx,\; sin(2x)=2sin(x)\,cos(x)$$
$$=2\int sin(8x).dx=2(-\frac{1}{8}cos(8x))=-\frac{1}{4}cos(8x)+c$$

الإجابة الصحيحة هي (a)

$$63-\int \frac{2\,tan(3x)}{1-tan^2(3x)}.dx=\int tan(6x).dx,\; tan(2x)=\frac{2\,tan(x)}{1-tan^2(x)}$$
$$=\int tan(6x).dx=\int \frac{sin(6x)}{cos(6x)}.dx=-\frac{1}{6}\int \frac{-6\,sin(6x)}{cos(6x)}.dx$$
$$=-\frac{1}{6}ln|cos(6x)|+c,\;\int \frac{g`(x)}{g(x)}.dx=ln|g(x)|+c$$

الإجابة الصحيحة هي (b)

$$64-\int \frac{1+cos(2x)}{1-cos(2x)}.dx=\int cot^2(x).dx,\; tan^2(x)=\frac{1-cos(2x)}{1+cos(2x)},\; cot^2(x)=\frac{1+cos(2x)}{1-cos(2x)}$$
$$\Rightarrow \int cot^2(x).dx=\int (csc^2(x)-1).dx,\; cot(x)=\frac{1}{tan(x)},\; csc^2(x)=cot^2(x)+1$$
$$=\int csc^2(x).dx-\int 1.dx=-\,cot(x)-x+c$$

الإجابة الصحيحة هي (b)

$$65-\int \frac{2}{1-cos(2x)}.dx=\int csc^2(x).dx,\; csc(x)=\frac{1}{sin(x)},\; sin^2(x)=\frac{1-cos(2x)}{2},\; csc^2(x)=\frac{2}{1-cos(2x)}$$
$$\Rightarrow \int csc^2(x).dx=-\,cot(x)+c$$

الإجابة الصحيحة هي (b)

$$66-\int \frac{4}{2-2\,cos(2x)}.dx=\int \frac{2(2)}{2(1-cos(2x))}.dx=\int \frac{2}{1-cos(2x)}.dx=\int csc^2(x).dx$$
$$=\int csc^2(x).dx,\; csc(x)=\frac{1}{sin(x)},\; sin^2(x)=\frac{1-cos(2x)}{2},\; csc^2(x)=\frac{2}{1-cos(2x)}$$
$$\Rightarrow \int csc^2(x).dx=-\,cot(x)+c$$

الإجابة الصحيحة هي (c)

$$67 - \int (sin^2(2x) - cos^2(2x)).dx = -\int -(sin^2(2x) - cos^2(2x)).dx = -\int (cos^2(2x) - sin^2(2x)).dx$$

$$= -\int (cos^2(2x) - sin^2(2x)).dx = -\int cos(2x).dx, \ cos(2x) = cos^2(x) - sin^2(x)$$

$$\Rightarrow -\int cos(2x).dx = -(\frac{1}{2}sin(2x)) = -\frac{1}{2}sin(2x)$$

الإجابة الصحيحة هي (d)

$$68 - \int (4 - 8\,sin^2(2x)).dx = 4\int (1 - 2\,sin^2(2x)).dx = 4\int (1 - (2(\frac{1 - cos(4x)}{2})).dx$$

$$= 4\int (1 - 1 + cos(4x)).dx = 4\int cos(4x).dx = 4(\frac{1}{4}sin(4x)) = sin(4x) + c, \ sin^2(x) = \frac{1 - cos(2x)}{2}$$

الإجابة الصحيحة هي (a)

$$69 - \int (3 - 6cos^2(3x)).dx = 3\int (1 - 2cos^2(3x)).dx = 3\int (1 - (2(\frac{1 + cos(6x)}{2})).dx$$

$$= 3\int (1 - 1 - cos(6x)).dx, \ cos^2(x) = \frac{1 + cos(2x)}{2}$$

$$= -3\int cos(6x).dx = -3(\frac{1}{6}sin(6x)) = -\frac{1}{2}sin(6x) + c$$

الإجابة الصحيحة هي (d)

$$70 - \int ((sin(3x)\,cos(5x) - cos(3x)\,sin(5x)) + ((cos(3x)\,cos(5x) - sin(3x)\,sin(5x))).dx$$

$$= \int (sin(-2x) + cos(8x)).dx, \ sin(x - y) = (sin(x)\,cos(y) - cos(x)\,sin(y))$$

$$, cos(x + y) = ((cos(x)\,cos(y) - sin(x)\,sin\,y)$$

$$= \int (-sin(2x) + cos(8x)).dx = -\int sin(2x).dx + \int cos(8x).dx, \ sin(-x) = -sin(x)$$

$$= -(-\frac{1}{2}cos(2x)) + \frac{1}{8}sin(8x) = \frac{1}{2}cos(2x) + \frac{1}{8}sin(8x) + c$$

الإجابة الصحيحة هي (b)

$$71 - \int (sin(3x)\,sin(4x) - cos(3x)\,sin(4x)).dx = \int (sin(3x)\,sin(4x)).dx - \int cos(3x)\,sin(4x)).dx$$

$$= \int \frac{1}{2}[cos(-x) - cos(7x)].dx - \int -\frac{1}{2}[sin(-x) - sin(7x)].dx$$

$$, sin(x)\,sin(y) = \frac{1}{2}[cos(x - y) - cos(x + y)], cos(x)\,sin(y) = -\frac{1}{2}[sin(x - y) - sin(x + y)]$$

$$= \frac{1}{2}[\int cos(x).dx - \int cos(7x).dx] + \frac{1}{2}[-\int sin(x).dx - \int sin(7x).dx]$$

$$, sin(-x) = -sin(x), cos(-x) = cos(x)$$

$$= \frac{1}{2}(sin(x) - \frac{1}{7}sin(7x)) + \frac{1}{2}(cos(x) + \frac{1}{7}cos(7x)) = \frac{1}{2}sin(x) - \frac{1}{14}sin(7x) + \frac{1}{2}cos(x) + \frac{1}{14}cos(7x)$$

$$= \frac{1}{2}(sin(x) + cos(x)) + \frac{1}{14}(cos(7x) - sin(7x))$$

الإجابة الصحيحة هي (c)

$$72 - \int (\sqrt{\frac{1 - cos\,\theta}{1 + cos\,\theta}}).d\theta = \int tan(\frac{\theta}{2}).d\theta = \int \frac{sin(\frac{\theta}{2})}{cos(\frac{\theta}{2})}.d\theta, tan(\frac{\theta}{2}) = \sqrt{\frac{1 - cos\,\theta}{1 + cos\,\theta}}, tan(x) = \frac{sin(x)}{cos(x)}$$

$$= \int \frac{sin(\frac{\theta}{2})}{cos(\frac{\theta}{2})}.d\theta = -2\int \frac{-\frac{1}{2}sin(\frac{\theta}{2})}{cos(\frac{\theta}{2})}.d\theta = -2\ln\left|cos(\frac{\theta}{2})\right| + c, \int \frac{g`(x)}{g(x)}.dx = ln|g(x)| + c$$

الإجابة الصحيحة هي (d)

$$73-\int(sin(3x)\,cos(4x)-cos(3x)\,cos(4x)).dx=\int(sin(3x)\,cos(4x)).dx-\int cos(3x)\,cos(4x)).dx$$

$$=\int\frac{1}{2}[sin(-x)+sin(7x)].dx-\int\frac{1}{2}[cos(-x)+cos(7x)].dx$$

$$,cos(x)\,cos(y)=\frac{1}{2}[cos(x-y)+cos(x+y)],sin(x)\,cos(y)=\frac{1}{2}[sin(x-y)+sin(x+y)]$$

$$=\frac{1}{2}[-\int sin(x).dx+\int sin(7x).dx]-\frac{1}{2}[\int cos(x).dx+\int cos(7x).dx]$$

$$,sin(-x)=-sin(x),cos(-x)=cos(x)$$

$$=\frac{1}{2}(cos(x)-\frac{1}{7}cos(7x))-\frac{1}{2}(sin(x)+\frac{1}{7}sin(7x))=\frac{1}{2}cos(x)-\frac{1}{14}cos(7x)-\frac{1}{2}sin(x)-\frac{1}{14}sin(7x)$$

$$=\frac{1}{2}(\cos(x)-sin(x))+\frac{1}{14}(-cos(7x)-sin(7x))+c$$

الإجابة الصحيحة هي (c)

$$74-\int_0^{2\pi}(sin^2(\frac{x}{4}))dx=\int_0^{2\pi}\frac{1-cos(\frac{x}{2})}{2}.dx=\frac{1}{2}x-(\frac{1}{2}\times 2sin(\frac{x}{2})),sin^2(x)=\frac{1-cos(2x)}{2}$$

$$=\frac{1}{2}x-sin(\frac{x}{2})\Big|_0^{2\pi}=(\frac{1}{2}\times 2\pi-\sin(\frac{2\pi}{2}))-(\frac{1}{2}\times 0-\sin(\frac{0}{2}))=(\pi-\sin(\pi))-(0-\sin(0))$$

$$=(\pi-0)-(0-0)=\pi-0=\pi$$

الإجابة الصحيحة هي (d)

$$75-\int\frac{1}{1-cos(x)}.dx=\int\left(\frac{1}{1-cos(x)}\times\frac{1+cos(x)}{1+cos(x)}\right).dx=\int\frac{1+cos(x)}{1-cos^2(x)}.dx$$

$$=\int\frac{1+cos(x)}{sin^2(x)}.dx=\int\frac{1}{sin^2(x)}.dx+\int\frac{cos(x)}{sin^2(x)}.dx=\int csc^2(x).dx+\int\frac{cos(x)}{sin(x)}\times\frac{1}{sin(x)}.dx$$

$$,csc(x)=\frac{1}{sin(x)},cot(x)=\frac{cos(x)}{sin(x)}$$

$$=\int csc^2(x).dx+\int cot(x)\,csc(x).dx=-cot(x)+(-csc(x))=-cot(x)-csc(x)+c$$

الإجابة الصحيحة هي (c)

$$76-\int\frac{1}{1+cos(x)}.dx=\int\left(\frac{1}{1+cos(x)}\times\frac{1-cos(x)}{1-cos(x)}\right).dx=\int\frac{1-cos(x)}{1-cos^2(x)}.dx$$

$$=\int\frac{1-cos(x)}{sin^2(x)}.dx=\int\frac{1}{sin^2(x)}.dx-\int\frac{cos(x)}{sin^2(x)}.dx=\int csc^2(x).dx-\int\frac{cos(x)}{sin(x)}\times\frac{1}{sin(x)}.dx$$

$$,csc(x)=\frac{1}{sin(x)},cot(x)=\frac{cos(x)}{sin(x)}$$

$$=\int csc^2(x).dx-\int cot(x)\,csc(x).dx=-cot(x)-(-csc(x))=-cot(x)+csc(x)+c$$

الإجابة الصحيحة هي (a)

$$77-\int\frac{1}{1-sin(x)}.dx=\int\left(\frac{1}{1-sin(x)}\times\frac{1+sin(x)}{1+sin(x)}\right).dx=\int\frac{1+sin(x)}{1-sin^2(x)}.dx$$

$$=\int\frac{1+sin(x)}{cos^2(x)}.dx=\int\frac{1}{cos^2(x)}.dx+\int\frac{sin(x)}{cos^2(x)}.dx=\int sec^2(x).dx+\int\frac{sin(x)}{cos(x)}\times\frac{1}{cos(x)}.dx$$

$$,sec(x)=\frac{1}{cos(x)},tan(x)=\frac{sin(x)}{cos(x)}$$

$$=\int sec^2(x).dx+\int tan(x)\,sec(x).dx=tan(x)+sec(x)+c$$

الإجابة الصحيحة هي (d)

$$78 - \int \frac{1}{1+sin(x)}.dx = \int \left(\frac{1}{1+sin(x)} \times \frac{1-sin(x)}{1-sin(x)}\right).dx = \int \frac{1-sin(x)}{1-sin^2(x)}.dx$$

$$= \int \frac{1-sin(x)}{cos^2(x)}.dx = \int \frac{1}{cos^2(x)}.dx - \int \frac{sin(x)}{cos^2(x)}.dx = \int sec^2(x).dx - \int \frac{sin(x)}{cos(x)} \times \frac{1}{cos(x)}.dx$$

$$, sec(x) = \frac{1}{cos(x)}, tan(x) = \frac{sin(x)}{cos(x)}$$

$$= \int sec^2(x).dx - \int tan(x)\,sec(x).dx = tan(x) - sec(x) + c$$

الإجابة الصحيحة هي (b)

$$79 - \int (cos(x)(1+csc^2(x)).dx = \int (cos(x) + cos(x)\,csc^2(x)).dx$$

$$= \int cos(x).dx + \int cos(x)\,csc^2(x).dx = \int cos(x).dx + \int \frac{cos(x)}{sin(x)} \times \frac{1}{sin(x)}.dx$$

$$, csc(x) = \frac{1}{sin(x)}, cot(x) = \frac{cos(x)}{sin(x)}$$

$$= \int cos(x).dx + \int cot(x)\,csc(x).dx = sin(x) - csc(x) + c$$

الإجابة الصحيحة هي (c)

$$80 - \int_0^{\frac{\pi}{4}} ((sin(x) - 3\cos(x))^2).dx = \int_0^{\frac{\pi}{4}} (sin^2(x) + 9\cos^2(x) - 6\sin(x)\cos(x)).dx$$

$$= \int_0^{\frac{\pi}{4}} sin^2(x).dx + \int_0^{\frac{\pi}{4}} 9\cos^2(x).dx - \int_0^{\frac{\pi}{4}} 6\sin(x)\cos(x).dx$$

$$= \int_0^{\frac{\pi}{4}} \frac{1-cos(2x)}{2}.dx + 9\int_0^{\frac{\pi}{4}} \frac{1+cos(2x)}{2}.dx - 3\int_0^{\frac{\pi}{4}} 2\sin(x)\cos(x).dx$$

$$, sin^2(x) = \frac{1-cos(2x)}{2}, \cos^2(x) = \frac{1+cos(2x)}{2}, sin(2x) = 2sin(x)\,cos(x)$$

$$= [(\frac{1}{2}x - \frac{1}{4}sin(2x)) + 9(\frac{1}{2}x + \frac{1}{4}sin(2x)) - 3(sin(2x))]\Big|_0^{\frac{\pi}{4}}$$

$$= \frac{1}{2}x - \frac{1}{4}sin(2x) + \frac{9}{2}x + \frac{9}{4}sin(2x) - 3\,sin(2x)\Big|_0^{\frac{\pi}{4}} = 5x + 2\,sin(2x) - 3\,sin(2x)\Big|_0^{\frac{\pi}{4}}$$

$$= 5x - sin(2x)\Big|_0^{\frac{\pi}{4}} = (5 \times \frac{\pi}{4} - sin(2 \times \frac{\pi}{4})) - (5 \times 0 - sin(2 \times 0)) = \frac{5\pi}{4} - sin(\frac{\pi}{2}) - 0 + sin(0)$$

$$= \frac{5\pi}{4} - 1 + 0 = \frac{5\pi}{4} - 1$$

الإجابة الصحيحة هي (b)

$$81 - \int_0^{\frac{\pi}{8}} (cos^2(2x) - 4\,sin^2(x)\,cos^2(x))\,dx = \int_0^{\frac{\pi}{8}} (cos^2(2x) - (2sin(x)\,cos(x))^2)\,dx$$

$$= \int_0^{\frac{\pi}{8}} \frac{1+cos(4x)}{2}.dx - \int_0^{\frac{\pi}{8}} (sin(2x))^2.dx = \int_0^{\frac{\pi}{8}} \frac{1+cos(4x)}{2}.dx - \int_0^{\frac{\pi}{8}} \frac{1-cos(4x)}{2}.dx$$

$$, sin^2(x) = \frac{1-cos(2x)}{2}, \cos^2(x) = \frac{1+cos(2x)}{2}, sin(2x) = 2sin(x)\,cos(x)$$

$$= [(\frac{1}{2}x + \frac{1}{8}sin(4x)) - (\frac{1}{2}x - \frac{1}{8}sin(4x))] \Big|_0^{\frac{\pi}{8}} = [\frac{1}{2}x + \frac{1}{8}sin(4x) - \frac{1}{2}x + \frac{1}{8}sin(4x)] \Big|_0^{\frac{\pi}{8}}$$

$$= \frac{1}{4}sin(4x) \Big|_0^{\frac{\pi}{8}} = (\frac{1}{4}sin(4 \times \frac{\pi}{8}) - (\frac{1}{4}sin(0)) = (\frac{1}{4}sin(\frac{\pi}{2})) - (\frac{1}{4}sin(0)) = \frac{1}{4} - 0 = \frac{1}{4}$$

الإجابة الصحيحة هي (a)

$$82 - \int_0^{\pi} 4cos^2(\frac{1}{2}x) . dx = 4\int_0^{\pi} \frac{1+cos(x)}{2} . dx = 4[\int_0^{\pi} \frac{1}{2} . dx + \int_0^{\pi} \frac{cos(x)}{2} . dx] = 4(\frac{1}{2}x + \frac{1}{2}sin(x)) \Big|_0^{\pi}$$

$$, cos^2(x) = \frac{1+cos(2x)}{2}$$

$$= 2x + 2\,sin(x)) \Big|_0^{\pi} = (2\pi + 2\,sin(\pi)) - (0 + 2\,sin(0) = (2\pi + 0) - (0+0) = 2\pi$$

الإجابة الصحيحة هي (b)

$$83 - \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \frac{cot^2(x)}{1+cot^2(x)} . dx = \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \frac{csc^2(x) - 1}{csc^2(x)} . dx = \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \frac{csc^2(x)}{csc^2(x)} - \frac{1}{csc^2(x)} . dx = \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} 1 - sin^2(x) . dx$$

$$, csc^2(x) = cot^2(x) + 1, csc(x) = \frac{1}{sin(x)}, sin^2(x) = \frac{1-cos(2x)}{2}$$

$$= \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} 1 . dx - \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \frac{1-cos(2x)}{2} . dx = [x - (\frac{1}{2}x - \frac{1}{4}sin(2x))] \Big|_{\frac{\pi}{4}}^{\frac{\pi}{2}} = (\frac{1}{2}x + \frac{1}{4}sin(2x)) \Big|_{\frac{\pi}{4}}^{\frac{\pi}{2}}$$

$$= (\frac{\pi}{4} + \frac{1}{4}sin(\pi)) - (\frac{\pi}{8} + \frac{1}{4}sin(\frac{\pi}{2})) = \left(\frac{\pi}{4} + 0\right) - \left(\frac{\pi}{8} + \frac{1}{4}\right) = \frac{\pi}{4} - \frac{\pi}{8} - \frac{1}{4} = \frac{\pi}{8} - \frac{1}{4} = \frac{\pi - 2}{8}$$

الإجابة الصحيحة هي (c)

$$84 - \int \frac{sin(2x)}{1+cos(2x)} . dx = -\frac{1}{2}\int \frac{-2\,sin(2x)}{1+cos(2x)} . dx = -\frac{1}{2}ln|1+cos(2x)| + c$$

$$, \int \frac{g`(x)}{g(x)} . dx = ln|g(x)| + c$$

الإجابة الصحيحة هي (a)

$$85 - \int cot(x) . dx = \int \frac{cos(x)}{sin(x)} . dx = ln|sin(x)| + c, \int \frac{g`(x)}{g(x)} . dx = ln|g(x)| + c, cot(x) = \frac{cos(x)}{sin(x)}$$

الإجابة الصحيحة هي (b)

$$86 - \int csc(x) . dx = \int csc(x) \times \frac{csc(x) + cot(x)}{csc(x) + cot(x)} . dx = \int \frac{csc^2(x) + csc(x)\,cot(x)}{csc(x) + cot(x)} . dx$$

$$= -\int \frac{-csc^2(x) - csc(x)\,cot(x)}{csc(x) + cot(x)} . dx = -\,ln|csc(x) + cot(x)| + c, \int \frac{g`(x)}{g(x)} . dx = ln|g(x)| + c$$

الإجابة الصحيحة هي (a)

$$87 - \int \frac{sec(x)}{sin(x) - cos(x)} . dx = \int \frac{sec(x)}{sin(x) - cos(x)} \times \frac{sec(x)}{sec(x)} . dx = \int \frac{sec^2(x)}{\tan(x) - 1} . dx$$

$$= ln|\tan(x) - 1| + c, sec(x) = \frac{1}{cos(x)}, tan(x) = \frac{sin(x)}{cos(x)}, \int \frac{g`(x)}{g(x)} . dx = ln|g(x)| + c$$

الإجابة الصحيحة هي (d)

$$88-\int \frac{sin^3(x)+cos^3(x)}{sin(x)+cos(x)}.dx=\int \frac{(sin(x)+cos(x))(sin^2(x)+cos^2(x)-sin(x)cos(x))}{sin(x)+cos(x)}.dx=$$
$$=\int (sin^2(x)+cos^2(x)-sin(x)cos(x)).dx=\int (1-sin(x)cos(x)).dx$$
$$=\int 1.dx-\frac{1}{2}\int 2\,sin(x)cos(x).dx=\int 1.dx-\frac{1}{2}\int sin(2x).dx=x+\frac{1}{4}cos(2x)+c$$
$$,sin(2x)=2sin(x)cos(x),x^3+y^3=(x+y)(x^2+y^2-xy)$$

الإجابة الصحيحة هي (d)

$$89-\int \frac{sin(2x)+2\,cos(x)}{sin^2(x)+2\,sin(x)+1}.dx=\int \frac{2sin(x)cos(x)+2\,cos(x)}{(sin(x)+1)^2}.dx=\int \frac{2\,cos(x)\,(sin(x)+1)}{(sin(x)+1)^2}.dx$$
$$=\int \frac{2\,cos(x)}{sin(x)+1}.dx=2\int \frac{cos(x)}{sin(x)+1}.dx=2\,2ln|sin(x)+1|+c$$
$$,sin(2x)=2sin(x)cos(x),(x+y)^2=(x^2+2xy+y^2)$$

الإجابة الصحيحة هي (b)

$$90-\int \frac{cos^2(x)}{1-sin(x)}.dx=\int \frac{1-sin^2(x)}{1-sin(x)}.dx=\int \frac{(1-sin(x))(1+sin(x))}{1-sin(x)}.dx$$
$$=\int (1+sin(x)).dx=x-cos(x)+c$$
$$,sin^2(x)+cos^2(x)=1\Rightarrow cos^2(x)=1-sin^2(x),x^2-y^2=(x+y)(x-y)$$

الإجابة الصحيحة هي (a)

$$91-\int (2+tan^2(x)).dx=\int (1+(1+tan^2(x))).dx=\int 1+sec^2(x).dx=\int 1.dx+\int sec^2(x).dx$$
$$=x+tan(x)+c\,,1+tan^2(x)=sec^2(x)$$

الإجابة الصحيحة هي (b)

$$92-\int (cos(\theta)(tan(\theta)+sec(\theta)))\,d\theta=\int (cos(\theta)\,tan(\theta)+cos(\theta)\,sec(\theta)))\,d\theta$$
$$=\int (cos(\theta).\frac{sin(\theta)}{cos(\theta)}+cos(\theta).\frac{1}{cos(\theta)}))\,d\theta=\int (sin(\theta)+1)\,d\theta=\int sin(\theta).d\theta+\int 1.d\theta$$
$$=-\cos(\theta)+\theta=\theta-\cos(\theta)+c\,,sec(x)=\frac{1}{cos(x)},tan(x)=\frac{sin(x)}{cos(x)}$$

الإجابة الصحيحة هي (a)

$$93-\int \frac{csc(\theta)}{csc(\theta)-sin(\theta)}d\theta=\int \frac{csc(\theta)}{csc(\theta)-sin(\theta)}\times\frac{csc(\theta)}{csc(\theta)}d\theta=\int \frac{csc^2(\theta)}{csc^2(\theta)-csc(\theta)\,sin(\theta)}d\theta$$
$$=\int \frac{csc^2(\theta)}{csc^2(\theta)-1}d\theta=\int \frac{csc^2(\theta)}{cot^2(\theta)}d\theta=\int \frac{\frac{1}{sin^2(\theta)}}{\frac{cos^2(\theta)}{sin^2(\theta)}}.d\theta=\int \frac{1}{cos^2(\theta)}.d\theta=\int \sec^2(\theta).d\theta$$
$$=tan(\theta)+c,csc^2(x)=cot^2(x)+1,csc(x)=\frac{1}{sin(x)},sec(x)=\frac{1}{cos(x)},cot(x)=\frac{cos(x)}{sin(x)}$$

الإجابة الصحيحة هي (b)

$$94-\int (1-cot^2(x)).dx=\int \left(1-(csc^2(x)-1)\right).dx=\int (1-csc^2(x)+1).dx$$
$$=\int 2.dx-\int csc^2(x).dx=2x+cot(x)+c\,,csc^2(x)=cot^2(x)+1$$

الإجابة الصحيحة هي (d)

$$95-\int \frac{1}{1+\cos(2x)}.dx=\int \frac{1}{(\sin^2(x)+\cos^2(x))+(\cos^2(x)-\sin^2(x))}.dx$$
$$=\int \frac{1}{\sin^2(x)+\cos^2(x)+\cos^2(x)-\sin^2(x)}.dx=\int \frac{1}{2\cos^2(x)}.dx=\frac{1}{2}\int \frac{1}{\cos^2(x)}.dx$$
$$=\frac{1}{2}\int \sec^2(x).dx=\frac{1}{2}\tan(x)+c$$
$$,\sin^2(x)+\cos^2(x)=1, \cos^2(x)-\sin^2(x)=\cos(2x), \sec(x)=\frac{1}{\cos(x)}$$

الإجابة الصحيحة هي (b)

$$96-\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}((\sin^2(x)+\cos^2(x))+(\tan^2(x)-\sec^2(x)))\,dx=\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}(1)+(-1)\,dx=\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}(0)\,dx=0\Big|_{-\frac{\pi}{2}}^{\frac{\pi}{2}}=0$$
$$,\sin^2(x)+\cos^2(x)=1, 1+\tan^2(x)=\sec^2(x)\Rightarrow \tan^2(x)-\sec^2(x)=-1$$

الإجابة الصحيحة هي (d)

$$97-\int_{-\pi}^{\pi}(\cos(2x)+2\sin^2(x))\,dx=\int_{-\pi}^{\pi}(\cos(2x)+2(\frac{1}{2}-\frac{\cos(2x)}{2})\,dx=\int_{-\pi}^{\pi}(\cos(2x)+1-\cos(2x))\,dx$$
$$=\int_{-\pi}^{\pi}(1)\,dx=x\Big|_{-\pi}^{\pi}=\pi-(-\pi)=\pi+\pi=2\pi, \sin^2(x)=\frac{1-\cos(2x)}{2}$$

الإجابة الصحيحة هي (a)

$$98-\int_0^{\frac{\pi}{4}}\frac{1}{1-\sin(x)}.dx=\int_0^{\frac{\pi}{4}}\left(\frac{1}{1-\sin(x)}\times\frac{1+\sin(x)}{1+\sin(x)}\right).dx=\int_0^{\frac{\pi}{4}}\frac{1+\sin(x)}{1-\sin^2(x)}.dx=\int_0^{\frac{\pi}{4}}\frac{1+\sin(x)}{\cos^2(x)}.dx$$
$$=\int_0^{\frac{\pi}{4}}\frac{1}{\cos^2(x)}.dx+\int_0^{\frac{\pi}{4}}\frac{\sin(x)}{\cos^2(x)}.dx=\int_0^{\frac{\pi}{4}}\sec^2(x).dx+\int_0^{\frac{\pi}{4}}\frac{\sin(x)}{\cos(x)}\times\frac{1}{\cos(x)}.dx$$
$$,\sec(x)=\frac{1}{\cos(x)}, \tan(x)=\frac{\sin(x)}{\cos(x)}$$
$$=\int_0^{\frac{\pi}{4}}\sec^2(x).dx+\int_0^{\frac{\pi}{4}}\tan(x)\sec(x).dx=(\tan(x)+\sec(x))\Big|_0^{\frac{\pi}{4}}$$
$$=(\tan\left(\frac{\pi}{4}\right)+\sec(\frac{\pi}{4}))-(\tan(0)+\sec(0))=(1+\sqrt{2})-(0+1)=1+\sqrt{2}-0-1=\sqrt{2}$$

الإجابة الصحيحة هي (b)

$$99-\int_0^{\frac{\pi}{4}}\frac{1+\sin(x)}{\cos^2(x)}.dx=\int_0^{\frac{\pi}{4}}\frac{1+\sin(x)}{1-\sin^2(x)}.dx=\int_0^{\frac{\pi}{4}}\frac{1+\sin(x)}{(1-\sin(x))(1+\sin(x))}.dx=\int_0^{\frac{\pi}{4}}\frac{1}{1-\sin(x)}.dx$$
$$=\sqrt{2}$$

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$$,\sin^2(x)+\cos^2(x)=1\Rightarrow \cos^2(x)=1-\sin^2(x), x^2-y^2=(x+y)(x-y)$$

الإجابة الصحيحة هي (b)

$$100-\int((\csc(x)-\sec(x))(\sin(x)+\cos(x))).dx$$
$$=\int(\csc(x)\sin(x)+\csc(x)\cos(x)-\sec(x)\sin(x)-\sec(x)\cos(x)).dx$$
$$=\int(1+\frac{\cos(x)}{\sin(x)}-\frac{\sin(x)}{\cos(x)}-1).dx=\int(\cot(x)-\tan(x)).dx$$

$$,\sec(x)=\frac{1}{\cos(x)},\tan(x)=\frac{\sin(x)}{\cos(x)},\csc(x)=\frac{1}{\sin(x)},\cot(x)=\frac{\cos(x)}{\sin(x)}$$

$$=\int\cot(x).dx-\int\tan(x).dx=\int\frac{\cos(x)}{\sin(x)}.dx+\int\frac{-\sin(x)}{\cos(x)}.dx=\ln|\sin(x)|+\ln|\cos(x)|$$

$$=\ln|\sin(x)\cos(x)|+c,\int\frac{g`(x)}{g(x)}.dx=\ln|g(x)|+c,\ln|x|+\ln|y|=\ln|xy|$$

الإجابة الصحيحة هي (c)

$$101-\int\frac{1+\tan^2(x)}{1-\tan(x)}.dx=-\int\frac{-\sec^2(x)}{1-\tan(x)}.dx=-\ln|1-\tan(x)|+c$$

$$,1+\tan^2(x)=\sec^2(x),\int\frac{g`(x)}{g(x)}.dx=\ln|g(x)|+c$$

الإجابة الصحيحة هي (c)

$$102-\int\frac{1-\tan^2(x)}{1+\tan(x)}.dx=\int\frac{(1+\tan(x))(1-\tan(x))}{1+\tan(x)}.dx=\int 1-\tan(x).dx$$

$$=\int 1\,dx-\int\tan(x).dx=\int 1\,dx+\int\frac{-\sin(x)}{\cos(x)}.dx=x+\ln|\cos(x)|+c$$

$$,\tan(x)=\frac{\sin(x)}{\cos(x)},x^2-y^2=(x+y)(x-y),\int\frac{g`(x)}{g(x)}.dx=\ln|g(x)|+c$$

الإجابة الصحيحة هي (a)

$$103-\int\frac{1}{\sin^2(x)\cos^2(x)}.dx=\int\frac{1}{(\sin(x)\cos(x))^2}.dx=\int\frac{1}{(\frac{\sin(2x)}{2})^2}.dx=\int\frac{1}{\frac{\sin^2(2x)}{4}}.dx$$

$$=\int\frac{4}{\sin^2(2x)}.dx=4\int\frac{1}{\sin^2(2x)}.dx=4\int\csc^2(2x).dx=4\times-\frac{1}{2}\cot(2x)=-2\cot(2x)+c$$

$$,\csc(x)=\frac{1}{\sin(x)},\sin(2x)=2\sin(x)\cos(x)\Longrightarrow\frac{\sin(2x)}{2}=\sin(x)\cos(x)$$

الإجابة الصحيحة هي (b)

$$104-\int(\sin^2(x)+\sin^2(x)\tan^2(x).dx=\int(\sin^2(x)(1+\tan^2(x))).dx=\int\sin^2(x)\sec^2(x).dx$$

$$=\int\frac{\sin^2(x)}{\cos^2(x)}.dx=\int\tan^2(x).dx=\int(\sec^2(x)-1).dx=\int\sec^2(x).dx-\int 1.dx=\tan(x)-x+c$$

$$,1+\tan^2(x)=\sec^2(x),\sec(x)=\frac{1}{\cos(x)},\tan(x)=\frac{\sin(x)}{\cos(x)}$$

الإجابة الصحيحة هي (c)

$$105-\int_{\frac{\pi}{4}}^{\frac{\pi}{2}}(\frac{\cos(x)}{\sin(x)})\,dx=\ln|\sin(x)|\Big|_{\frac{\pi}{4}}^{\frac{\pi}{2}}=\ln\left|\sin(\frac{\pi}{2})\right|-\ln\left|\sin(\frac{\pi}{4})\right|=\ln(1)-\ln(\frac{1}{\sqrt{2}})=0-\ln(\frac{1}{\sqrt{2}})$$

$$=-\ln(\frac{1}{\sqrt{2}})=-\ln\left(2^{-\frac{1}{2}}\right)=\ln\left(2^{\frac{1}{2}}\right)=\ln\sqrt{2},\int\frac{g`(x)}{g(x)}.dx=\ln|g(x)|+c,$$

الإجابة الصحيحة هي (b)

$$106-\int\tan^2(x).dx=\int(\sec^2(x)-1).dx=\int\sec^2(x).dx-\int 1.dx=\tan(x)-x+c$$

$$,1+\tan^2(x)=\sec^2(x),\sec(x)=\frac{1}{\cos(x)},\tan(x)=\frac{\sin(x)}{\cos(x)},$$

الإجابة الصحيحة هي (d)

$$107-\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}(\sin(x).dx=-\cos(x)\Big|_{-\frac{\pi}{2}}^{\frac{\pi}{2}}=(-\cos(\frac{\pi}{2}))-(-\cos(-\frac{\pi}{2}))=0-0=0,\cos(-x)=\cos(x)$$

الإجابة الصحيحة هي (b)

$$108-\int \tan^3(x)\cos(x).dx=\int \tan^2(x)\tan(x)\cos(x).dx=\int (\sec^2(x)-1)\sin(x).dx$$
$$=\int (\sec^2(x)\sin(x).dx-\int \sin(x).dx=\int \tan(x)\sec(x).dx-\int \sin(x).dx=\sec(x)+\cos(x)+c$$
$$,1+\tan^2(x)=\sec^2(x),\sec(x)=\frac{1}{\cos(x)},\tan(x)=\frac{\sin(x)}{\cos(x)}$$

الإجابة الصحيحة هي (d)

$$109-\int \cot^3(x)\sin(x).dx=\int \cot^2(x)\cot(x)\sin(x).dx=\int (\csc^2(x)-1)\sin(x).dx$$
$$=\int \csc^2(x)\sin(x).dx-\int \sin(x).dx=\int \csc(x).dx-\int \sin(x).dx$$
$$=\int \csc(x)\times\frac{\csc(x)+\cot(x)}{\csc(x)+\cot(x)}.dx-\int \sin(x).dx=\int \frac{\csc^2(x)+\csc(x)\cot(x)}{\csc(x)+\cot(x)}.dx-\int \sin(x).dx$$
$$=-\int \frac{-\csc^2(x)-\csc(x)\cot(x)}{\csc(x)+\cot(x)}.dx-\int \sin(x).dx=-\ln|\csc(x)+\cot(x)|+\cos(x)+c$$
$$,\csc^2(x)=\cot^2(x)+1,\csc(x)=\frac{1}{\sin(x)},\int \frac{g`(x)}{g(x)}.dx=\ln|g(x)|+c$$

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$$\Rightarrow \int \cot^3(x)\sin(x).dx=\int \frac{\cos^3(x)}{\sin^3(x)}.\sin(x).dx=\int \frac{\cos^3(x)}{\sin^2(x)}.dx=\int \frac{\cos^2(x)\cos(x)}{\sin^2(x)}.dx$$
$$=\int \frac{(1-\sin^2(x)).\cos(x)}{\sin^2(x)}.dx=\int \frac{\cos(x)-\cos(x)\sin^2(x)}{\sin^2(x)}.dx$$
$$=\int \frac{\cos(x)}{\sin^2(x)}.dx-\int \frac{\cos(x)\sin^2(x)}{\sin^2(x)}.dx=\int \cot(x)\csc(x).dx-\int \cos(x).dx$$
$$=-\csc(x)-\sin(x)+c$$
$$,\cot(x)=\frac{\cos(x)}{\sin(x)},\sin^2(x)+\cos^2(x)=1\Rightarrow \cos^2(x)=1-\sin^2(x),\csc(x)=\frac{1}{\sin(x)}$$

الإجابة الصحيحة هي (a)

$$110-\int \frac{4}{\sin^2(x)}.dx=4\int \frac{1}{\sin^2(x)}.dx=4\int (\csc^2(x).dx=-4\cot(x)+c,\csc(x)=\frac{1}{\sin(x)}$$

الإجابة الصحيحة هي (b)

$$111-\int_0^{\frac{\pi}{2}}(2\sec\left(\frac{1}{2}x\right)-\tan\left(\frac{1}{2}x\right).dx=2\int_0^{\frac{\pi}{2}}\sec\left(\frac{1}{2}x\right).dx-\int_0^{\frac{\pi}{2}}\tan\left(\frac{1}{2}x\right).dx$$
$$=2\int_0^{\frac{\pi}{2}}\sec(\frac{1}{2}x)\times\frac{\sec(\frac{1}{2}x)+\tan(\frac{1}{2}x)}{\sec(\frac{1}{2}x)+\tan(\frac{1}{2}x)}.dx-\int_0^{\frac{\pi}{2}}\frac{\sin(\frac{1}{2}x)}{\cos(\frac{1}{2}x)}.dx,\tan(x)=\frac{\sin(x)}{\cos(x)}$$
$$=2\int_0^{\frac{\pi}{2}}\frac{\sec^2(\frac{1}{2}x)+\sec(\frac{1}{2}x)\tan(\frac{1}{2}x)}{\sec(\frac{1}{2}x)+\tan(\frac{1}{2}x)}.dx+\int_0^{\frac{\pi}{2}}\frac{-\sin(\frac{1}{2}x)}{\cos(\frac{1}{2}x)}.dx$$
$$=4\int_0^{\frac{\pi}{2}}\frac{\frac{1}{2}\sec^2(\frac{1}{2}x)+\frac{1}{2}\sec(\frac{1}{2}x)\tan(\frac{1}{2}x)}{\sec(\frac{1}{2}x)+\tan(\frac{1}{2}x)}.dx+\int_0^{\frac{\pi}{2}}\frac{-\sin(\frac{1}{2}x)}{\cos(\frac{1}{2}x)}.dx$$

$$= 4\ln\left|sec(\frac{1}{2}x) + tan(\frac{1}{2}x)\right| + \ln\left|cos(\frac{1}{2}x)\right|\Big|_0^{\frac{\pi}{2}}, \int \frac{g`(x)}{g(x)}.dx = \ln|g(x)| + c$$

$$= \left(4\ln\left|sec\left(\frac{\pi}{4}\right) + tan\left(\frac{\pi}{4}\right)\right| + \ln\left|cos\left(\frac{\pi}{4}\right)\right|\right) - (4\ln|sec(0) + tan(0)| + \ln|cos(0)|$$

$$= (4\ln|\sqrt{2}+1| + \ln\left|\frac{1}{\sqrt{2}}\right|) - (4\ln|1+0| + \ln|1|) = (4\ln(\sqrt{2}+1) + \ln(\frac{1}{\sqrt{2}})) - (0)$$

$$= 4\ln(\sqrt{2}+1) + \ln(\frac{1}{\sqrt{2}}) = 4\ln((\sqrt{2}+1) \times \frac{1}{\sqrt{2}}) = 4\ln(1 + \frac{1}{\sqrt{2}})$$

$$, sec(x) = \frac{1}{cos(x)}, \ln|x| + \ln|y| = \ln|xy|$$

الإجابة الصحيحة هي (b)

$$112 - \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} (sec^2(x)\, csc^2(x).dx = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} (\frac{1}{cos^2(x)} \times \frac{1}{sin^2(x)}.dx = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{1}{cos^2(x)\, sin^2(x)}.dx$$

$$= \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{1}{(cos(x)\, sin(x))^2}.dx = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{1}{(cos(x)\, sin(x))^2}.dx = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{1}{\left(\frac{sin(2x)}{2}\right)^2}.dx = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{4}{sin^2(2x)}.dx$$

$$= 4\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{4}{sin^2(2x)}.dx = 4\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{4}{sin^2(2x)}.dx = 4\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} csc^2(2x).dx = 4 \times -\frac{1}{2}cot(2x) = -2\,cot(2x)\Big|_{\frac{\pi}{6}}^{\frac{\pi}{3}}$$

$$, csc(x) = \frac{1}{sin(x)}, sec(x) = \frac{1}{cos(x)}, sin(2x) = 2sin(x)\, cos(x) \Rightarrow \frac{sin(2x)}{2} = sin(x)\, cos(x)$$

$$= \left(-2\,cot(\frac{2\pi}{3})\right) - \left(-2\,cot(\frac{2\pi}{6})\right) = \left(-2\,cot(\frac{2\pi}{3})\right) - \left(-2\,cot(\frac{\pi}{3})\right) = (-2(-\sqrt{3}) - (-2\sqrt{3})$$

$$= 2\sqrt{3} + 2\sqrt{3} = 4\sqrt{3}$$

الإجابة الصحيحة هي (b)

$$113 - \int sin^2(x).dx = \int \frac{1-cos(2x)}{2}.dx = \frac{1}{2}x - \frac{1}{4}sin(2x) + c, sin^2(x) = \frac{1-cos(2x)}{2}$$

الإجابة الصحيحة هي (b)

$$114 - \int cos^2(x).dx = \int \frac{1+cos(2x)}{2}.dx = \frac{1}{2}x + \frac{1}{4}sin(2x) + c, cos^2(x) = \frac{1+cos(2x)}{2}$$

الإجابة الصحيحة هي (a)

$$115 - \int sin^4(x).dx = \int (sin^2(x))^2.dx = \int \left(\frac{1}{2} - \frac{cos(2x)}{2}\right)^2.dx = \int \frac{1}{4} + \frac{cos^2(2x)}{4} - \frac{cos(2x)}{2}.dx$$

$$= \int \frac{1}{4} + \frac{1}{4}(\frac{1}{2} + \frac{cos(4x)}{2}) - \frac{cos(2x)}{2}.dx = \int \frac{1}{4} + \frac{1}{8} + \frac{cos(4x)}{8} - \frac{cos(2x)}{2}.dx$$

$$= \int \frac{3}{8} + \frac{cos(4x)}{8} - \frac{cos(2x)}{2}.dx = \int \frac{3}{8}.dx + \int \frac{cos(4x)}{8}.dx - \int \frac{cos(2x)}{2}.dx$$

$$= \frac{3}{8}x + \frac{1}{32}sin(4x) - \frac{1}{4}sin(2x) + c, sin^2(x) = \frac{1-cos(2x)}{2}, cos^2(x) = \frac{1+cos(2x)}{2}$$

الإجابة الصحيحة هي (d)

$$116-\int cos^4(x)\,.dx=\int (cos^2(x))^2\,.dx=\int\left(\frac{1}{2}+\frac{cos(2x)}{2}\right)^2.dx=\int\frac{1}{4}+\frac{cos^2(2x)}{4}+\frac{cos(2x)}{2}.dx$$

$$=\int\frac{1}{4}+\frac{1}{4}\left(\frac{1}{2}+\frac{cos(4x)}{2}\right)+\frac{cos(2x)}{2}.dx=\int\frac{1}{4}+\frac{1}{8}+\frac{cos(4x)}{8}+\frac{cos(2x)}{2}.dx$$

$$=\int\frac{3}{8}+\frac{cos(4x)}{8}+\frac{cos(2x)}{2}.dx=\int\frac{3}{8}.dx+\int\frac{cos(4x)}{8}.dx+\int\frac{cos(2x)}{2}.dx$$

$$=\frac{3}{8}x+\frac{1}{32}sin(4x)+\frac{1}{4}sin(2x)+c\,,\ cos^2(x)=\frac{1+cos(2x)}{2}$$

الإجابة الصحيحة هي (d)

$$117-\int_0^{\frac{\pi}{12}}\frac{1}{3}tan(3x)\,.dx=\frac{1}{3}\int_0^{\frac{\pi}{12}}tan(3x).dx=\frac{1}{3}\int_0^{\frac{\pi}{12}}\frac{sin(3x)}{cos(3x)}.dx=\frac{1}{-9}\int_0^{\frac{\pi}{12}}\frac{-3\,sin(3x)}{cos(3x)}.dx$$

$$=-\frac{1}{9}ln|cos(3x)|\Big|_0^{\frac{\pi}{12}}=\left(-\frac{1}{9}ln\left|cos\left(3\times\frac{\pi}{12}\right)\right|\right)-\left(-\frac{1}{9}ln|cos(3\times 0)|\right)$$

$$=\left(-\frac{1}{9}ln\left|cos\left(\frac{\pi}{4}\right)\right|\right)-\left(-\frac{1}{9}ln|cos(0)|\right)=-\frac{1}{9}ln\left(\frac{1}{\sqrt{2}}\right)+\frac{1}{9}ln(1)=-\frac{1}{9}ln\left(\frac{1}{\sqrt{2}}\right)=-\frac{1}{9}ln\left(\frac{1}{2^{\frac{1}{2}}}\right)$$

$$=-\frac{1}{9}ln\left(2^{-\frac{1}{2}}\right)=\frac{1}{9}ln\left(2^{\frac{1}{2}}\right)=\frac{ln\sqrt{2}}{9}\,,\ tan(x)=\frac{sin(x)}{cos(x)},\int\frac{g`(x)}{g(x)}.dx=ln|g(x)|+c$$

الإجابة الصحيحة هي (c)

$$118-\int\frac{1-tan(x)}{1+tan(x)}.dx=\int\frac{1-\frac{sin(x)}{cos(x)}}{1+\frac{sin(x)}{cos(x)}}.dx=\int\frac{\frac{cos(x)-sin(x)}{cos(x)}}{\frac{cos(x)+sin(x)}{cos(x)}}.dx=\int\frac{cos(x)-sin(x)}{cos(x)+sin(x)}.dx$$

$$=ln|cos(x)+sin(x)|\,,\ tan(x)=\frac{sin(x)}{cos(x)},\int\frac{g`(x)}{g(x)}.dx=ln|g(x)|+c$$

الإجابة الصحيحة هي (d)

$$119-\int\frac{1+tan(x)}{1-tan(x)}.dx=\int\frac{1+\frac{sin(x)}{cos(x)}}{1-\frac{sin(x)}{cos(x)}}.dx=\int\frac{\frac{cos(x)+sin(x)}{cos(x)}}{\frac{cos(x)-sin(x)}{cos(x)}}.dx=\int\frac{cos(x)+sin(x)}{cos(x)-sin(x)}.dx$$

$$=-\int\frac{-cos(x)-sin(x)}{cos(x)-sin(x)}.dx=ln|cos(x)-sin(x)|$$

$$,tan(x)=\frac{sin(x)}{cos(x)},\int\frac{g`(x)}{g(x)}.dx=ln|g(x)|+c$$

الإجابة الصحيحة هي (b)

$$120-\int cos(3x)\,cos(x)\,.dx=\int\frac{1}{2}[cos(2x)+cos(4x)].dx,\ cos(x)\,cos(y)=\frac{1}{2}[cos(x-y)+cos(x+y)]$$

$$=\frac{1}{2}\left[\int cos(2x).dx+\int cos(4x)\,.dx\right]=\frac{1}{2}\left(\frac{1}{2}sin(2x)+\frac{1}{4}sin(4x)\right)=\frac{1}{4}sin(2x)+\frac{1}{8}sin(4x)+c$$

الإجابة الصحيحة هي (a)

$$121-\int\frac{x^2-x-1}{x+1}.dx=\int x-2+\frac{1}{x+1}.dx,\ \frac{x^2-x-1}{x+1}=x-2+\frac{1}{x+1}$$ بالقسمة الطويلة

$$=\int x.dx-\int 2\,.dx+\int\frac{1}{x+1}\,.dx=\frac{x^2}{2}-2x+ln|x+1|+c,\int\frac{g`(x)}{g(x)}.dx=ln|g(x)|+c$$

الإجابة الصحيحة هي (c)

$$122-\int \frac{x^2+x+1}{x^2+1}.dx=\int 1+\frac{x}{x^2+1}.dx,\ \frac{x^2+x+1}{x^2+1}=1+\frac{x}{x^2+1}$$ بالقسمة الطويلة

$$=\int 1.dx+\int \frac{x}{x^2+1}.dx=\int 1.dx+\frac{1}{2}\int \frac{2x}{x^2+1}.dx=x+\frac{1}{2}ln|x^2+1|+c$$

$$,\int \frac{g`(x)}{g(x)}.dx=ln|g(x)|+c$$

الإجابة الصحيحة هي (c)

$$123-\int \frac{x^3+x}{x+1}.dx=\int x^2-x+2-\frac{2}{x+1}.dx,\ \frac{x^3+x}{x+1}=x^2-x+2-\frac{2}{x+1}$$ بالقسمة الطويلة

$$=\int x^2.dx-\int x.dx+\int 2.dx-\int \frac{2}{x+1}.dx=\int x^2.dx-\int x.dx+\int 2.dx-2\int \frac{1}{x+1}.dx$$

$$=\frac{x^3}{3}-\frac{x^2}{2}+2x-2\,ln|x+1|+c,\int \frac{g`(x)}{g(x)}.dx=ln|g(x)|+c$$

الإجابة الصحيحة هي (d)

$$124-f(x)=|4x^2-1|,\int_0^1 f(x).dx$$

$$\Rightarrow 4x^2-1=0\Rightarrow 4x^2=1\Rightarrow x^2=\frac{1}{4}\Rightarrow x=\pm\frac{1}{2}$$

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$-\frac{1}{2}$ 0 $\frac{1}{2}$ 1

$$\therefore |4x^2-1|=\begin{cases}4x^2-1, x\le -\frac{1}{2}\\ 1-4x^2, -\frac{1}{2}<x<\frac{1}{2}\\ 4x^2-1, x\ge \frac{1}{2}\end{cases}$$

$$\therefore \int_0^1 f(x).dx=\int_0^1 |4x^2-1|.dx=\int_0^{\frac{1}{2}}(1-4x^2).dx+\int_{\frac{1}{2}}^1(4x^2-1).dx$$

$$=(\int_0^{\frac{1}{2}}1.dx-4\int_0^{\frac{1}{2}}x^2.dx)+(4\int_{\frac{1}{2}}^1 x^2.dx-\int_{\frac{1}{2}}^1 1.dx)=(x-4\frac{x^3}{3})\Big|_0^{\frac{1}{2}}+(4\frac{x^3}{3}-x)\Big|_{\frac{1}{2}}^{1}=(1-\frac{4}{3})$$

$$=((\frac{1}{2}-\frac{1}{6})-(0))+((\frac{4}{3}-1)-(\frac{1}{6}-\frac{1}{2}))=\frac{1}{3}+(\frac{1}{3}+\frac{1}{3})=1$$

الإجابة الصحيحة هي (d)

$$125-f(x)=|x^2-4x+3|,\int_0^4 f(x).dx$$

$$\Rightarrow x^2-4x+3=0\Rightarrow (x-3)(x-1)=0\Rightarrow x=1,3$$

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0 1 3 4

$$\therefore |x^2-4x+3|=\begin{cases}x^2-4x+3, x\le 1\\ -x^2+4x-3, 1<x<3\\ x^2-4x+3, x\ge 3\end{cases}$$

$$\therefore \int_0^4 f(x).dx=\int_0^4 |x^2-4x+3|.dx=\int_0^1(x^2-4x+3).dx+\int_1^3(-x^2+4x-3).dx+\int_3^4(x^2-4x+3).dx$$

$$= (\int_0^1 x^2.dx - 4\int_0^1 x.dx + \int_0^1 3.dx) + (\int_1^3 -x^2.dx + 4\int_1^3 x.dx - \int_1^3 3.dx) + (\int_3^4 x^2.dx - 4\int_3^4 x.dx + \int_3^4 3.dx)$$

$$= (\frac{x^3}{3} - 4\frac{x^2}{2} + 3x)\Big|_0^1 + (-\frac{x^3}{3} + 4\frac{x^2}{2} - 3x)\Big|_1^3 + (\frac{x^3}{3} - 4\frac{x^2}{2} + 3x)\Big|_3^4 +$$

$$= ((\frac{1}{3} - 2 + 3) - (0)) + ((-9 + 18 - 9) - (-\frac{1}{3} + 2 - 3)) + ((\frac{64}{3} - 32 + 12) - (9 - 18 + 9))$$

$$= (-\frac{2}{3}) + (0 - (-\frac{4}{3})) + ((\frac{64}{3} - 20) - (0)) = -\frac{2}{3} + \frac{4}{3} + \frac{64}{3} - 20 = \frac{66}{3} - 20 = 22 - 20 = 2$$

الإجابة الصحيحة هي (a)

$$126 - f(x) = |sin(x)|, \int_0^{2\pi} f(x).dx$$

$$\Rightarrow sin(x) = 0 \Rightarrow x = 0, \pi, 2\pi$$

++++++−−−−−−

0 π 2π

$$\therefore |sin(x)| = \begin{cases} sin(x), 0 < x < \pi \\ -sin(x), \pi < x < 2\pi \end{cases}$$

$$\therefore \int_0^{2\pi} f(x).dx = \int_0^1 |sin(x)|.dx = \int_0^{\pi} (sin(x)).dx + \int_{\pi}^{2\pi} (-sin(x)).dx$$

$$= (-\cos(x))\Big|_0^{\pi} + (\cos(x))\Big|_{\pi}^{2\pi} = (-\cos(\pi) - (-\cos(0))) + (\cos(2\pi) - \cos(\pi))$$

$$= \big(1 - (-1)\big) + (1 - (-1) = 2 + 2 = 4$$

الإجابة الصحيحة هي (a)

$$127 - f(x) = \begin{cases} |1 - x|, -2 \le x < 2 \\ 2x \quad , x \ge 2 \end{cases}, \int_{-2}^{3} f(x).dx$$

$$\Rightarrow 1 - x = 0 \Rightarrow x = 1$$

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$$\therefore |x^2 - 4x + 3| = \begin{cases} 1 - x, x < 1 \\ x - 1, 1 < x < 2 \\ 2x, x \ge 2 \end{cases}$$

$$\therefore \int_{-2}^{3} f(x).dx = \int_{-2}^{1} (1 - x).dx + \int_1^2 (x - 1).dx + \int_2^3 (2x).dx$$

$$= (\int_{-2}^{1} 1.dx - \int_{-2}^{1} x.dx) + (\int_1^2 x.dx - \int_{-2}^{1} 1.dx) + \int_2^3 (2x).dx$$

$$= (x - \frac{x^2}{2})\Big|_{-2}^{1} + (\frac{x^2}{2} - x)\Big|_1^2 + (x^2)\Big|_2^3$$

$$= ((1 - \frac{1}{2}) - (-2 - 2)) + ((2 - 2) - (\frac{1}{2} - 1)) + (9 - 4) = \frac{1}{2} + 4 + 0 + \frac{1}{2} + 5 = 10$$

الإجابة الصحيحة هي (c)

$$128-f(x)=\begin{cases}2\cos^2\left(\frac{1}{2}x\right),0\le x<\frac{\pi}{2}\\ 2\sin^2(\frac{1}{2}x)\ ,\frac{\pi}{2}<x\le\pi\end{cases}=,\int_0^{\pi}f(x).dx$$

$$\Rightarrow f(x)=\begin{cases}1+\cos(x),0\le x<\frac{\pi}{2}\\ 1-\cos(x)\ ,\frac{\pi}{2}<x\le\pi\end{cases},\cos^2(x)=\frac{1+\cos(2x)}{2}\Rightarrow 2\cos^2(x)=1+\cos(2x)$$

$$,\sin^2(x)=\frac{1-\cos(2x)}{2}\Rightarrow 2\sin^2(x)=1-\cos(2x)$$

$$\therefore\int_0^{\pi}f(x).dx=\int_0^{\frac{\pi}{2}}(1+\cos(x)).dx+\int_{\frac{\pi}{2}}^{\pi}(1-\cos(x)).dx$$

$$=(\int_0^{\frac{\pi}{2}}1.dx+\int_0^{\frac{\pi}{2}}\cos(x).dx)+(\int_{\frac{\pi}{2}}^{\pi}1.dx-\int_{\frac{\pi}{2}}^{\pi}\cos(x).dx)=(x+\sin(x))\Big|_0^{\frac{\pi}{2}}+(x-\sin(x))\Big|_{\frac{\pi}{2}}^{\pi}$$

$$=((\frac{\pi}{2}+\sin(\frac{\pi}{2}))-(0+\sin(0)))+((\pi-\sin(\pi))-(\frac{\pi}{2}-\sin(\frac{\pi}{2})))$$

$$=\frac{\pi}{2}+1-0+\pi-0-\frac{\pi}{2}+1=\pi+2$$

الإجابة الصحيحة هي (c)

$$129-f(x)=\begin{cases}\frac{4}{x}\quad ,1\le x<2\\ 2^{0.5x}-2,2\le x<4\end{cases}=,\int_1^{4}f(x).dx$$

$$\therefore\int_0^{\pi}f(x).dx=\int_1^{2}\frac{4}{x}.dx+\int_2^{4}(2^{0.5x}-2).dx=(4\int_1^{2}\frac{1}{x}.dx)+(\int_2^{4}2^{0.5x}.dx-\int_2^{4}2.dx)$$

$$=4\ln|x|\Big|_1^2+(2\frac{2^{0.5x}}{\ln(2)}-2x)\Big|_2^4=4(\ln|2|-\ln|1|)+((2\frac{2^2}{\ln(2)}-4)-(2\frac{2^1}{\ln(2)}-2))$$

$$=4\ln(2)+\left(\frac{8}{\ln(2)}-4-\frac{4}{\ln(2)}+2\right)=4\ln(2)+\left(\frac{4}{\ln(2)}-2\right)=4\ln(2)+\left(\frac{4}{\ln(2)}-2\right)$$

الإجابة الصحيحة هي (c)

$$130-\int_a^{2a}(\frac{2x-1}{x}).dx=\ln(2)),a>0$$

$$\therefore\int_a^{2a}(\frac{2x-1}{x}).dx=\int_a^{2a}(2-\frac{1}{x}).dx=\int_a^{2a}2.dx-\int_a^{2a}\frac{1}{x}.dx=2x\Big|_a^{2a}-\ln|x|\Big|_a^{2a}$$

$$=(4a-2a)-(\ln(2a)-\ln(a))=2a-\ln(\frac{2a}{a})=2a-\ln(2)\Rightarrow\therefore 2a-\ln(2)=\ln(2)$$

$$\Rightarrow 2a=2\ln(2)\Rightarrow a=\ln(2)$$

الإجابة الصحيحة هي (d)

$$131-\int_a^{2a}(\frac{x}{x^2+4}).dx=\ln(\sqrt{2})),a\ne 0$$

$$\therefore\int_a^{2a}\frac{x}{x^2+4}.dx=\frac{1}{2}\int_a^{2a}\frac{2x}{x^2+4}.dx=\frac{1}{2}\ln|x^2+4|\Big|_a^{2a}=\frac{1}{2}(\ln(4a^2+4)-\ln(a^2+4))$$

$$=\frac{1}{2}\ln(\frac{4a^2+4}{a^2+4})=\ln(\sqrt{2})\Rightarrow\ln(\frac{4a^2+4}{a^2+4})=2\ln(\sqrt{2})\Rightarrow\ln(\frac{4a^2+4}{a^2+4})=\ln((2^{\frac{1}{2}})^2)$$

$$\Rightarrow ln(\frac{4a^2+4}{a^2+4}) = ln(2) \Rightarrow \therefore \frac{4a^2+4}{a^2+4} = 2 \Rightarrow 2a^2+8 = 4a^2+4 \Rightarrow 2a^2 = 4 \Rightarrow a^2 = 2$$
$$\Rightarrow \therefore a = \pm\sqrt{2}$$

الإجابة الصحيحة هي (a)

$$132 - \int_1^a (\frac{dx}{2x}) = 1, a > 0$$
$$\therefore \int_1^a (\frac{1}{2x}).dx = \frac{1}{2}\int_1^a (\frac{1}{x}).dx = \frac{1}{2} ln|x|\Big|_1^a = \frac{1}{2}(ln(a) - ln(1)) = \frac{1}{2} ln(a) \Rightarrow \frac{1}{2} ln(a) = 1$$
$$\therefore ln(a) = 2 \Rightarrow e^2 = a$$

الإجابة الصحيحة هي (c)

$$133 - \int_1^{2a} (\frac{2}{2x-1}).dx = 1, a > 0$$
$$\therefore \int_1^{2a} (\frac{2}{2x-1}).dx = ln|2x-1|\Big|_1^{2a} = ln(2a-1) - ln(1) = ln(2a-1) \Rightarrow ln(2a-1) = 1$$
$$\therefore ln(a) = 1 \Rightarrow e^1 = 2a - 1 \Rightarrow 2a = e + 1 \Rightarrow a = \frac{e+1}{2}$$

الإجابة الصحيحة هي (b)

$$134 - y = \int (sin(\frac{\pi}{2} - 2x))\,dx, y = 1, x = \frac{\pi}{4}$$
$$y = \int (sin(\frac{\pi}{2} - 2x))\,dx = -(-\frac{1}{2}cos(\frac{\pi}{2} - 2x)) = \frac{1}{2}cos\left(\frac{\pi}{2} - 2x\right) = \frac{1}{2}sin(2x) + c$$
$$\therefore \frac{1}{2}sin(2x) + c = 1 \Rightarrow \frac{1}{2}sin\left(2 \times \frac{\pi}{4}\right) + c = 1 \Rightarrow \frac{1}{2}sin\left(\frac{\pi}{2}\right) + c = 1 \Rightarrow \frac{1}{2} + c = 1 \Rightarrow c = \frac{1}{2}$$
$$\Rightarrow y = \frac{1}{2}sin(2x) + \frac{1}{2} = \frac{1 + sin(2x)}{2}$$

الإجابة الصحيحة هي (c)

$$134 - y = \int (sin(\frac{\pi}{2} - 2x))\,dx, y = 1, x = \frac{\pi}{4}, cos\left(\frac{\pi}{2} - x\right) = sin(x)$$
$$y = \int (sin(\frac{\pi}{2} - 2x))\,dx = -(-\frac{1}{2}cos(\frac{\pi}{2} - 2x)) = \frac{1}{2}cos\left(\frac{\pi}{2} - 2x\right) = \frac{1}{2}sin(2x) + c$$
$$\therefore \frac{1}{2}sin(2x) + c = 1 \Rightarrow \frac{1}{2}sin\left(2 \times \frac{\pi}{4}\right) + c = 1 \Rightarrow \frac{1}{2}sin\left(\frac{\pi}{2}\right) + c = 1 \Rightarrow \frac{1}{2} + c = 1 \Rightarrow c = \frac{1}{2}$$
$$\Rightarrow y = \frac{1}{2}sin(2x) + \frac{1}{2} = \frac{1 + sin(2x)}{2}$$

الإجابة الصحيحة هي (c)

$$135 - f`(x) = cos^2(x) - sin^2(x), (0,0), cos(2x) = cos^2(x) - sin^2(x)$$
$$f(x) = \int (cos^2(x) - sin^2(x)).dx = \int cos(2x).dx = \frac{1}{2}sin(2x) + c \Rightarrow \therefore \frac{1}{2}sin(2 \times 0) + c = 0$$
$$\Rightarrow \frac{1}{2}sin(0) + c = 0 \Rightarrow 0 + c = 0 \Rightarrow c = 0 \Rightarrow f(x) = \frac{1}{2}sin(2x)$$

الإجابة الصحيحة هي (a)

$$136 - f`(x) = e^{-x} + x^2, (0,4)$$
$$f(x) = \int (e^{-x} + x^2).dx = \int e^{-x}.dx + \int x^2.dx = -e^{-x} + \frac{x^3}{3} + c \Rightarrow \therefore -e^{-0} + \frac{0^3}{3} + c = 4$$
$$\Rightarrow -\frac{1}{e^0} + 0 + c = 4 \Rightarrow -1 + c = 4 \Rightarrow c = 5 \Rightarrow f(x) = e^{-x} + \frac{1}{3}x^3 + 5$$

الإجابة الصحيحة هي (b)

$137 - f`(x) = 2\pi\, cos(\pi x) - 2\pi\, sin(\pi x)\,,(1,-2)$

$$f(x) = \int (2\pi\, cos(\pi x) - 2\pi\, sin(\pi x)).dx = 2\pi \int cos(\pi x)\,.dx - 2\pi \int sin(\pi x)\,.dx$$
$$= 2\pi \times \frac{1}{\pi} sin(\pi x) + 2\pi \times \frac{1}{\pi} cos(\pi x) = 2\, sin(\pi x) + 2\, cos(\pi x) + c$$
$$\Rightarrow 2\, sin(\pi) + 2\, cos(\pi) + c = -2 \Rightarrow 0 - 2 + c = -2 \Rightarrow c = 0$$
$$\Rightarrow f(x) = 2\, sin(\pi x) + 2\, cos(\pi x)$$

الإجابة الصحيحة هي (c)

$138 - f`(x) = -a\, csc^2(x)\,,\left(\frac{3\pi}{4},1\right).(\frac{\pi}{4},5)$

$$f(x) = \int (-a\, csc^2(x)).dx = -a \int csc^2(x)\,.dx = a\cot(x) + c \Rightarrow \therefore\ a\cot\left(\frac{3\pi}{4}\right) + c = 1$$
$$\Rightarrow -a + c = 1 \qquad (1), \Rightarrow a\cot\left(\frac{\pi}{4}\right) + c = 5 \Rightarrow a + c = 5 \qquad (2)$$

بجمع المعادلتين (1) و (2)

$$\Rightarrow 2c = 6 \Rightarrow c = 3, a + 3 = 5 \Rightarrow a = 2 \Rightarrow f(x) = 2\cot(x) + 3$$

الإجابة الصحيحة هي (d)

$139 - f`(x) = 2e^x - 2, f(ln(8)) = 2, e^{ln(x)} = x$

$$f(x) = \int (2e^x - 2).dx = 2\int e^x.dx - \int 2.dx = 2e^x - 2x + c \Rightarrow \because f(ln(8)) = 2$$
$$\therefore 2e^{ln(8)} - 2\, ln(8) + c = 2 \Rightarrow 16 - 2\, ln(8) + c = 2 \Rightarrow -2\, ln(8) + c = -14$$
$$\Rightarrow c = -14 + 2\, ln(8) \Rightarrow f(x) = 2e^x - 2x + 2\, ln(8) - 14$$

الإجابة الصحيحة هي (c)

$140 - f'^{(x)} = 3^x + \frac{4}{x}, f(4) = 3, f(-4) = 5$

$$f(x) = \int \left(3^x + \frac{4}{x}\right).dx = \int 3^x.dx + 4\int \frac{1}{x}.dx = \frac{3^x}{ln(3)} + 4\, ln|x| + c \Rightarrow \because f(4) = 3, f(-4) = 5$$
$$\therefore \frac{3^4}{ln(3)} + 4\, ln|4| + c = 3 \Rightarrow \frac{81}{ln(3)} + 4\, ln(4) + c = 3 \Rightarrow c = 3 - \frac{81}{ln(3)} - 4\, ln(4), x > 0$$
$$\therefore \frac{3^{-4}}{ln(3)} + 4\, ln|-4| + c = 5 \Rightarrow \frac{1}{81\, ln(3)} + 4\, ln(4) + c = 5 \Rightarrow c = 5 - \frac{1}{81 ln(3)} - 4\, ln(4), x < 0$$
$$\therefore f(x) = \begin{cases} \frac{3^x}{ln(3)} + 4\, ln|x| + 5 - \frac{1}{81 ln(3)} - 4\, ln(4), x < 0 \\ \frac{3^x}{ln(3)} + 4\, ln|x| + 3 - \frac{81}{ln(3)} - 4\, ln(4)\,, x > 0 \end{cases}$$

الإجابة الصحيحة هي (b)

$141 - f`(x) = -a\, sin(x)\,,$ عند $(x = 0)$ قيمة عظمى تساوي (2) وعند $(x = \pi)$ قيمة صغرى تساوي (-2)

$$f(x) = \int -a\, sin(x)\,.dx = -a\int sin(x)\,.dx = -a(-\,cos(x) = a\, cos(x) + c$$
$$\therefore f(0) = 2 \Rightarrow a\, cos(0) + c = 2 \Rightarrow a + c = 2 \qquad (1)$$
$$\therefore f(\pi) = -2 \Rightarrow a\, cos(\pi) + c = -2 \Rightarrow -a + c = -2 \qquad (2)$$

بجمع المعادلتين (1) و (2)

$$\Rightarrow 2c = 0 \Rightarrow c = 0 \Rightarrow a + c = 2 \Rightarrow a + 0 = 2 \Rightarrow a = 2$$
$$\Rightarrow f(x) = 2\, cos(x)$$

الإجابة الصحيحة هي (a)

$142 - v(t) = 6\sin(3t), [0.2\pi]$

$$\Rightarrow v(t) = 6\sin(3t) = 0 \Rightarrow \sin(3t) = 0 \Rightarrow 3t = 0, \pi, 2\pi \Rightarrow t = 0, \frac{\pi}{3}, \frac{2\pi}{3}$$

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$0 \quad \frac{\pi}{3} \quad \frac{2\pi}{3} \quad 2\pi$

$$\therefore v(t) = \begin{cases} 6\sin(3t) & , 0 < x < \frac{\pi}{3} \\ -6\sin(3t) & , \frac{\pi}{3} < x < \frac{2\pi}{3} \\ 6\sin(3t) & , \frac{2\pi}{3} < x \geq 2\pi \end{cases}$$

$$\therefore \int_0^{2\pi} |v(t)|.dt = \int_0^{\frac{\pi}{3}} 6\sin(3t).dt + \int_{\frac{\pi}{3}}^{\frac{2\pi}{3}} -6\sin(3t).dt + \int_{\frac{2\pi}{3}}^{2\pi} 6\sin(3t).dt$$

$$\therefore \int_0^{2\pi} |v(t)|.dt = -2\cos(3t)\Big|_0^{\frac{\pi}{3}} + 2\cos(3t)\Big|_{\frac{\pi}{3}}^{\frac{2\pi}{3}} - 2\cos(3t)\Big|_{\frac{2\pi}{3}}^{2\pi}$$

$$= (-2\cos(\pi) - (-2\cos(0)) + (2\cos(2\pi) - (2\cos(\pi)) + (-2\cos(6\pi) - (-2\cos(2\pi))$$

الإجابة الصحيحة هي (d)

$143 - v(t) = \cos(3t), [0.2\pi]$

$$\therefore S(t_2) - S(t_1) = \int_{t_1}^{t_2} v(t).dt \Rightarrow S(2\pi) - S(0) = \int_0^{2\pi} v(t).dt = \int_0^{2\pi} \cos(3t).dt = \frac{1}{3}\sin(3t)\Big|_0^{2\pi}$$

$$= \frac{1}{3}\sin(6\pi) - \frac{1}{3}\sin(0) = 0 - 0 = 0m$$

الإجابة الصحيحة هي (a)

$144 - a(t) = -\sin(t) - \cos(t), v(0) = 1, s(0) = 1$

$$\therefore v(t) = \int a(t).dt = \int -\sin(t) - \cos(t).dt = \int -\sin(t).dt + \int -\cos(t).dt$$

$$= \cos(t) - \sin(t) + c_1 \Rightarrow \because v(0) = 1 \Rightarrow \therefore v(0) = \cos(0) - \sin(0) + c_1 \Rightarrow 1 - 0 + c_1 = 1$$

$$\therefore c_1 = 0 \Rightarrow v(t) = \cos(t) - \sin(t)$$

$$\therefore S(t) = \int v(t).dt = \int \cos(t) - \sin(t).dt = \int \cos(t).dt + \int -\sin(t).dt$$

$$= \sin(t) + \cos(t) + c_2 \Rightarrow \because S(0) = 1 \Rightarrow \therefore S(0) = \sin(0) + \cos(0) + c_2 \Rightarrow 0 + 1 + c_2 = 1$$

$$\therefore c_2 = 0 \Rightarrow S(t) = \sin(t) + \cos(t) \Rightarrow S\left(\frac{\pi}{2}\right) = \sin\left(\frac{\pi}{2}\right) + \cos\left(\frac{\pi}{2}\right) \Rightarrow S\left(\frac{\pi}{2}\right) = 1 + 0 = 1m$$

الإجابة الصحيحة هي (b)

$145 - v(t) = \cos(2t), s(0) = 4$

$$\therefore S(t) = \int v(t).dt = \int \cos(2t).dt = \frac{1}{2}\sin(2t) + c$$

$$\Rightarrow \because S(0) = 4 \Rightarrow \therefore S(0) = \frac{1}{2}\sin(0) + c = 4 \Rightarrow 0 + c = 4 \Rightarrow c = 4 \Rightarrow S(t) = \frac{1}{2}\sin(2t) + 4$$

$$\Rightarrow S\left(\frac{\pi}{4}\right) = \frac{1}{2}\sin\left(\frac{\pi}{2}\right) + 4 = 4.5m \Rightarrow \therefore S(t_2) - S(t_1) = S(t_2) - S(0) = 4.5 - 4 = 0.5m$$

الإجابة الصحيحة هي (c)

$146 - v(t) = e^{-\frac{1}{2}t}, s(0) = 2$

$$\therefore S(t) = \int v(t).dt = \int e^{-\frac{1}{2}t}.dt = -2e^{-\frac{1}{2}t} + c$$

$$\Rightarrow \because S(0) = 2 \Rightarrow \therefore S(0) = -2e^0 + c = 2 \Rightarrow -2 + c = 4 \Rightarrow c = 4 \Rightarrow S(t) = 4 - 2e^{-\frac{1}{2}t}$$

$$\Rightarrow S(10) = 4 - 2e^{-5}m$$

الإجابة الصحيحة هي (b)

$$147 - v(t) = \frac{-t}{1+t^2}, [0,1], \int \frac{g`(x)}{g(x)}.dx = ln|g(x)| + c$$

$$\therefore \int_0^1 |v(t)|.dt = \int_0^1 \left|\frac{-t}{1+t^2}\right|.dt = \int_0^1 \frac{t}{1+t^2}.dt = \frac{1}{2}\int_0^1 \frac{2t}{1+t^2}.dt = \frac{1}{2} ln|1+t^2| \Big|_0^1$$

$$= \frac{1}{2} ln|2| - \frac{1}{2} ln|1| = ln\left(2^{\frac{1}{2}}\right) - 0 = ln(\sqrt{2})$$

الإجابة الصحيحة هي (d)

$$148 - v(t) = \frac{-3t}{t^2+2}, [0,4], \int \frac{g`(x)}{g(x)}.dx = ln|g(x)| + c$$

$$\therefore \int_0^4 v(t).dt = \int_0^1 \frac{-3t}{t^2+2}.dt = -\frac{3}{2}\int_0^1 \frac{2t}{1+t^2}.dt = -\frac{3}{2} ln|t^2+2| \Big|_0^4$$

$$= -\frac{3}{2} ln|18| + \frac{3}{2} ln|2| = \frac{3}{2} ln|2| - \frac{3}{2} ln|18| = \frac{3}{2} ln(\frac{2}{18}) = \frac{3}{2} ln(\frac{1}{9})\, m$$

الإجابة الصحيحة هي (b)

$$149 - v(t) = 3\sqrt{2}e^{-\sqrt{2}t}, s(0) = 0$$

$$\therefore S(t) = \int v(t).dt = \int 3\sqrt{2}e^{-\sqrt{2}t}.dt = \int 3\sqrt{2}e^{-\sqrt{2}t}.dt = 3\sqrt{2}\int e^{-\sqrt{2}t}.dt = -\frac{3\sqrt{2}e^{-\sqrt{2}t}}{\sqrt{2}}$$

$$\therefore S(t) = -3e^{-\sqrt{2}t} + c \Rightarrow \because S(0) = 0 \Rightarrow \therefore S(0) = -3e^0 + c = 0 \Rightarrow -3 + c = 0 \Rightarrow c = 3$$

$$\Rightarrow S(t) = -3e^{-\sqrt{2}t} + 3 \Rightarrow \therefore S(\sqrt{6}) = -3e^{-\sqrt{2}\times\sqrt{6}} + 3 = -3e^{-\sqrt{12}} + 3 = 3 - 3e^{-2\sqrt{3}} m$$

الإجابة الصحيحة هي (c)

$150 - v(t) = \begin{cases} t+2 & ,0 \le t \le 1 \\ 4-(t-2)^2, & 1 < t \le 3 \end{cases}$, $(0 \le t \le 1)$ في الفترة $s(0) = 0, (1 < t \le 3)$ في الفترة $s(1)$

$$\therefore S(t) = \int v(t).dt = \int (t+2).dt = \int t.dt + \int 2.dt = \frac{t^2}{2} + 2t + c_1$$

$$\therefore S(t) = \frac{t^2}{2} + 2t + c_1 \Rightarrow \because S(0) = 0 \Rightarrow \therefore S(0) = 0 + 0 + c_1 = 0 \Rightarrow c_1 = 0 \Rightarrow \therefore S(t) = \frac{t^2}{2} + 2t$$

$$\Rightarrow \therefore S(1) = \frac{1}{2} + 2 = \frac{5}{2} m, 0 \le t \le 1$$

$$\therefore S(t) = \int v(t).dt = \int (4-(t-2)^2).dt = \int (4-(t^2+4-4t).dt = \int (4t - t^2).dt$$

$$= 4\int t.dt - \int t^2.dt = 4\frac{t^2}{2} - \frac{t^3}{3} + c_2 \Rightarrow S(t) = 4\frac{t^2}{2} - \frac{t^3}{3} + c_2 \Rightarrow \because S(1) = \frac{5}{2} \Rightarrow S(t)$$

$$\Rightarrow S(1) = 4\frac{1}{2} - \frac{1}{3} + c_2 = 2.5 \Rightarrow 2 - \frac{1}{3} + c_2 = 2.5 \Rightarrow \frac{5}{3} + c_2 = \frac{5}{2} \Rightarrow c_2 = \frac{5}{2} - \frac{5}{3} = \frac{15-10}{6} = \frac{5}{6}$$

$$\Rightarrow \therefore S(t) = 4\frac{t^2}{2} - \frac{t^3}{3} + \frac{5}{6} \Rightarrow S(2) = 4\frac{4}{2} - \frac{8}{3} + \frac{5}{6} = 8 - \frac{8}{3} + \frac{5}{6} = \frac{48-16+5}{6} = \frac{37}{6}$$

الإجابة الصحيحة هي (c)

$$151 - \int cos(x)e^{sin(x)}.dx = e^{sin(x)} + c, \int g`(x).e^{g(x)}.dx = e^{g(x)} + c$$

او بالتكامل بالتعويض

$$\int cos(x)e^{sin(x)}.dx, u = sin(x) \Rightarrow du = cos(x)\,dx \Rightarrow dx = \frac{du}{cos(x)}$$

$$= \int cos(x)e^{sin(x)}.dx = \int cos(x)e^u.\frac{du}{cos(x)} = \int e^u.du = e^u + c = e^{sin(x)} + c$$

الإجابة الصحيحة هي (a)

$$152-\int_{1}^{e}\left(\frac{(\ln(x))^2}{x}\right)dx=\int_{1}^{e}\left(\frac{1}{x}.(\ln(x))^2\right)dx,\int g`(x).(g(x))^n.dx=\frac{(g(x))^{n+1}}{n+1}+c$$

$$=\int_{1}^{e}\left(\frac{1}{x}.(\ln(x))^2\right)dx=\frac{(\ln(x))^3}{3}\Big|_1^e=\frac{(\ln(e))^3}{3}-\frac{(\ln(1))^3}{3}=\frac{1}{3}-0=\frac{1}{3}$$

او بالتكامل بالتعويض

$$\int_{1}^{e}\left(\frac{(\ln(x))^2}{x}\right)dx,u=\ln(x)\Rightarrow du=\frac{1}{x}dx\Rightarrow dx=x.du$$

$$\because x=1\Rightarrow\therefore u=0,\because x=e\Rightarrow\therefore u=1\Rightarrow\int_{1}^{e}\left(\frac{(\ln(x))^2}{x}\right)dx=\int_{0}^{1}\left(\frac{(u)^2}{x}\right)x.du=\int_{0}^{1}(u)^2.du=\frac{(u)^3}{3}\Big|_0^1$$

$$=\frac{(1)^3}{3}-\frac{(0)^3}{3}=\frac{1}{3}-0=\frac{1}{3}$$

الإجابة الصحيحة هي (a)

$$153-\int_{\frac{\pi}{4}}^{\frac{\pi}{2}}\left(\frac{e^{\cot(x)}}{\sin^2(x)}\right)dx=\int_{\frac{\pi}{4}}^{\frac{\pi}{2}}\left(\frac{1}{\sin^2(x)}.e^{\cot(x)}\right)dx=\int_{\frac{\pi}{4}}^{\frac{\pi}{2}}(\csc^2(x).e^{\cot(x)})dx=-\int_{\frac{\pi}{4}}^{\frac{\pi}{2}}-(\csc^2(x).e^{\cot(x)})dx$$

$$,\csc(x)=\frac{1}{\sin(x)},\int g`(x).e^{g(x)}.dx=e^{g(x)}+c$$

$$=-\int_{\frac{\pi}{4}}^{\frac{\pi}{2}}-(\csc^2(x).e^{\cot(x)})dx=-e^{\cot(x)}\Big|_{\frac{\pi}{4}}^{\frac{\pi}{2}}=-e^{\cot\left(\frac{\pi}{2}\right)}-\left(-e^{\cot\left(\frac{\pi}{4}\right)}\right)=-e^0+e^1=-1+e=e-1$$

او بالتكامل بالتعويض

$$\int_{\frac{\pi}{4}}^{\frac{\pi}{2}}\left(\frac{e^{\cot(x)}}{\sin^2(x)}\right)dx=\int_{\frac{\pi}{4}}^{\frac{\pi}{2}}\left(\frac{1}{\sin^2(x)}.e^{\cot(x)}\right)dx=\int_{\frac{\pi}{4}}^{\frac{\pi}{2}}(\csc^2(x).e^{\cot(x)})dx$$

$$,u=\cot(x)\Rightarrow du=-\csc^2(x)\,dx\Rightarrow dx=\frac{du}{-\csc^2(x)}$$

$$\because x=\frac{\pi}{4}\Rightarrow\therefore u=1,\because x=\frac{\pi}{2}\Rightarrow\therefore u=0\Rightarrow\int_{\frac{\pi}{4}}^{\frac{\pi}{2}}(\csc^2(x).e^{\cot(x)})dx=\int_{1}^{0}(\csc^2(x).e^u).\frac{du}{-\csc^2(x)}$$

$$=-\int_{1}^{0}e^u.du=-e^u\Big|_1^0=-e^0-(-e^1)=-1+e=e-1$$

الإجابة الصحيحة هي (c)

$$154-\int_{0}^{1}\left(\frac{2e^{\sqrt{x}}}{\sqrt{x}}\right)dx=4\int_{0}^{1}\left(\frac{1}{2\sqrt{x}}.e^{\sqrt{x}}\right).dx=4e^{\sqrt{x}}\Big|_0^1=4e^{\sqrt{1}}-4e^{\sqrt{0}}=4e^1-4e^0=4e-4$$

$$,\int g`(x).e^{g(x)}.dx=e^{g(x)}+c$$

او بالتكامل بالتعويض

$$\int_{0}^{1}\left(\frac{2e^{\sqrt{x}}}{\sqrt{x}}\right)dx,u=\sqrt{x}\Rightarrow du=\frac{1}{2\sqrt{x}}dx\Rightarrow dx=2\sqrt{x}.du$$

$\because x=0 \Rightarrow \therefore u=0, \because x=1 \Rightarrow \therefore u=1 \Rightarrow \int_0^1 (\frac{2e^{\sqrt{x}}}{\sqrt{x}})\,dx = \int_0^1 (\frac{2e^u}{\sqrt{x}}).2\sqrt{x}.du = 4\int_0^1 e^u.du$

$= 4e^u \Big|_0^1 = 4e^1 - 4e^0 = 4e-4$

الإجابة الصحيحة هي (b)

$155 - \int_{\frac{1}{2}}^{\frac{e}{2}} (\frac{cos(ln(4x^2))}{x})\,dx, u = ln(4x^2) \Rightarrow du = \frac{8x}{4x^2}dx = \frac{2x}{x^2}dx \Rightarrow dx = \frac{x^2}{2x}.du$, التكامل بالتعويض

$\because x = \frac{1}{2} \Rightarrow \therefore u=0, \because x=\frac{e}{2} \Rightarrow \therefore u=2 \Rightarrow \int_{\frac{1}{2}}^{\frac{e}{2}} (\frac{cos(ln(4x^2))}{x})\,dx = \int_0^2 (\frac{cos(u)}{x}).\frac{x^2}{2x}.du$

$= \frac{1}{2}\int_0^2 cos(u).du = \frac{1}{2}sin(u)\Big|_0^2 = \frac{1}{2}sin(2) - \frac{1}{2}sin(0) = \frac{1}{2}sin(2)$

الإجابة الصحيحة هي (a)

$156 - \int (\sqrt{x}\,sin^2(x^{\frac{3}{2}}-1))\,dx, u = x^{\frac{3}{2}}-1 \Rightarrow du = \frac{3}{2}x^{\frac{1}{2}}.dx = \frac{3}{2}\sqrt{x}.dx \Rightarrow dx = \frac{2}{3\sqrt{x}}.du$, التكامل بالتعويض

$= \int (\sqrt{x}\,sin^2(x^{\frac{3}{2}}-1))\,dx = \int (\sqrt{x}\,sin^2(u)).\frac{2}{3\sqrt{x}}.du = \frac{2}{3}\int sin^2(u).du, sin^2(x) = \frac{1-cos(2x)}{2}$

$= \frac{2}{3}\int sin^2(u).du = \frac{2}{3}\int (\frac{1}{2} - \frac{cos(2u)}{2}).du = \frac{2}{3}(\int \frac{1}{2}.du - \frac{1}{2}\int cos(2u).du)$

$= \frac{2}{3}(\frac{1}{2}u - \frac{1}{2}(\frac{1}{2}sin(2u))) = \frac{1}{3}u - \frac{1}{6}sin(2u) = \frac{1}{3}(x^{\frac{3}{2}}-1) - \frac{1}{6}sin\left(2x^{\frac{3}{2}}-2\right) + c$

الإجابة الصحيحة هي (c)

$157 - \int (\frac{2^{\frac{1}{x^2}}}{x^3})\,dx, u = \frac{1}{x^2} \Rightarrow du = \frac{-2x}{x^4}.dx = \frac{-2}{x^3}.dx \Rightarrow dx = \frac{x^3}{-2}.du$, التكامل بالتعويض

$= \int (\frac{2^{\frac{1}{x^2}}}{x^3})\,dx = \int (\frac{2^u}{x^3})\frac{x^3}{-2}.du = -\frac{1}{2}\int 2^u.du = -\frac{1}{2}\times\frac{2^u}{ln(2)} = -\frac{2^u}{2ln(2)}$

$= -\frac{2^u}{ln(2^2)} = -\frac{2^u}{ln(4)} = -\frac{2^{\frac{1}{x^2}}}{ln(4)} + c$

الإجابة الصحيحة هي (d)

$158 - \int (csc^2(2x)\,cot(2x))\,dx, \quad u = cot(2x) \Rightarrow du = -\frac{1}{2}csc^2(2x)\,dx \Rightarrow dx = \frac{2du}{-csc^2(2x)}$, التكامل بالتعويض

$= \int (csc^2(2x)\,cot(2x))\,dx = \int csc^2(2x).u.\frac{2du}{-csc^2(2x)} = -2\int u.du = -2\frac{u^2}{2} = -u^2$

$= -cot^2(2x) + c$

الإجابة الصحيحة هي (c)

$159 - \int (\frac{e^{2x}}{1-e^x})\,dx, \quad u = 1-e^x \Rightarrow du = -e^x\,dx \Rightarrow dx = \frac{du}{-e^x}, -e^x = u-1$, التكامل بالتعويض

$\int (\frac{e^{2x}}{1-e^x})\,dx = \int \frac{e^{2x}}{u}.\frac{du}{-e^x} = \int -\frac{e^x}{u}.du = \int \frac{-e^x}{u}.du = \int (u-1)\frac{1}{u}.du = \int (1-\frac{1}{u}).du$

$= \int 1.du - \int \frac{1}{u}.du = u - ln|u| = 1 - e^x - ln|1-e^x| + c$

الإجابة الصحيحة هي (a)

$$160-\int(\frac{2^{2x}}{2^x+1})\,dx\,,\quad u=2^x+1\Rightarrow du=\frac{2^x}{\ln(2)}dx\Rightarrow dx=\frac{\ln(2)\,du}{2^x}\,,2^x=u-1\,,$$ التكامل بالتعويض

$$\int(\frac{2^{2x}}{2^x+1})\,dx=\int\frac{2^{2x}}{u}.\frac{\ln(2)\,du}{2^x}=\ln(2)\int\frac{2^x}{u}.du=\ln(2)\int\frac{u-1}{u}.du=\ln(2)\int(u-1)\frac{1}{u}.du$$

$$=\ln(2)\int(1-\frac{1}{u}).du=\ln(2)\,(\int 1.du-\int\frac{1}{u}.du)=\ln(2)\,(u-ln|u|)$$

$$=\ln(2)\,(2^x+1-ln|2^x+1|)+c$$

الإجابة الصحيحة هي (b)

$$161-\int((x^3+x)(3x^2+1))\,dx\,,\quad u=x^3+x\Rightarrow du=(3x^2+1).dx\Rightarrow dx=\frac{du}{3x^2+1}\,,$$ التكامل بالتعويض

$$=\int((x^3+x)(3x^2+1))\,dx=\int(u)(3x^2+1)).\frac{du}{3x^2+1}=\int u.du=\frac{u^2}{2}=\frac{1}{2}(x^3+x)^2+c$$

$$=-cot^2(2x)+c$$

الإجابة الصحيحة هي (c)

$$162-\int(x\sqrt{2x+1})\,dx\,,\qquad u=\sqrt{2x+1}\Rightarrow du=\frac{2}{2\sqrt{2x+1}}.dx=\frac{1}{\sqrt{2x+1}}.dx\Rightarrow dx=\sqrt{2x+1}.du,$$

$$,u^2=2x+1\Rightarrow x=\frac{u^2}{2}-\frac{1}{2}\,,$$ التكامل بالتعويض

$$=\int(x\sqrt{2x+1})\,dx=\int xu^2.du=\int(\frac{u^2}{2}-\frac{1}{2})u^2.du=\int\frac{u^4}{2}-\frac{u^2}{2}.du=\frac{1}{2}\int u^4.du-\frac{1}{2}\int u^2.du$$

$$=\frac{1}{2}(\frac{u^5}{5})-\frac{1}{2}(\frac{u^3}{3})=\frac{1}{2}(\frac{u^5}{5}-\frac{u^3}{3})=\frac{1}{2}(\frac{\left(\sqrt{2x+1}\right)^5}{5}-\frac{\left(\sqrt{2x+1}\right)^3}{3})=\frac{1}{2}(\frac{\sqrt{(2x+1)^5}}{5}+\frac{\sqrt{(2x+1)^3}}{3})+c$$

$$,x^{\frac{m}{n}}=\sqrt[n]{x^m}=(\sqrt[n]{x})^m$$

الإجابة الصحيحة هي (d)

$$163-\int_0^4(\frac{1}{\sqrt{x}\,(1+\sqrt{x})^2})\,dx\,,\qquad u=1+\sqrt{x}\Rightarrow du=\frac{1}{2\sqrt{x}}.dx\Rightarrow dx=2\sqrt{x}.du,\,,$$ التكامل بالتعويض

$$\because x=0\Rightarrow\therefore u=1,\because x=4\Rightarrow\therefore u=3\Rightarrow\int_0^4(\frac{1}{\sqrt{x}\,(1+\sqrt{x})^2})\,dx=\int_1^3(\frac{1}{\sqrt{x}\,(u)^2}).2\sqrt{x}.du$$

$$=2\int_1^3\frac{1}{(u)^2}.du=2\int_1^3u^{-2}.du=-2\frac{1}{u}\Big|_1^3=-2\frac{1}{3}-(-2\frac{1}{1})=\frac{-2}{3}+2=\frac{4}{3}$$

الإجابة الصحيحة هي (c)

$$164-\int_{-1}^1(x\sqrt{(x+1)^2})\,dx,\;u=x+1\Rightarrow du=dx,x=u-1\,,$$ التكامل بالتعويض

$$\because x=-1\Rightarrow\therefore u=0,\because x=1\Rightarrow\therefore u=2\Rightarrow\int_{-1}^1(x\sqrt{(x+1)^2})\,dx=\int_0^2((u-1)\sqrt{(u)^2}).du$$

$$=\int_0^2((u-1)u).du=\int_0^2(u^2-u).du=\int_0^2u^2.du-\int_0^2u.du=\left(\frac{u^3}{3}-\frac{u^2}{2}\right)\Big|_0^2=\left(\frac{8}{3}-\frac{4}{2}\right)-\left(\frac{0}{3}-\frac{0}{2}\right)$$

$$=\left(\frac{8}{3}-\frac{4}{2}\right)-0=\frac{16-12}{6}=\frac{4}{6}=\frac{2}{3}$$

الإجابة الصحيحة هي (d)

$$165-\int (x^2(x+2)^3)\,dx\,,u=x+2 \Rightarrow du=dx\,,x=u-2\,,$$ التكامل بالتعويض

$$=\int (x^2(x+2)^3)\,dx=\int (u-2)^2(u)^3).du=\int (u^2-4u+4)u^3\,.du=\int (u^5-4u^4+4u^3)\,.du$$

$$=\int u^5\,.du-4\int u^4\,.du+4\int u^3\,.du=(\frac{u^6}{6})-4(\frac{u^5}{5})+4(\frac{u^4}{4})=(\frac{u^6}{6})-4(\frac{u^5}{5})+(u^4)$$

$$=\frac{(x+2)^6}{6}-\frac{4(x+2)^5}{5}+(x+2)^4+c$$

الإجابة الصحيحة هي (d)

$$166-\int (e^{2x}\sqrt{e^x+1})\,dx\,,\qquad u=\sqrt{e^x+1}\Rightarrow du=\frac{e^x}{2\sqrt{e^x+1}}.dx\Rightarrow dx=\frac{2\sqrt{e^x+1}}{e^x}.du,$$

$$,u^2=e^x+1\Rightarrow e^x=u^2-1\,,$$ التكامل بالتعويض

$$=\int (e^{2x}\sqrt{e^x+1})\,dx=\int (e^x u)\,.2u.du=2\int ((u^2-1)u^2)\,.du=2\int (u^4-u^2)\,.du$$

$$=2\left(\int u^4\,.du-\int u^2\,.du\right)=2(\frac{u^5}{5}-\frac{u^3}{3})=\frac{2u^5}{5}-\frac{2u^3}{3}=\frac{2\left(\sqrt{e^x+1}\right)^5}{5}-\frac{2\left(\sqrt{e^x+1}\right)^3}{3}$$

$$=\frac{2}{5}(e^x+1)^{\frac{5}{2}}-\frac{2}{3}(e^x+1)^{\frac{3}{2}}+c,x^{\frac{m}{n}}=\sqrt[n]{x^m}=(\sqrt[n]{x})^m$$

الإجابة الصحيحة هي (a)

$$167-\int (3^{2x}\sqrt{3^x+1})\,dx\,,\qquad u=\sqrt{3^x+1}\Rightarrow du=\frac{3^x\,ln(3)}{2\sqrt{3^x+1}}.dx\Rightarrow dx=\frac{2\sqrt{3^x+1}}{3^x\,ln(3)}.du,$$

$$,u^2=3^x+1\Rightarrow 3^x=u^2-1\,,$$ التكامل بالتعويض

$$=\int (3^{2x}\sqrt{3^x+1})\,dx=\int (3^x u).\frac{2u}{ln(3)}.du=\frac{2}{ln(3)}\int ((u^2-1)u^2)\,.du=\frac{2}{ln(3)}\int (u^4-u^2)\,.du$$

$$=\frac{2}{ln(3)}\left(\int u^4\,.du-\int u^2\,.du\right)=\frac{2}{ln(3)}(\frac{u^5}{5}-\frac{u^3}{3})=\frac{2u^5}{5\,ln(3)}-\frac{2u^3}{3\,ln(3)}=\frac{2\left(\sqrt{3^x+1}\right)^5}{5\,ln(3)}-\frac{2\left(\sqrt{3^x+1}\right)^3}{3\,ln(3)}$$

$$=\frac{2}{5\,ln(3)}(3^x+1)^{\frac{5}{2}}-\frac{2}{3\,ln(3)}(3^x+1)^{\frac{3}{2}}+c,x^{\frac{m}{n}}=\sqrt[n]{x^m}=(\sqrt[n]{x})^m$$

الإجابة الصحيحة هي (c)

$$168-\int (\sqrt{1+\sqrt{x}})\,dx\,,u=\sqrt{1+\sqrt{x}}\Rightarrow du=\frac{0+\frac{1}{2\sqrt{x}}}{2\sqrt{1+\sqrt{x}}}.dx=\frac{\frac{1}{2\sqrt{x}}}{2\sqrt{1+\sqrt{x}}}.dx\Rightarrow dx=\frac{2\sqrt{1+\sqrt{x}}}{\frac{1}{2\sqrt{x}}}.du,$$

$$dx=4u.\sqrt{x}.du\,,u^2=1+\sqrt{x}\Rightarrow \sqrt{x}=u^2-1\,,$$ التكامل بالتعويض

$$=\int (\sqrt{1+\sqrt{x}})\,dx=\int (u)\,.4u.\sqrt{x}.du=4\int ((u^2-1)u^2)\,.du=4\int (u^4-u^2)\,.du$$

$$=4\left(\int u^4\,.du-\int u^2\,.du\right)=4(\frac{u^5}{5}-\frac{u^3}{3})=\frac{2u^5}{5}-\frac{2u^3}{3}=\frac{4\left(\sqrt{1+\sqrt{x}}\right)^5}{5}-\frac{4\left(\sqrt{1+\sqrt{x}}\right)^3}{3}$$

$$=\frac{4}{5}(1+\sqrt{x})^{\frac{5}{2}}-\frac{4}{3}(1+\sqrt{x})^{\frac{3}{2}}+c,x^{\frac{m}{n}}=\sqrt[n]{x^m}=(\sqrt[n]{x})^m$$

الإجابة الصحيحة هي (b)

$$169-\int\left(\frac{1+x}{\sqrt{1-x}}\right)dx\,,\qquad u=\sqrt{1-x}\Rightarrow du=\frac{-1}{2\sqrt{1-x}}.dx\Rightarrow dx=-2\sqrt{1-x}.du,$$

$,u^2=1-x\Rightarrow x=1-u^2$, التكامل بالتعويض

$$=\int\left(\frac{1+x}{\sqrt{1-x}}\right)dx=\int\left(\frac{1+1-u^2}{u}\right).-2u.du=\int(-2(2-u^2).du=\int(-4+2u^2).du$$

$$=\int -4.du+2\int u^2.du=-4u+\frac{2u^3}{3}=\frac{2u^3}{3}-4u=\frac{2(\sqrt{1-x})^3}{3}-4\sqrt{1-x}$$

$$=\frac{2}{3}\sqrt{(1-x)^3}+4\sqrt{1-x}+c,x^{\frac{m}{n}}=\sqrt[n]{x^m}=(\sqrt[n]{x})^m$$

الإجابة الصحيحة هي (a)

$$170-\int\left(\frac{x}{\sqrt{1+x}}\right)dx\,,\qquad u=\sqrt{1+x}\Rightarrow du=\frac{1}{2\sqrt{1+x}}.dx\Rightarrow dx=2\sqrt{1+x}.du,$$

$,u^2=1+x\Rightarrow x=u^2-1$, التكامل بالتعويض

$$=\int\left(\frac{x}{\sqrt{1+x}}\right)dx=\int\left(\frac{u^2-1}{u}\right).2u.du=\int(2(u^2-1).du=\int(2u^2-2).du$$

$$=2\int u^2.du-\int 2.du=\frac{2u^3}{3}-2u=\frac{2(\sqrt{1+x})^3}{3}-2\sqrt{1+x}$$

$$=\frac{2}{3}\sqrt{(1+x)^3}-2\sqrt{1+x}+c,x^{\frac{m}{n}}=\sqrt[n]{x^m}=(\sqrt[n]{x})^m$$

الإجابة الصحيحة هي (c)

$$171-\int\left(\frac{x}{(x-1)^2}\right)dx\,,u=x-1\Rightarrow du=dx\,,x=u+1\,,$$ التكامل بالتعويض

$$=\int\left(\frac{x}{(x-1)^2}\right)dx=\int\left(\frac{u+1}{u^2}\right)du=\int(u+1)u^{-2}.du=\int(u^{-1}+u^{-2}).du$$

$$=\int\frac{1}{u}.du+\int u^{-2}.du=ln|u|-\frac{1}{u}=ln|x-1|-\frac{1}{x-1}+c$$

حل آخر بالتكامل بالكسور الجزيئية

$$\int\left(\frac{x}{(x-1)^2}\right)dx\Rightarrow\frac{x}{(x-1)^2}=\frac{A}{x-1}+\frac{B}{(x-1)^2}\Rightarrow\frac{x}{(x-1)^2}=\frac{A(x-1)}{x-1}+\frac{B}{(x-1)^2}$$

$$\Rightarrow x=A(x-1)+B\Rightarrow x=1\Rightarrow 0+B=1\Rightarrow B=1\quad,\Rightarrow x=2\Rightarrow A+1=2\Rightarrow A=1$$

$$\therefore\int\left(\frac{x}{(x-1)^2}\right)dx=\int\frac{1}{x-1}.dx+\int(x-1)^{-2}.dx=ln|x-1|-(x-1)^{-1}$$

$$=ln|x-1|-\frac{1}{x-1}+c$$

الإجابة الصحيحة هي (d)

$$172-\int\left(\frac{e^{4x}}{(1+e^{2x})^{\frac{2}{3}}}\right)dx\,,u=1+e^{2x}\Rightarrow du=0+2e^{2x}.dx\Rightarrow dx=\frac{1}{2e^{2x}}.du,$$

$,u=1+e^{2x}\Rightarrow e^{2x}=u-1$, التكامل بالتعويض

$$=\int\left(\frac{e^{4x}}{(1+e^{2x})^{\frac{2}{3}}}\right)dx=\int\left(\frac{e^{4x}}{(u)^{\frac{2}{3}}}\right).\frac{1}{2e^{2x}}.du=\frac{1}{2}\int((u-1)u^{-\frac{2}{3}}).du=\frac{1}{2}\int(u^{\frac{1}{3}}-u^{-\frac{2}{3}}).du$$

$$=\frac{1}{2}\left(\int u^{\frac{1}{3}}.du-\int u^{-\frac{2}{3}}.du\right)=\frac{1}{2}\left(\frac{3u^{\frac{4}{3}}}{4}-3u^{\frac{1}{3}}\right)=\frac{3u^{\frac{4}{3}}}{8}-\frac{3u^{\frac{1}{3}}}{2}=\frac{3(1+e^{2x})^{\frac{4}{3}}}{8}-\frac{3(1+e^{2x})^{\frac{1}{3}}}{2}+c$$

الإجابة الصحيحة هي (d)

$173-\int\left(\frac{e^{2x}}{\sqrt[3]{1+e^{x}}}\right)dx\,,u=1+e^{x}\Rightarrow du=0+e^{x}.dx\Rightarrow dx=\frac{du}{e^{x}},$

$,u=1+e^{x}\Rightarrow e^{x}=u-1\,,$ التكامل بالتعويض

$$=\int\left(\frac{e^{2x}}{\sqrt[3]{1+e^{x}}}\right)dx=\int\left(\frac{e^{2x}}{\sqrt[3]{u}}\right).\frac{du}{e^{x}}=\int\left(\frac{u-1}{u^{\frac{1}{3}}}\right).du=\int(u-1)u^{-\frac{1}{3}}.du=\int\left(u^{\frac{2}{3}}-u^{-\frac{1}{3}}\right).du$$

$$=\int u^{\frac{2}{3}}.du-\int u^{-\frac{1}{3}}.du=\frac{3u^{\frac{5}{3}}}{5}-\frac{3u^{\frac{2}{3}}}{2}=\frac{3(1+e^{x})^{\frac{5}{3}}}{5}-\frac{3(1+e^{x})^{\frac{2}{3}}}{2}$$

$$=\frac{3}{5}\sqrt[3]{(1+e^{x})^{5}}-\frac{3}{2}\sqrt[3]{(1+e^{x})^{2}}+c,x^{\frac{m}{n}}=\sqrt[n]{x^{m}}=(\sqrt[n]{x})^{m}$$

الإجابة الصحيحة هي (d)

$174-\int\left(\frac{2x}{\sqrt[3]{x^{2}+1}}\right)dx\,,u=x^{2}+1\Rightarrow du=2x.dx\Rightarrow dx=\frac{du}{2x},$ التكامل بالتعويض

$$=\int\left(\frac{2x}{\sqrt[3]{x^{2}+1}}\right)dx=\int\left(\frac{2x}{\sqrt[3]{u}}\right).\frac{du}{2x}=\int\left(\frac{1}{u^{\frac{1}{3}}}\right).du=\int u^{-\frac{1}{3}}.du=\frac{3u^{\frac{2}{3}}}{2}=\frac{3(x^{2}+1)^{\frac{2}{3}}}{2}$$

$$=\frac{3}{2}\sqrt[3]{(x^{2}+1)^{2}}+c,x^{\frac{m}{n}}=\sqrt[n]{x^{m}}=(\sqrt[n]{x})^{m}$$

الإجابة الصحيحة هي (c)

$175-\int\left(\cos^{3}(x)\sqrt{\sin(x)}\right)dx\,,u=\sqrt{\sin(x)}\Rightarrow du=\frac{\cos(x)}{2\sqrt{\sin(x)}}.dx\Rightarrow dx=\frac{2\sqrt{\sin(x)}.du}{\cos(x)},$ التكامل بالتعويض

$,u=\sqrt{\sin(x)}\Rightarrow \sin(x)=u^{2}\Rightarrow \sin^{2}(x)=u^{4}$

$$=\int\left(\cos^{3}(x)\sqrt{\sin(x)}\right)dx=\int(\cos^{2}(x).u).2u.du=2\int(u^{2})(\cos^{2}(x)).du$$

$$=2\int(u^{2})(1-\sin^{2}(x)).du=2\int(u^{2})(1-u^{4}).du=2\int(u^{2}-u^{6}).du=2\left(\int u^{2}.du-\int u^{6}.du\right)$$

$$=2\left(\frac{u^{3}}{3}-\frac{u^{7}}{7}\right)=\frac{2u^{3}}{3}-\frac{2u^{7}}{7}=\frac{2\left(\sqrt{\sin(x)}\right)^{3}}{3}-\frac{2\left(\sqrt{\sin(x)}\right)^{7}}{7}=\frac{2}{3}\sin^{\frac{3}{2}}(x)-\frac{2}{7}\sin^{\frac{7}{2}}(x)+c$$

$,\sin^{2}(x)+\cos^{2}(x)=1\Rightarrow \cos^{2}(x)=1-\sin^{2}(x)\,,x^{\frac{m}{n}}=\sqrt[n]{x^{m}}=(\sqrt[n]{x})^{m}$

الإجابة الصحيحة هي (d)

$176-\int\left(\sin^{3}(x)\cos^{\frac{1}{2}}(x)\right)dx\,,u=\cos^{\frac{1}{2}}(x)\Rightarrow du=-\frac{\cos^{-\frac{1}{2}}(x)\sin(x)}{2}.dx=-\frac{\sin(x)}{2\cos^{\frac{1}{2}}(x)}.dx$

$\Rightarrow dx=-\frac{2\cos^{\frac{1}{2}}(x).du}{\sin(x)},$ التكامل بالتعويض$,u=\cos^{\frac{1}{2}}(x)\Rightarrow \cos(x)=u^{2}\Rightarrow \cos^{2}(x)=u^{4}$

$$=\int\left(\sin^{3}(x)\cos^{\frac{1}{2}}(x)\right)dx=\int(\sin^{2}(x).u).-2u.du=-2\int(u^{2})(\sin^{2}(x)).du$$

$$=-2\int(u^{2})(1-\cos^{2}(x)).du=-2\int(u^{2})(1-u^{4}).du=-2\int(u^{2}-u^{6}).du$$

$$=-2\left(\int u^{2}.du-\int u^{6}.du\right)=-2\left(\frac{u^{3}}{3}-\frac{u^{7}}{7}\right)=-\frac{2u^{3}}{3}+\frac{2u^{7}}{7}=-\frac{2\left(\cos^{\frac{1}{2}}(x)\right)^{3}}{3}+\frac{2\left(\cos^{\frac{1}{2}}(x)\right)^{7}}{7}$$

$$=\frac{2}{7}\cos^{\frac{7}{2}}(x)+\frac{2}{3}\cos^{\frac{3}{2}}(x)+c,\sin^{2}(x)+\cos^{2}(x)=1\Rightarrow \sin^{2}(x)=1-\cos^{2}(x)$$

الإجابة الصحيحة هي (c)

$$177-\int(\sin^3(x)\cos^2(x))\,dx\,,u=\cos^2(x)\Rightarrow du=-2\cos(x)\sin(x)\,.dx=-2\sin(x)\cos(x)\,.dx$$

$$\Rightarrow dx=-\frac{du}{2\cos(x)\sin(x)}$$, التكامل بالتعويض, $u=\cos^2(x)\Rightarrow\cos(x)=\sqrt{u}=u^{\frac{1}{2}}$

$$=\int(\sin^3(x)\cos^2(x))\,dx=\int(\sin(x)\sin^2(x)\cos^2(x))\,dx=-\frac{1}{2}\int(\sin^2(x))u^{\frac{1}{2}}\,.du$$

$$=-\frac{1}{2}\int(1-\cos^2(x))u^{\frac{1}{2}}\,.du=-\frac{1}{2}\int(1-u)u^{\frac{1}{2}}.du=-\frac{1}{2}\int(u^{\frac{1}{2}}-u^{\frac{3}{2}}).du$$

$$=-\frac{1}{2}\left(\int u^{\frac{1}{2}}\,.du-\int u^{\frac{3}{2}}\,.du\right)=-\frac{1}{2}\left(\frac{2u^{\frac{3}{2}}}{3}-\frac{2u^{\frac{5}{2}}}{5}\right)=-\frac{u^{\frac{3}{2}}}{3}+\frac{u^{\frac{5}{2}}}{5}=-\frac{(\cos^2(x))^{\frac{3}{2}}}{3}+\frac{(\cos^2(x))^{\frac{5}{2}}}{5}$$

$$=-\frac{1}{3}\cos^3(x)+\frac{1}{5}\cos^5(x)+c,\ \sin^2(x)+\cos^2(x)=1\Rightarrow\sin^2(x)=1-\cos^2(x)$$

الإجابة الصحيحة هي (a)

$$178-\int(\cos^5(x)\sin^2(x))\,dx\,,u=\sin^2(x)\Rightarrow du=2\sin(x)\cos(x)\,.dx$$

$$\Rightarrow dx=\frac{du}{2\sin(x)\cos(x)}$$, التكامل بالتعويض, $u=\sin^2(x)\Rightarrow\sin(x)=\sqrt{u}=u^{\frac{1}{2}}$

$$=\int(\cos^5(x)\sin^2(x))\,dx=\int(\cos(x)\sin^2(x)(\cos^2(x))^2)\,dx=\int(\cos(x)\sin^2(x)(1-\sin^2(x))^2)\,dx$$

$$=\int(\cos(x)\,.u(1-u)^2)\,.\frac{du}{2\sin(x)\cos(x)}=\frac{1}{2}\int(u(1-u)^2)\,.\frac{du}{u^{\frac{1}{2}}}=\frac{1}{2}\int(u^{\frac{1}{2}}(1-u)^2)\,.du$$

$$=\frac{1}{2}\int u^{\frac{1}{2}}(u^2-2u+1)\,.du=\frac{1}{2}\int(u^{\frac{5}{2}}-2u^{\frac{3}{2}}+u^{\frac{1}{2}})\,.du$$

$$=\frac{1}{2}\left(\int u^{\frac{5}{2}}\,.du-2\int u^{\frac{3}{2}}\,.du+\int u^{\frac{1}{2}}\,.du\right)=\frac{1}{2}\left(\frac{2u^{\frac{7}{2}}}{7}-\frac{4u^{\frac{5}{2}}}{5}+\frac{2u^{\frac{3}{2}}}{3}\right)=\frac{u^{\frac{7}{2}}}{7}-\frac{2u^{\frac{5}{2}}}{5}+\frac{u^{\frac{3}{2}}}{3}$$

$$=\frac{(\sin^2(x))^{\frac{7}{2}}}{7}-\frac{2(\sin^2(x))^{\frac{5}{2}}}{5}+\frac{(\sin^2(x))^{\frac{3}{2}}}{3}=\frac{1}{3}\sin^3(x)-\frac{2}{5}\sin^5(x)+\frac{1}{7}\sin^7(x)+c$$

$$,\sin^2(x)+\cos^2(x)=1\Rightarrow\cos^2(x)=1-\sin^2(x)$$

الإجابة الصحيحة هي (a)

$$179-\int\left(\frac{1}{x^2}\sin\left(\frac{1}{x}\right)\cos\left(\frac{1}{x}\right)\right)dx=\frac{1}{2}\int2\left(\frac{1}{x^2}\sin\left(\frac{1}{x}\right)\cos\left(\frac{1}{x}\right)\right)dx=\frac{1}{2}\int\left(\frac{1}{x^2}\sin\left(\frac{2}{x}\right)\right)dx$$

$$,u=\frac{2}{x}\Rightarrow du=\frac{-2}{x^2}.dx\Rightarrow dx=\frac{x^2du}{-2}$$, التكامل بالتعويض, $\sin(2x)=2\sin(x)\cos(x)$

$$=\frac{1}{2}\int\left(\frac{1}{x^2}\sin\left(\frac{2}{x}\right)\right)dx=\frac{1}{2}\int\left(\frac{1}{x^2}\sin(u)\right).\frac{x^2du}{-2}=-\frac{1}{4}\int\sin(u)\,.du=\frac{1}{4}\cos(u)=\frac{1}{4}\cos\left(\frac{2}{x}\right)+c$$

الإجابة الصحيحة هي (a)

$$180-\int\left(\sin^3\left(\frac{x}{3}\right)\cos\left(\frac{x}{3}\right)\right)dx\,,u=\sin\left(\frac{x}{3}\right)\Rightarrow du=\frac{1}{3}\cos\left(\frac{x}{3}\right).dx\Rightarrow dx=\frac{3.du}{\cos\left(\frac{x}{3}\right)}$$, التكامل بالتعويض

$$=\int\left(\sin^3\left(\frac{x}{3}\right)\cos\left(\frac{x}{3}\right)\right)dx=3\int u^3\,.du=3\frac{u^4}{4}=3\frac{\sin^4\left(\frac{x}{3}\right)}{4}+c$$

الإجابة الصحيحة هي (b)

$$181-\int \frac{1}{x^2}cos^2(\frac{1}{x})\,dx\,,u=\frac{1}{x}\Rightarrow du=-\frac{1}{x^2}.dx\Rightarrow dx=-x^2.du,$$ التكامل بالتعويض

$$=\int \frac{1}{x^2}cos^2(\frac{1}{x})\,dx=\int \frac{1}{x^2}cos^2(u).-x^2.du=-\int cos^2(u).du=-\int(\frac{1}{2}+\frac{1}{2}cos(2u)).du$$

$$=-(\int\frac{1}{2}.du+\frac{1}{2}\int cos(2u).du)=-(\frac{1}{2}u+\frac{1}{2}(\frac{1}{2}sin(2u))=-\frac{1}{2}u-\frac{1}{4}sin(2u)$$

$$=-\frac{1}{2}.\frac{1}{x}-\frac{1}{4}sin(2.\frac{1}{x})=-\frac{1}{2x}-\frac{1}{4}sin(\frac{2}{x})+c\,,cos^2(x)=\frac{1}{2}+\frac{1}{2}cos(2x))$$

الإجابة الصحيحة هي (b)

$$182-\int(\frac{cos(\sqrt{x})}{\sqrt{x}\,sin^2(\sqrt{x})})\,dx=\int(\frac{cos(\sqrt{x})}{\sqrt{x}\,sin(\sqrt{x})\,sin(\sqrt{x})})\,dx=\int(\frac{cot(\sqrt{x})\,csc(\sqrt{x})}{\sqrt{x}})\,dx,cot(x)=\frac{cos(x)}{sin(x)}$$

$$,csc(x)=\frac{1}{sin(x)},u=\sqrt{x}\Rightarrow du=\frac{1}{2\sqrt{x}}.dx\Rightarrow dx=2\sqrt{x}.du,$$ التكامل بالتعويض

$$=\int(\frac{cot(\sqrt{x})\,csc(\sqrt{x})}{\sqrt{x}})\,dx=\int(\frac{cot(u)\,csc(u)}{\sqrt{x}}).2\sqrt{x}.du=2\int(cot(u)\,csc(u)).du$$

$$=-2\,csc(u)=-2\,csc(\sqrt{x})+c$$

الإجابة الصحيحة هي (c)

$$183-\int(3\,sin^3(3x))\,dx=3\int(sin^3(3x))\,dx=3\int sin^2(3x)\,sin(3x).dx$$

$$=3\int(1-cos^2(3x))\,sin(3x).dx=3\int(sin(3x)-cos^2(3x)\,sin(3x)).dx$$

$$=3(\int(sin(3x).dx-\int cos^2(3x)\,sin(3x).dx)$$

$$,u=cos(3x)\Rightarrow du=-3\,sin(3x).dx\Rightarrow dx=\frac{du}{-3\,sin(3x)},$$ التكامل بالتعويض

$$=3\left(\int(sin(3x).dx-\int u^2\,sin(3x).\frac{du}{-3\,sin(3x)}\right)=3\left(\int(sin(3x).dx+\frac{1}{3}\int u^2.du\right)$$

$$=3\left(-\frac{1}{3}cos(3x)+(\frac{1}{3}.\frac{u^3}{3})\right)=-cos(3x)+\frac{1}{3}u^3=\frac{1}{3}u^3-cos(3x)=\frac{1}{3}cos^3(3x)-cos(3x)+c$$

$$,sin^2(x)+cos^2(x)=1\Rightarrow sin^2(x)=1-cos^2(x)$$

الإجابة الصحيحة هي (d)

$$184-\int(sin^5(x)\,cos^7(x))\,dx\,,u=cos(x)\Rightarrow du=-sin(x).dx\Rightarrow dx=\frac{du}{-sin(x)},$$ التكامل بالتعويض,

$$u=cos(x)\Rightarrow cos^2(x)=u^2$$

$$=\int(sin^5(x)\,cos^7(x))\,dx=\int(sin^5(x)\,u^7).\frac{du}{-sin(x)}=-\int(u^7\,sin^4(x).du$$

$$=-\int(u^7\,(sin^2(x))^2.du=-\int(u^7(1-cos^2(x))^2.du=-\int(u^7(1-u^2)^2.du$$

$$=-\int u^7(u^4-2u^2+1).du=-\int(u^{11}-2u^9+u^7).du$$

$$=-(\int u^{11}.du-2\int u^9.du+\int u^7.du)=-\left(\frac{u^{12}}{12}-\frac{2u^{10}}{10}+\frac{u^8}{8}\right)=-\frac{u^{12}}{12}+\frac{u^{10}}{5}-\frac{u^8}{8}$$

$$=-\frac{cos^{12}(x)}{12}+\frac{cos^{10}(x)}{5}-\frac{cos^8(x)}{8}+c,sin^2(x)+cos^2(x)=1\Rightarrow sin^2(x)=1-cos^2(x)$$

الإجابة الصحيحة هي (b)

$$185-\int cos^6(x)\,sin^3(x)\,dx\,,u=cos(x)\Rightarrow du=-sin(x).dx\Rightarrow dx=\frac{du}{-sin(x)}$$, التكامل بالتعويض,

$$u=cos(x)\Rightarrow cos^2(x)=u^2$$

$$=\int (cos^6(x)\,sin^3(x)\,dx=\int (sin^3(x)\,u^6).\frac{du}{-sin(x)}=-\int (u^6\,sin^2(x).du$$

$$=-\int (u^6\,(sin^2(x).du=-\int (u^6(1-cos^2(x)).du=-\int (u^6(1-u^2).du$$

$$=-\int (u^6-u^8).du=-(\int u^6.du-\int u^8.du)=-\left(\frac{u^7}{7}-\frac{u^9}{9}\right)=-\frac{u^7}{7}+\frac{u^9}{9}$$

$$=-\frac{cos^7(x)}{7}+\frac{cos^9(x)}{9}+c,\ sin^2(x)+cos^2(x)=1\Rightarrow sin^2(x)=1-cos^2(x)$$

الإجابة الصحيحة هي (c)

$$186-\int cot^3(x)\,dx\,,u=cot(x)\Rightarrow du=-csc^2(x).dx\Rightarrow dx=\frac{du}{-csc^2(x)}$$, التكامل بالتعويض,

$$=\int cot^3(x)\,dx=\int cot(x)\,cot^2(x).dx=\int cot(x)\,(csc^2(x)-1).dx$$

$$=\int cot(x)\,csc^2(x)-cot(x).dx=\int cot(x)\,csc^2(x).dx-\int cot(x).dx$$

$$=\int cot(x)\,csc^2(x).dx-\int cot(x).dx=\int u.csc^2(x).\frac{du}{-csc^2(x)}-\int \frac{cos(x)}{sin(x)}.dx$$

$$=-\int u.du-\int\frac{cos(x)}{sin(x)}.dx=-\frac{u^2}{2}-ln|sin(x)|=-\frac{1}{2}cot^2(x)-ln|sin(x)|+c$$

$$,csc^2(x)-1=cot^2(x),cot(x)=\frac{cos(x)}{sin(x)},\int\frac{g`(x)}{g(x)}.dx=ln|g(x)|+c$$

الإجابة الصحيحة هي (d)

$$187-\int cot^6(x)\,dx\,,u=cot(x)\Rightarrow du=-csc^2(x).dx\Rightarrow dx=\frac{du}{-csc^2(x)}$$, التكامل بالتعويض,

$$,csc^2(x)-1=cot^2(x)$$

$$=\int cot^6(x)\,dx=\int cot^2(x)\,cot^4(x).dx=\int cot^4(x)\,(csc^2(x)-1).dx$$

$$=\int cot^4(x)\,csc^2(x)-cot^4(x).dx=\int cot^4(x)\,csc^2(x).dx-\int cot^2(x)\,(csc^2(x)-1).dx$$

$$=\int cot^4(x)\,csc^2(x).dx-(\int cot^2(x)\,csc^2(x)-\int cot^2(x).dx)$$

$$=\int cot^4(x)\,csc^2(x).dx-(\int cot^2(x)\,csc^2(x)-\int csc^2(x)-1)$$

$$=\int cot^4(x)\,csc^2(x).dx-(\int cot^2(x)\,csc^2(x)-(\int csc^2(x).dx-\int 1.dx))$$

$$=\int u^4.csc^2(x).\frac{du}{-csc^2(x)}-(\int u^2.csc^2(x).\frac{du}{-csc^2(x)}-(\int csc^2(x).dx-\int 1.dx))$$

$$=\int -u^4.du-(\int -u^2.du-(\int csc^2(x).dx-\int 1.dx))$$

$$=\frac{-u^5}{5}-\left(\frac{-u^3}{3}-(-cot(x)-x)\right)=\frac{-u^5}{5}+\frac{u^3}{3}-cot(x)-x=\frac{-cot^5(x)}{5}+\frac{cot^3(x)}{3}-cot(x)-x+c$$

الإجابة الصحيحة هي (b)

$$188-\int csc^2(2x)\,cot(2x)\,dx\,,u=csc(2x)\Rightarrow du=-2\,csc(2x)\,cot(2x)\,.\,dx$$

$$\Rightarrow dx=\frac{du}{-2\,csc(2x)\,cot(2x)}\,,$$ التكامل بالتعويض

$$=\int csc^2(2x)\,cot(2x)\,dx=\int u^2\,cot(2x)\,.\frac{du}{-2\,csc(2x)\,cot(2x)}=-\frac{1}{2}\int u\,.\,du=-\frac{1}{2}\times\frac{u^2}{2}$$

$$=-\frac{1}{4}u^2=-\frac{1}{4}csc^2(2x)+c$$

حل آخر

$$\Rightarrow\int csc^2(2x)\,cot(2x)\,dx\,,u=cot(2x)\Rightarrow du=-2\,csc^2(2x)\,.\,dx$$

$$\Rightarrow dx=\frac{du}{-2\,csc^2(2x)}\,,$$ التكامل بالتعويض

$$=\int \csc^2(2x)\cot(2x)\,dx=\int csc^2(2x)\,cot(2x)\,.\frac{du}{-2\,csc^2(2x)}=-\frac{1}{2}\int u\,.\,du=-\frac{1}{2}\times\frac{u^2}{2}$$

$$=-\frac{1}{4}u^2=-\frac{1}{4}cot^2(2x)+c$$

الإجابة الصحيحة هي (d)

$$189-\int\frac{sin(\sqrt{x})}{\sqrt{x\,cos^3(\sqrt{x})}}dx=\int\frac{sin(\sqrt{x})}{\sqrt{x\,cos^2(\sqrt{x})\,cos(\sqrt{x})}}dx=\int\frac{sin(\sqrt{x})}{cos(\sqrt{x})\sqrt{x\,cos(\sqrt{x})}}dx$$

$$=\int\frac{sin(\sqrt{x})}{cos(\sqrt{x})\,\sqrt{x}\sqrt{cos(\sqrt{x})}}dx$$

$$,u=cos(\sqrt{x})\Rightarrow du=-\frac{sin(\sqrt{x})}{2\sqrt{x}}.\,dx\Rightarrow dx=\frac{2\sqrt{x}.\,du}{-\,sin(\sqrt{x})}\,,$$ التكامل بالتعويض

$$=\int\frac{sin(\sqrt{x})}{cos(\sqrt{x})\,\sqrt{x}\sqrt{cos(\sqrt{x})}}dx=\int\frac{sin(\sqrt{x})}{u\sqrt{x}\sqrt{u}}.\frac{2\sqrt{x}.\,du}{-\,sin(\sqrt{x})}$$

$$=-2\int\frac{1}{u\sqrt{u}}.\,du=-2\int\frac{1}{u.\,u^{\frac{1}{2}}}.\,du=-2\int\frac{1}{u^{\frac{3}{2}}}.\,du-2\int u^{-\frac{3}{2}}.\,du=-2\times\frac{u^{-\frac{1}{2}}}{-\frac{1}{2}}$$

$$=\frac{4}{u^{\frac{1}{2}}}=\frac{4}{\sqrt{u}}=\frac{4}{\sqrt{u}}=\frac{4}{\sqrt{cos(\sqrt{x})}}+c=4cos^{-\frac{1}{2}}(\sqrt{x})+c$$

الإجابة الصحيحة هي (c)

$$190-\int\frac{sec(x)\,tan(x)}{\sqrt{sec(x)}}dx\,,u=\sqrt{sec(x)}\Rightarrow du=\frac{sec(x)\,tan(x)}{2\sqrt{sec(x)}}.\,dx\Rightarrow dx=\frac{2\sqrt{sec(x)}\,du}{sec(x)\,tan(x)}\,,$$ التكامل بالتعويض

$$=\int\frac{sec(x)\,tan(x)}{\sqrt{sec(x)}}dx=\int\frac{sec(x)\,tan(x)}{\sqrt{sec(x)}}.\frac{2\sqrt{sec(x)}\,du}{sec(x)\,tan(x)}=\int 2\,.\,du=2u=2\sqrt{sec(x)}+c$$

الإجابة الصحيحة هي (c)

$$191-\int sec^2(x)\,tan^2(x)\,dx\,,u=tan(x)\Rightarrow du=sec^2(x)\,.\,dx\Rightarrow dx=\frac{du}{sec^2(x)}\,,$$ التكامل بالتعويض

$$=\int sec^2(x)\,tan^2(x)\,dx=\int sec^2(x)\,tan^2(x)\,.\frac{du}{sec^2(x)}=\int u^2\,.\,du=\frac{u^3}{3}=\frac{1}{3}u^3=\frac{1}{3}tan^3(x)+c$$

الإجابة الصحيحة هي (d)

$$192-\int tan^3(x)\,sec^3(x)\,dx\,, u=sec(x)\Rightarrow du=sec(x)\,tan(x)\,.dx\Rightarrow dx=\frac{du}{sec(x)\,tan(x)}$$, التكامل بالتعويض

$$=\int tan^3(x)\,sec^3(x)\,dx=\int tan^3(x)\,sec^3(x)\,.\frac{du}{sec(x)\,tan(x)}=\int tan^2(x)\,u^2\,.du$$

$$=\int (sec^2(x)-1)u^2\,.du=\int (u^2-1)u^2\,.du=\int (u^4-u^2)\,.du=\int u^4\,.du-\int u^2\,.du=\frac{u^5}{5}-\frac{u^3}{3}$$

$$=\frac{1}{5}sec^5(x)-\frac{1}{3}sec^3(x)+c, sec^2(x)=tan^2(x)+1\Rightarrow tan^2(x)=sec^2(x)-1$$

الإجابة الصحيحة هي (d)

$$193-\int tan^5(x)\,sec^4(x)\,dx\,, u=tan(x)\Rightarrow du=sec^2(x)\,.dx\Rightarrow dx=\frac{du}{sec^2(x)}$$, التكامل بالتعويض

$$=\int tan^5(x)\,sec^4(x)\,dx=\int tan^5(x)\,sec^4(x)\,.\frac{du}{sec^2(x)}$$

$$=\int u^5\,sec^2(x)\,.du=\int u^5(tan^2(x)+1)\,.du=\int u^5(u^2+1)\,.du=\int (u^7+u^5)\,.du$$

$$\int u^7\,.du+\int u^5\,.du=\frac{u^8}{8}+\frac{u^5}{5}=\frac{1}{8}tan^8(x)+\frac{1}{5}tan^5(x)+c, sec^2(x)=tan^2(x)+1$$

الإجابة الصحيحة هي (c)

$$194-\int \csc^5(x)\cot^3(x)\,dx\,, u=csc(x)\Rightarrow du=-csc(x)\,cot(x)\,.dx$$

$$\Rightarrow dx=\frac{du}{-csc(x)\,cot(x)}$$, التكامل بالتعويض

$$=\int \csc^5(x)\cot^3(x)\,dx=\int \csc^5(x)\cot^3(x)\,.\frac{du}{-csc(x)\,cot(x)}=-\int u^4\cot^2(x)\,.du$$

$$=-\int u^4(csc^2(x)-1)\,.du=-\int u^4(u^2-1)\,.du=-\int (u^6-u^4)\,.du=-\int u^6\,.du+\int u^4\,.du$$

$$=-\frac{u^7}{7}+\frac{u^5}{5}=-\frac{1}{7}csc^7(x)+\frac{1}{5}csc^5(x)+c=\frac{1}{5}csc^5(x)-\frac{1}{7}csc^7(x)+c, csc^2(x)-1=cot^2(x)$$

الإجابة الصحيحة هي (b)

$$195-\int cos(x)\,sin^2(2x)\,dx\,, u=sin(x)\Rightarrow du=cos(x)\,.dx\Rightarrow dx=\frac{du}{cos(x)}$$, التكامل بالتعويض,

$$=\int cos(x)\,sin^2(2x)\,dx=\int cos(x)\,(2\,sin(x)\,cos(x))^2\,dx, sin(2x)=2\,sin(x)\,cos(x)$$

$$=\int 4\,cos(x)\,sin^2(x)\,cos^2(x)\,dx=4\int sin^2(x)\,cos^3(x)\,dx=4\int sin^2(x)\,cos^3(x)\,.\frac{du}{cos(x)}$$

$$=4\int u^2\,cos^2(x)\,.du=4\int (u^2(1-sin^2(x)).du=4\int (u^2(1-u^2).du$$

$$=4\int (u^2-u^4).du=4(\int u^2\,.du-\int u^4\,.du)=4\left(\frac{u^3}{3}-\frac{u^5}{5}\right)=\frac{4}{3}sin^3(x)-\frac{4}{5}sin^5(x)+c$$

$$, sin^2(x)+cos^2(x)=1\Rightarrow cos^2(x)=1-sin^2(x)$$

الإجابة الصحيحة هي (c)

$$196-\int \frac{sec^4(x)}{tan^2(x)}dx\,, u=tan(x)\Rightarrow du=sec^2(x)\,.dx\Rightarrow dx=\frac{du}{sec^2(x)}$$, التكامل بالتعويض

$$=\int \frac{sec^4(x)}{tan^2(x)}dx=\int \frac{sec^4(x)}{tan^2(x)}\,.\frac{du}{sec^2(x)}=\int \frac{sec^2(x)}{u^2}\,.du=\int \frac{tan^2(x)+1}{u^2}\,.du=\int \frac{u^2+1}{u^2}\,.du$$

$$=\int 1+\frac{1}{u^2}.du=\int 1.du+\int u^{-2}.du=u-u^{-1}=u-\frac{1}{u}=tan(x)-\frac{1}{tan(x)}+c$$

$$, sec^2(x)=tan^2(x)+1$$

الإجابة الصحيحة هي (d)

$$197-\int \frac{cot^3(x)}{csc^2(x)}dx\,,u=csc(x)\Rightarrow du=-csc(x)\,cot(x)\,.dx$$

$$\Rightarrow dx=\frac{du}{-csc(x)\,cot(x)}$$ ، التكامل بالتعويض

$$=\int \frac{cot^3(x)}{csc^2(x)}dx=\int \frac{cot^3(x)}{csc^2(x)}.\frac{du}{-csc(x)\,cot(x)}=-\int \frac{cot^2(x)}{u^3}.du=-\int \frac{csc^2(x)-1}{u^3}.du$$

$$=-\int \frac{u^2-1}{u^3}.du=-\int (\frac{u^2}{u^3}-\frac{1}{u^3}).du=-\int \left(\frac{1}{u}-u^{-3}\right).du=-\int \frac{1}{u}.du+\int u^{-3}.du$$

$$=-ln|u|+\frac{u^{-2}}{-2}=-ln|u|-\frac{1}{2}u^{-2}=-ln|csc(x)|-\frac{1}{2}csc^{-2}(x)=-ln|csc(x)|-\frac{1}{2\,csc^{-2}(x)}$$

$$=-ln|csc(x)|-\frac{1}{2}sin^2(x)+c\,,csc^2(x)-1=cot^2(x),sin(x)=\frac{1}{csc(x)}$$

الإجابة الصحيحة هي (a)

$$198-\int \frac{sin^3(x)}{\sqrt{cos(x)}}dx\,,u=cos(x)\Rightarrow du=-sin(x\,).dx\Rightarrow dx=\frac{du}{-sin(x\,)}$$ ، التكامل بالتعويض

$$=\int \frac{sin^3(x)}{\sqrt{cos(x)}}dx=\int \frac{sin^3(x)}{\sqrt{cos(x)}}.\frac{du}{-sin(x\,)}=-\int \frac{sin^2(x)}{\sqrt{u}}.du=-\int \frac{1-cos^2(x)}{u^{\frac{1}{2}}}.du$$

$$=-\int \frac{1-u^2}{u^{\frac{1}{2}}}.du=-\int \frac{1}{u^{\frac{1}{2}}}.du+\int u^{\frac{3}{2}}.du=-\int u^{-\frac{1}{2}}.du+\int u^{\frac{3}{2}}.du=-2u^{\frac{1}{2}}+\frac{2u^{\frac{5}{2}}}{5}$$

$$=-2\,cos^{\frac{1}{2}}(x)+\frac{2}{5}cos^{\frac{5}{2}}(x)+c\,,sin^2(x)+cos^2(x)=1\Rightarrow sin^2(x)=1-cos^2(x)$$

الإجابة الصحيحة هي (b)

$$199-\int \frac{sin(2x)}{2+cos(x)}dx\,,u=2+cos(x)\Rightarrow du=-sin(x).dx\Rightarrow dx=\frac{du}{-sin(x)}$$ ، التكامل بالتعويض,

$$u=2+cos(x)\Rightarrow cos(x)=u-2$$

$$=\int \frac{2\,sin(x)\,cos(x)}{2+cos(x)}dx=\int \frac{2\,sin(x)\,cos(x)}{2+cos(x)}.\frac{du}{-sin(x)},sin(2x)=2\,sin(x)\,cos(x)$$

$$=-\int \frac{2\,cos(x)}{u}.du=-\int \frac{2(u-2)}{u}.du=-\int \frac{2u-4}{u}.du=-\int \frac{2u}{u}.du+4\int \frac{1}{u}.du$$

$$=-\int 2\,.du+4\int \frac{1}{u}.du=-2u+4\ln|u|=-2(2+cos(x))+4\ln|2+cos(x)|$$

$$=-4-2\,cos(x)+4\ln|2+cos(x)|+c=\ 4\,ln|2+cos(x)|-4-2\,cos(x)+c$$

الإجابة الصحيحة هي (d)

$$200-\int \frac{cos(x)}{sin^2(x)}dx\,,u=sin(x)\Rightarrow du=cos(x)\,.dx\Rightarrow dx=\frac{du}{cos(x)}$$ ، التكامل بالتعويض,

$$=\int \frac{cos(x)}{sin^2(x)}dx=\int \frac{cos(x)}{sin^2(x)}.\frac{du}{cos(x)}=\int \frac{1}{u^2}.du=\int u^{-2}\,.du=-u^{-1}=-sin^{-1}(x)=-\frac{1}{sin(x)}+c$$

الإجابة الصحيحة هي (b)

$$201-\int \frac{sin^3(x)}{cos^2(x)}dx\,,u=cos(x)\Rightarrow du=-sin(x\,).dx\Rightarrow dx=\frac{du}{-sin(x\,)}$$ ، التكامل بالتعويض

$$=\int \frac{sin^3(x)}{cos^2(x)}dx=\int \frac{sin^3(x)}{cos^2(x)}.\frac{du}{-sin(x\,)}=-\int \frac{sin^2(x)}{u^2}.du=-\int \frac{1-cos^2(x)}{u^2}.du$$

$$=-\int \frac{1-u^2}{u^2}.du=-\int \frac{1}{u^2}.du+\int 1\,.du=-\int u^{-2}\,.du+\int 1\,.du=u^{-1}+u=\frac{1}{u}+u$$

$$=\frac{1}{cos(x)}+cos(x)+c,sin^2(x)+cos^2(x)=1\Rightarrow sin^2(x)=1-cos^2(x)$$

الإجابة الصحيحة هي (a)

$$202-\int\left(\frac{tan(x)}{cos^2(x)}+\frac{sin(x)}{cos^3(x)}\right)dx\,,u=cos(x)\Rightarrow du=-sin(x).dx\Rightarrow dx=\frac{du}{-sin(x)}$$ ،التكامل بالتعويض

$$=\int\left(\frac{tan(x)}{cos^2(x)}+\frac{sin(x)}{cos^3(x)}\right)dx=\int\left(\frac{\frac{sin(x)}{cos(x)}}{cos^2(x)}+\frac{sin(x)}{cos^3(x)}\right)dx=\int\left(\frac{sin(x)}{cos^3(x)}+\frac{sin(x)}{cos^3(x)}\right)dx$$

$$=2\int\frac{sin(x)}{cos^3(x)}dx=2\int\frac{sin(x)}{cos^3(x)}.\frac{du}{-sin(x)}=-2\int\frac{1}{u^3}.du=-2\int u^{-3}.du=-2\frac{u^{-2}}{-2}=\frac{1}{u^2}$$

$$=\frac{1}{cos^2(x)}=sec^2(x)+c\,,tan(x)=\frac{sin(x)}{cos(x)}\,,sec(x)=\frac{1}{cos(x)}$$

الإجابة الصحيحة هي (b)

$$203-\int\left(\frac{cot(x)}{sin^2(x)}\right)dx\,,u=sin(x)\Rightarrow du=cos(x).dx\Rightarrow dx=\frac{du}{cos(x)}$$ ،التكامل بالتعويض

$$=\frac{\frac{cos(x)}{sin(x)}}{sin^2(x)}dx=\int\frac{cos(x)}{sin^3(x)}dx=\int\frac{cos(x)}{sin^3(x)}.\frac{du}{cos(x)}=\int\frac{1}{u^3}.du=\int u^{-3}.du=\frac{u^{-2}}{-2}=-\frac{1}{2u^2}$$

$$=-\frac{1}{2sin^2(x)}=-\frac{1}{2}csc^2(x)+c\,,cot(x)=\frac{cos(x)}{sin(x)}\,,sin(x)=\frac{1}{csc(x)}$$

الإجابة الصحيحة هي (d)

$$204-\int sec^6(x)\,dx\,,u=tan(x)\Rightarrow du=sec^2(x).dx\Rightarrow dx=\frac{du}{sec^2(x)}$$ ،التكامل بالتعويض

$$=\int sec^2(x)\,sec^4(x)\,dx=\int(tan^2(x)+1)\,sec^4(x)\,dx=\int(tan^2(x)\,sec^4(x)+sec^4(x))dx$$

$$=\int tan^2(x)\,sec^4(x).dx+\int sec^4(x)\,dx=\int tan^2(x)\,sec^4(x).dx+\int sec^2(x)\,sec^2(x)\,dx$$

$$=\int tan^2(x)\,sec^4(x).dx+\int sec^2(x)\,(tan^2(x)+1)dx$$

$$=\int tan^2(x)\,sec^4(x).dx+\int(sec^2(x)\,tan^2(x)+sec^2(x)).dx$$

$$=\int tan^2(x)\,sec^4(x).dx+\int sec^2(x)\,tan^2(x).dx+\int sec^2(x)\,dx$$

$$=\int tan^2(x)\,sec^4(x).\frac{du}{sec^2(x)}+\int sec^2(x)\,tan^2(x).\frac{du}{sec^2(x)}+\int sec^2(x)\,dx$$

$$=\int u^2\,sec^2(x).du+\int u^2.du+\int sec^2(x)\,dx=\int u^2(tan^2(x)+1).du+\int u^2.du+\int sec^2(x)\,dx$$

$$=\int u^2(u^2+1).du+\int u^2.du+\int sec^2(x)\,dx=\int(u^4+u^2).du+\int u^2.du+\int sec^2(x)\,dx$$

$$=\int u^4.du+\int u^2.du+\int u^2.du+\int sec^2(x)\,dx=\frac{1}{5}u^5+\frac{1}{3}u^3+\frac{1}{3}u^3+tan(x)$$

$$=tan(x)+\frac{2}{3}tan^3(x)+\frac{1}{5}tan^5(x)+c\,,sec^2(x)=tan^2(x)+1$$

الإجابة الصحيحة هي (d)

$$205-\int tan^5(x)\,dx\,,u=sec(x)\Rightarrow du=sec(x)\,tan(x).dx\Rightarrow dx=\frac{du}{sec(x)\,tan(x)}$$ ،التكامل بالتعويض

$$=\int tan(x)\,tan^2(x)\,tan^2(x)\,dx=\int tan(x)\,(sec^2(x)-1)^2\,dx$$

$$=\int tan(x)\,(sec^2(x)-1)^2.\frac{du}{sec(x)\,tan(x)}=\int(u^2-1)^2.\frac{du}{u}=\int\left(\frac{u^4-2u^2+1}{u}\right).du$$

$$=\int\frac{u^4}{u}.du-2\int\frac{u^2}{u}.du+\int\frac{1}{u}.du=\int u^3.du-2\int u.du+\int\frac{1}{u}.du$$

$$=\frac{1}{4}u^4-\frac{2}{2}u^2+\ln|u|=\frac{1}{4}u^4-u^2+\ln|u|=\frac{1}{4}sec^4(x)-sec^2(x)+\ln|sec(x)|+c$$

$ln|sec(x)| + \frac{1}{4}sec^4(x) - sec^2(x) + c, sec^2(x) = tan^2(x) + 1, \int \frac{g`(x)}{g(x)}.dx = ln|g(x)| + c$

الإجابة الصحيحة هي (c)

$206 - \int csc^6(x)\, dx, u = cot(x) \Rightarrow du = -csc^2(x).dx \Rightarrow dx = \frac{du}{-csc^2(x)}$, التكامل بالتعويض

$= \int csc^2(x)\, csc^4(x)\, dx = \int (cot^2(x) + 1)\, csc^4(x)\, dx = \int (cot^2(x)\, csc^4(x) + csc^4(x))dx$

$= \int cot^2(x)\, csc^4(x)\, dx + \int csc^4(x)\, dx = \int cot^2(x)\, csc^4(x).dx + \int csc^2(x)\, csc^2(x).dx$

$= \int cot^2(x)\, csc^4(x).dx + \int csc^2(x)\,(cot^2(x) + 1)dx$

$= \int cot^2(x)\, csc^4(x).dx + \int (csc^2(x)\, cot^2(x) + csc^2(x)).dx$

$= \int cot^2(x)\, csc^4(x).dx + \int csc^2(x)\, cot^2(x).dx + \int csc^2(x)\, dx$

$= \int cot^2(x)\, csc^4(x).\frac{du}{-csc^2(x)} + \int csc^2(x)\, cot^2(x).\frac{du}{-csc^2(x)} + \int csc^2(x)\, dx$

$= -\int u^2\, csc^2(x).du - \int u^2.du + \int csc^2(x)\, dx$

$= -\int u^2\,(cot^2(x) + 1).du - \int u^2.du + \int csc^2(x)\, dx$

$= -\int u^2(u^2 + 1).du - \int u^2.du + \int sec^2(x)\, dx = -\int (u^4 + u^2).du - \int u^2.du + \int csc^2(x)\, dx$

$= -\int u^4.du - \int u^2.du - \int u^2.du + \int csc^2(x)\, dx = -\frac{1}{5}u^5 - \frac{1}{3}u^3 - \frac{1}{3}u^3 - cot(x)$

$= -cot(x) - \frac{2}{3}cot^3(x) - \frac{1}{5}cot^5(x), csc^2(x) = cot^2(x) + 1$

الإجابة الصحيحة هي (b)

$207 - \int cos^5(x)\, dx, u = sin(x) \Rightarrow du = cos(x).dx \Rightarrow dx = \frac{du}{cos(x)}$, التكامل بالتعويض

$= \int cos^5(x)\, dx = \int cos^2(x)\, cos^2(x)\, cos(x)\, dx = \int ((1 - sin^2(x))(1 - sin^2(x))\, cos(x))\, dx$

$= \int ((1 - sin^2(x))(1 - sin^2(x))\, cos(x)).\frac{du}{cos(x)} = \int ((1 - u^2)(1 - u^2)).du$

$= \int (u^4 - 2u^2 + 1).du = \int u^4.du - 2\int u^2.du + \int 1.du = \frac{1}{5}u^5 - \frac{2}{3}u^3 + u$

$= \frac{1}{5}sin^5(x) - \frac{2}{3}sin^3(x) + sin(x) + c = sin(x) - \frac{2}{3}sin^3(x) + \frac{1}{5}sin^5(x) + c$

$, sin^2(x) + cos^2(x) = 1 \Rightarrow cos^2(x) = 1 - sin^2(x)$

الإجابة الصحيحة هي (c)

$208 - \int sin^5(x)\, dx, u = cos(x) \Rightarrow du = -sin(x).dx \Rightarrow dx = \frac{du}{-sin(x)}$, التكامل بالتعويض

$= \int sin^5(x)\, dx = \int sin^2(x)\, sin^2(x)\, sin(x)\, dx = \int ((1 - cos^2(x))(1 - cos^2(x))\, sin(x))\, dx$

$= \int ((1 - cos^2(x))(1 - cos^2(x))\, sin(x)).\frac{du}{-sin(x)} = -\int ((1 - u^2)(1 - u^2)).du$

$= -\int (u^4 - 2u^2 + 1).du = -\int u^4.du + 2\int u^2.du - \int 1.du = -\frac{1}{5}u^5 + \frac{2}{3}u^3 - u$

$= -\frac{1}{5}cos^5(x) + \frac{2}{3}cos^3(x) - cos(x) + c = -cos(x) + \frac{2}{3}cos^3(x) - \frac{1}{5}cos^5(x) + c$

$, sin^2(x) + cos^2(x) = 1 \Rightarrow sin^2(x) = 1 - cos^2(x)$

الإجابة الصحيحة هي (c)

$$209-\int \frac{x+14}{(x-4)(x+2)}dx \Rightarrow \frac{x+14}{(x-4)(x+2)} = \frac{A}{x-4}+\frac{B}{x+2}$$
$$\Rightarrow \frac{x+14}{(x-4)(x+2)} = \frac{A(x+2)}{x-4}+\frac{B(x-4)}{x+2}$$
$$\Rightarrow x+14 = A(x+2)+B(x-4) \Rightarrow x=-2 \Rightarrow 0-6B=12 \Rightarrow B=-2$$
$$, \Rightarrow x=4 \Rightarrow 6A+0=18 \Rightarrow A=3$$
$$\therefore \int (\frac{x+14}{(x-4)(x+2)})\,dx = \int \frac{3}{x-4}.dx + \int \frac{-2}{x+2}.dx = 3\int \frac{1}{x-4}.dx - 2\int \frac{1}{x+2}.dx$$
$$= 3\,ln|x-4| - 2\,ln|x+2| + c$$

الإجابة الصحيحة هي (b)

$$210-\int \frac{2x-13}{x^2-x-2}dx = \int \frac{2x-13}{(x-2)(x+1)}dx \Rightarrow \frac{2x-13}{(x-2)(x+1)} = \frac{A}{x-2}+\frac{B}{x+1}$$
$$\Rightarrow \frac{2x-13}{(x-2)(x+1)} = \frac{A(x+1)}{x-2}+\frac{B(x-2)}{x+1}$$
$$\Rightarrow 2x-13 = A(x+1)+B(x-2) \Rightarrow x=-1 \Rightarrow 0-3B=-15 \Rightarrow B=5$$
$$, \Rightarrow x=2 \Rightarrow 3A+0=-9 \Rightarrow A=-3$$
$$\therefore \int (\frac{2x-13}{(x-2)(x+1)})\,dx = \int \frac{-3}{x-2}.dx + \int \frac{5}{x+1}.dx = -3\int \frac{1}{x-2}.dx + 5\int \frac{1}{x+1}.dx$$
$$= -3\,ln|x-2| + 5\,ln|x+1| + c$$

الإجابة الصحيحة هي (d)

$$211-\int \frac{x}{x^2-5x+6}dx = \int \frac{x}{(x-2)(x-3)}dx \Rightarrow \frac{x}{(x-2)(x-3)} = \frac{A}{x-2}+\frac{B}{x-3}$$
$$\Rightarrow \frac{x}{(x-2)(x-3)} = \frac{A(x-3)}{x-2}+\frac{B(x-2)}{x-3}$$
$$\Rightarrow x = A(x-3)+B(x-2) \Rightarrow x=3 \Rightarrow 0+B=3 \Rightarrow B=3$$
$$, \Rightarrow x=2 \Rightarrow -A+0=2 \Rightarrow A=-2$$
$$\therefore \int (\frac{x}{(x-2)(x-3)})\,dx = \int \frac{-2}{x-2}.dx + \int \frac{3}{x-3}.dx = -2\int \frac{1}{x-2}.dx + 3\int \frac{1}{x-3}.dx$$
$$= -2\,ln|x-2| + 3\,ln|x-3| + c$$

الإجابة الصحيحة هي (d)

$$212-\int \frac{-x^2+2x+4}{x^3-4x^2+4x}dx = \int \frac{-x^2+2x+4}{x(x^2-4x+4)}dx = \frac{-x^2+2x+4}{x(x-2)(x-2)} = \frac{-x^2+2x+4}{x(x-2)^2}$$
$$\Rightarrow \frac{-x^2+2x+4}{x(x-2)^2} = \frac{A}{x}+\frac{B}{x-2}+\frac{C}{(x-2)^2} \Rightarrow \frac{-x^2+2x+4}{x(x-2)^2} = \frac{A(x-2)^2}{x}+\frac{B(x(x-2))}{x-2}+\frac{C(x)}{(x-2)^2}$$
$$\Rightarrow -x^2+2x+4 = A(x-2)^2+B(x^2-x)+C(x) \Rightarrow x=0 \Rightarrow 4A+0+0=4 \Rightarrow A=1$$
$$, \Rightarrow x=1 \Rightarrow 1+0+C=5 \Rightarrow C=4, \Rightarrow x=3 \Rightarrow 1+6B+12=1 \Rightarrow B=-2$$
$$\therefore \int (\frac{-x^2+2x+4}{x(x-2)(x-2)})\,dx = \int \frac{1}{x}.dx + \int \frac{-2}{x-2}.dx + \int \frac{4}{(x-2)^2}.dx$$
$$= \int \frac{1}{x}.dx - 2\int \frac{1}{x-2}.dx + 4\int (x-2)^{-2}.dx = \ln|x| - 2\,ln|x-2| - \frac{4}{x-2} + c$$

الإجابة الصحيحة هي (c)

$$213-\int \frac{x^2+8x+4}{x^3-2x^2}dx = \int \frac{x^2+8x+4}{x^2(x-2)}dx \Rightarrow \frac{x^2+8x+4}{x^2(x-2)} = \frac{A}{x}+\frac{B}{x^2}+\frac{C}{x-2}$$
$$\Rightarrow \frac{x^2+8x+4}{x^2(x-2)} = \frac{A\big(x(x-2)\big)}{x}+\frac{B(x-2)}{x^2}+\frac{C(x^2)}{x-2}$$
$$\Rightarrow x^2+8x+4 = A(x^2-2x)+B(x-2)+C(x^2)$$
$$\Rightarrow x=0 \Rightarrow 0-2B+0=4 \Rightarrow B=-2, \Rightarrow x=2 \Rightarrow 0+0+4C=24 \Rightarrow C=6$$
$$, \Rightarrow x=1 \Rightarrow -A+2+6=13 \Rightarrow A=-5$$
$$\therefore \int (\frac{x^2+8x+4}{x^2(x-2)})\,dx = \int \frac{-5}{x}.dx + \int \frac{-2}{x^2}.dx + \int \frac{6}{x-2}.dx$$

$$= -5\int \frac{1}{x}.dx - 2\int \frac{1}{x^2}.dx + 6\int \frac{1}{x-2}.dx = -5\,ln|x| + \frac{2}{x} + 6\,ln|x-2| + c$$
$$= \frac{2}{x} + 6\,ln|x-2| - 5\,ln|x| + C$$

الإجابة الصحيحة هي (b)

$$214 - \int \frac{5x-12}{x^2+4x}dx = \int \frac{5x-12}{x(x+4)}dx \Rightarrow \frac{5x-12}{x(x+4)} = \frac{A}{x} + \frac{B}{x+4} \Rightarrow \frac{5x-12}{x(x+4)} = \frac{A(x+4)}{x} + \frac{B(x)}{x+4}$$
$$\Rightarrow 5x-12 = A(x+4) + B(x) \Rightarrow x = 0 \Rightarrow 4A + 0 = -12 \Rightarrow A = -3$$
$$, \Rightarrow x = 1 \Rightarrow -15 + B = -7 \Rightarrow B = 8$$
$$\therefore \int (\frac{5x-12}{x(x+4)})\,dx = \int \frac{-3}{x}.dx + \int \frac{8}{x+4}.dx = -3\int \frac{1}{x}.dx + 8\int \frac{1}{x+4}.dx$$
$$= -3\,ln|x| + 8\,ln|x+4| + c$$

الإجابة الصحيحة هي (c)

$$215 - \int \frac{x^2-2}{x^3-x^2-2x}dx = \int \frac{x^2-2}{x(x^2-x-2)}dx = \int \frac{x^2-2}{x(x-2)(x+1)}dx$$
$$\Rightarrow \frac{x^2-2}{x(x-2)(x+1)} = \frac{A}{x} + \frac{B}{x+1} + \frac{C}{x-2}$$
$$\Rightarrow \frac{x^2-2}{x(x-2)(x+1)} = \frac{A(x^2-x-2)}{x} + \frac{B(x(x-2))}{x+1} + \frac{C(x(x+1))}{x-2}$$
$$\Rightarrow x^2-2 = A(x^2-x-2) + B(x^2-2x) + C(x^2+x)$$
$$\Rightarrow x = 2 \Rightarrow 0 + 0 + 6C = 2 \Rightarrow C = \frac{1}{3}, \Rightarrow x = -1 \Rightarrow 0 + 3B + 0 = -1 \Rightarrow B = -\frac{1}{3}$$
$$, \Rightarrow x = 1 \Rightarrow -2A + \frac{1}{3} + \frac{2}{3} = -1 \Rightarrow A = 1$$
$$\therefore \int (\frac{x^2-2}{x(x-2)(x+1)})\,dx = \int \frac{1}{x}.dx + \int \frac{-\frac{1}{3}}{x+1}.dx + \int \frac{\frac{1}{3}}{x-2}.dx$$
$$= \int \frac{1}{x}.dx - \frac{1}{3}\int \frac{1}{x+1}.dx + \frac{1}{3}\int \frac{1}{x-2}.dx = ln|x| + \frac{1}{3}ln|x-2| - \frac{1}{3}ln|x+1| + c$$

الإجابة الصحيحة هي (d)

$$216 - \int \frac{5x^2-10x-8}{x^3-4x}dx = \int \frac{5x^2-10x-8}{x(x^2-4)}dx = \int \frac{5x^2-10x-8}{x(x-2)(x+2)}dx$$
$$\Rightarrow \frac{5x^2-10x-8}{x(x-2)(x+2)} = \frac{A}{x} + \frac{B}{x+2} + \frac{C}{x-2}$$
$$\Rightarrow \frac{5x^2-10x-8}{x(x-2)(x+2)} = \frac{A(x^2-4)}{x} + \frac{B(x(x-2))}{x+2} + \frac{C(x(x+2))}{x-2}$$
$$\Rightarrow 5x^2-10x-8 = A(x^2-4) + B(x^2-2x) + C(x^2+2x)$$
$$\Rightarrow x = 2 \Rightarrow 0 + 0 + 8C = -8 \Rightarrow C = -1, \Rightarrow x = -2 \Rightarrow 0 + 8B + 0 = 32 \Rightarrow B = 4$$
$$, \Rightarrow x = 1 \Rightarrow -3A - 4 - 3 = -13 \Rightarrow A = 2$$
$$\therefore \int (\frac{5x^2-10x-8}{x(x-2)(x+2)})\,dx = \int \frac{2}{x}.dx + \int \frac{4}{x+2}.dx + \int \frac{-1}{x-2}.dx$$
$$= 2\int \frac{1}{x}.dx + 4\int \frac{1}{x+2}.dx - \int \frac{1}{x-2}.dx = 2\,ln|x| + 4ln|x+2| - ln|x-2| + c$$

الإجابة الصحيحة هي (a)

$$217-\int \frac{x^2+x}{(x^2-4)(x+4)}dx=\int \frac{x^2+x}{(x+2)(x-2)(x+4)}dx$$

$$\Rightarrow \frac{x^2+x}{(x+2)(x-2)(x+4)}=\frac{A}{x+2}+\frac{B}{x-2}+\frac{C}{x+4}$$

$$\Rightarrow \frac{x^2+x}{(x+2)(x-2)(x+4)}=\frac{A((x-2)(x+4))}{x+2}+\frac{B((x+2)(x+4))}{x-2}+\frac{C((x-2)(x+2))}{x+4}$$

$$\Rightarrow x^2+x=\ A(x^2+2x-8)+B(x^2+6x+8)+C(x^2-4)$$

$$\Rightarrow x=2\Rightarrow 0+24B+0=6\Rightarrow B=\frac{1}{4},\Rightarrow x=-2\Rightarrow -8A+0+0=2\Rightarrow A=-\frac{1}{4}$$

$$,\Rightarrow x=1\Rightarrow \frac{5}{4}+\frac{15}{4}-3C=2\Rightarrow C=1$$

$$\therefore \int (\frac{x^2+x}{(x+2)(x-2)(x+4)})\,dx=\int \frac{-\frac{1}{4}}{x+2}.dx+\int \frac{\frac{1}{4}}{x-2}.dx+\int \frac{1}{x+4}.dx$$

$$=-\frac{1}{4}\int \frac{1}{x+2}.dx+\frac{1}{4}\int \frac{1}{x-2}.dx+\int \frac{1}{x+4}.dx=-\frac{1}{4}ln|x+2|+\frac{1}{4}ln|x-2|+ln|x+4|+c$$

$$=\frac{1}{4}ln|x-2|-\frac{1}{4}ln|x+2|+ln|x+4|+c$$

الإجابة الصحيحة هي (b)

$$218-\int \frac{2x-3}{(x^2-x-2)(x+2)}dx=\int \frac{2x-3}{(x+1)(x-2)(x+2)}dx$$

$$\Rightarrow \frac{2x-3}{(x+1)(x-2)(x+2)}=\frac{A}{x+1}+\frac{B}{x-2}+\frac{C}{x+2}$$

$$\Rightarrow \frac{2x-3}{(x+1)(x-2)(x+2)}=\frac{A((x-2)(x+2))}{x+1}+\frac{B((x+1)(x+2))}{x-2}+\frac{C((x-2)(x+1))}{x+2}$$

$$\Rightarrow 2x-3=\ A(x^2-4)+B(x^2+3x+2)+C(x^2-x-2)$$

$$\Rightarrow x=2\Rightarrow 0+12B+0=1\Rightarrow B=\frac{1}{12},\Rightarrow x=-1\Rightarrow -3A+0+0=-5\Rightarrow A=\frac{5}{3}$$

$$,\Rightarrow x=0\Rightarrow \frac{-20}{3}+\frac{1}{6}-2C=-3\Rightarrow C=-\frac{7}{4}$$

$$\therefore \int (\frac{2x-3}{(x+1)(x-2)(x+2)})\,dx=\int \frac{\frac{5}{3}}{x+1}.dx+\int \frac{\frac{1}{12}}{x-2}.dx+\int \frac{-\frac{7}{4}}{x+2}.dx$$

$$=\frac{5}{3}\int \frac{1}{x+1}.dx+\frac{1}{12}\int \frac{1}{x-2}.dx-\frac{7}{4}\int \frac{1}{x+2}.dx=\frac{5}{3}ln|x+1|+\frac{1}{12}ln|x-2|-\frac{7}{4}ln|x+2|+c$$

$$=\frac{1}{12}ln|x-2|-\frac{7}{4}ln|x+2|+\frac{5}{3}ln|x+1|+c$$

الإجابة الصحيحة هي (a)

$$219-\int \frac{2x^2-5x-1}{x^3-2x^2-x+2}dx=\int \frac{2x^2-5x-1}{(x-2)(x^2-1)}dx=\int \frac{2x^2-5x-1}{(x-2)(x-1)(x+1)}dx$$

$$\Rightarrow \frac{2x^2-5x-1}{(x-2)(x-1)(x+1)}=\frac{A}{x-2}+\frac{B}{x-1}+\frac{C}{x+1}$$

$$\Rightarrow \frac{2x^2-5x-1}{(x-2)(x-1)(x+1)}=\frac{A(x^2-1)}{x-2}+\frac{B((x+1)(x-2))}{x-1}+\frac{C((x-2)(x-1))}{x+1}$$

$$\Rightarrow 2x^2-5x-1=\ A(x^2-1)+B(x^2-x-2)+C(x^2-3x+2)$$

$$\Rightarrow x=1\Rightarrow 0-2B+0=-4\Rightarrow B=2,\Rightarrow x=-1\Rightarrow 0+0+6C=6\Rightarrow C=1$$

$$,\Rightarrow x=0\Rightarrow -A-4+2=-1\Rightarrow A=-1$$

$$\therefore \int (\frac{2x^2-5x-1}{(x-2)(x-1)(x+1)})\,dx=\int \frac{-1}{x-2}.dx+\int \frac{2}{x-1}.dx+\int \frac{1}{x+1}.dx$$

$$=-\int \frac{1}{x-2}.dx+2\int \frac{1}{x-1}.dx+\int \frac{1}{x+1}.dx=-ln|x-2|+2ln|x-1|+ln|x+1|+c$$

الإجابة الصحيحة هي (c)

$$220-\int \frac{x-2}{4x^2+4x+1}dx=\int \frac{x-2}{(2x+1)(2x+1)}dx=\int \frac{x-2}{(2x+1)^2}dx$$

$$\Rightarrow \frac{x-2}{(2x+1)^2}=\frac{A}{2x+1}+\frac{B}{(2x+1)^2}\Rightarrow \frac{x-2}{(2x+1)^2}=\frac{A(2x+2)}{2x+1}+\frac{B}{(2x+1)^2}$$

$$\Rightarrow x-2= A(2x+1)+B\Rightarrow x=-\frac{1}{2}\Rightarrow 0+B+0=\frac{-5}{2}\Rightarrow B=\frac{-5}{2}$$

$$,\Rightarrow x=0\Rightarrow A-\frac{5}{2}=-2\Rightarrow A=\frac{1}{2}$$

$$\therefore \int (\frac{x-2}{(2x+1)^2})\,dx=\int \frac{\frac{1}{2}}{2x+1}.dx+\int \frac{-\frac{5}{2}}{(2x+1)^2}.dx=\frac{1}{2}\int \frac{1}{2x+1}.dx-\frac{5}{2}\int \frac{1}{(2x+1)^2}.dx$$

$$=\frac{1}{4}\int \frac{2}{2x+1}.dx-\frac{5}{2}\int (2x+1)^{-2}.dx=\frac{1}{4}ln|2x+1|-\frac{5}{2}\left(\frac{(2x+1)^{-1}}{-2}\right)$$

$$=\frac{1}{4}ln|2x+1|+\frac{5}{4}(\frac{1}{2x+1})+c=\frac{1}{4}ln|2x+1|+\frac{5}{8x+4}+c=$$

الإجابة الصحيحة هي (d)

$$221-\int \frac{2x+2}{(x+1)^3}dx\Rightarrow \frac{2x+2}{(x+1)^3}=\frac{A}{x+1}+\frac{B}{(x+1)^2}+\frac{C}{(x+1)^3}$$

$$\Rightarrow \frac{2x+2}{(x+1)^3}=\frac{A((x+1)^2)}{x+1}+\frac{B(x+1)}{(x+1)^2}+\frac{C}{(x+1)^3}$$

$$\Rightarrow 2x+2=A((x+1)^2)+B(x+1)+C\Rightarrow x=-1\Rightarrow 0+0+C=0\Rightarrow C=0$$

$$,\Rightarrow x=0\Rightarrow A+B=2\Rightarrow -2A-2B=-4 \quad (1)$$

$$,\Rightarrow x=1\Rightarrow 4A+2B=4 \quad (2)$$

بحل المعادلتين $(1,2)$ ينتج

$$,\Rightarrow 2A=0\Rightarrow A=0\,,B=2$$

$$\therefore \int (\frac{2x+2}{(x+1)^3})\,dx=\int \frac{0}{x+1}.dx+\int \frac{2}{(x+1)^2}.dx+\int \frac{0}{(x+1)^3}.dx=\int \frac{2}{(x+1)^2}.dx$$

$$=2\int (x+1)^{-2}.dx=2\left(\frac{(x+1)^{-1}}{-1}\right)=\frac{-2}{x+1}+c$$

الإجابة الصحيحة هي (c)

$$222-\int \frac{2x-4}{(x-1)^2}dx\Rightarrow \frac{2x-4}{(x-1)^2}=\frac{A}{x-1}+\frac{B}{(x-1)^2}\Rightarrow \frac{2x-4}{(x-1)^2}=\frac{A(x-1)}{x-1}+\frac{B}{(x-1)^2}$$

$$\Rightarrow 2x-4=A(x-1)+B\Rightarrow x=1\Rightarrow 0+B=-2\Rightarrow B=-2$$

$$,\Rightarrow x=0\Rightarrow -A-2=-4\Rightarrow A=2$$

$$\therefore \int (\frac{2x-4}{(x-1)^2})\,dx=\int \frac{2}{x-1}.dx+\int \frac{-2}{(x-1)^2}.dx=2\int \frac{1}{x-1}.dx-2\int (x-1)^{-2}.dx$$

$$=2\,ln|x-1|-2\left(\frac{(x-1)^{-1}}{-1}\right)=2\,ln|x-1|+\frac{2}{x+1}+c$$

الإجابة الصحيحة هي (d)

$$223-\int \frac{x^2}{(x+2)^3}dx\Rightarrow \frac{x^2}{(x+2)^3}=\frac{A}{x+2}+\frac{B}{(x+2)^2}+\frac{C}{(x+2)^3}$$

$$\Rightarrow \frac{x^2}{(x+2)^3}=\frac{A((x+2)^2)}{x+2}+\frac{B(x+2)}{(x+2)^2}+\frac{C}{(x+2)^3}$$

$$\Rightarrow x^2=A((x+2)^2)+B(x+2)+C\Rightarrow x=-2\Rightarrow 0+0+C=4\Rightarrow C=4$$

$$,\Rightarrow x=0\Rightarrow 4A+2B+4=0\Rightarrow 4A+2B=-4\Rightarrow -12A-6B=12 \quad (1)$$

$$,\Rightarrow x=1\Rightarrow 9A+3B+4=1\Rightarrow 9A+3B=-3\Rightarrow 18A+6B=-6 \quad (2)$$

بحل المعادلتين $(1,2)$ ينتج

$$,\Rightarrow 6A=6\Rightarrow A=1\,,B=-4$$

$$\therefore \int (\frac{x^2}{(x+2)^3})\,dx=\int \frac{1}{x+2}.dx+\int \frac{-4}{(x+2)^2}.dx+\int \frac{4}{(x+2)^3}.dx$$

$$= \int \frac{1}{x+2}.dx - 4\int (x+2)^{-2}.dx + 4\int (x+2)^{-3}.dx$$

$$= ln|x+2| - 4\left(\frac{(x+2)^{-1}}{-1}\right) + 4\left(\frac{(x+2)^{-2}}{-2}\right) = ln|x+2| + \frac{4}{x+2} - \frac{2}{(x+2)^2} + c$$

الإجابة الصحيحة هي (a)

$$224 - \int \frac{2x^2+3}{x^4-2x^2+1}dx = \int \frac{2x^2+3}{(x^2-1)(x^2-1)}dx = \int \frac{2x^2+3}{(x-1)(x+1)(x-1)(x+1)}dx$$

$$= \int \frac{2x^2+3}{(x-1)^2(x+1)^2}dx \Rightarrow \frac{2x^2+3}{(x-1)^2(x+1)^2} = \frac{A}{x-1} + \frac{B}{x+1} + \frac{C}{(x-1)^2} + \frac{D}{(x+1)^2}$$

$$\Rightarrow \frac{2x^2+3}{(x-1)^2(x+1)^2} = \frac{A((x-1)(x+1)^2)}{x-1} + \frac{B((x+1)(x-1)^2)}{x+1} + \frac{C(x+1)^2}{(x-1)^2} + \frac{D(x-1)^2}{(x+1)^2}$$

$$\Rightarrow 2x^2+3 = A\left((x-1)(x+1)^2\right) + B\left((x+1)(x-1)^2\right) + C(x+1)^2 + D(x-1)^2$$

$$\Rightarrow x = 1 \Rightarrow 0+0+4C+0 = 5 \Rightarrow C = \frac{5}{4}$$

$$, \Rightarrow x = -1 \Rightarrow 0+0+0+4D = 5 \Rightarrow D = \frac{5}{4}$$

$$, \Rightarrow x = 0 \Rightarrow -A+B+\frac{5}{4}+\frac{5}{4} = 3 \Rightarrow -A+B = 3-\frac{10}{4} \Rightarrow -A+B = \frac{1}{2} \Rightarrow 3A-3B = -\frac{3}{2} \qquad (1)$$

$$, \Rightarrow x = 2 \Rightarrow 9A+3B+\frac{45}{4}+\frac{5}{4} = 11 \Rightarrow 9A+3B = 11-\frac{50}{4} \Rightarrow 9A+3B = -\frac{3}{2} \quad (2)$$

بحل المعادلتين $(1,2)$ ينتج

$$, \Rightarrow 12A = -3 \Rightarrow A = -\frac{1}{4}, B = \frac{1}{4}$$

$$\therefore \int \left(\frac{2x^2+3}{(x-1)^2(x+1)^2}\right)dx = \int \frac{-\frac{1}{4}}{x-1}.dx + \int \frac{\frac{1}{4}}{x+1}.dx + \int \frac{\frac{5}{4}}{(x-1)^2}.dx + \int \frac{\frac{5}{4}}{(x+1)^2}.dx$$

$$= -\frac{1}{4}\int \frac{1}{x-1}.dx + \frac{1}{4}\int \frac{1}{x+1}.dx + \frac{5}{4}\int (x-1)^{-2}.dx + \frac{5}{4}\int (x+1)^{-2}.dx$$

$$= -\frac{1}{4}ln|x-1| + \frac{1}{4}ln|x+1| + \frac{5}{4}\left(\frac{(x-1)^{-1}}{-1}\right) + \frac{5}{4}\left(\frac{(x+1)^{-2}}{-1}\right)$$

$$= \frac{1}{4}ln|x+1| - \frac{1}{4}ln|x-1| - \frac{5}{4x-4} - \frac{5}{4x+4} + c$$

الإجابة الصحيحة هي (a)

$$225 - \int \frac{2x^3}{x^2+1}dx, \frac{2x^3}{x^2+1} = 2x + \frac{-2x}{x^2+1}$$ بالقسمة الطويلة

$$\therefore \int \left(\frac{2x^3}{x^2+1}\right)dx = \int 2x.dx - \int \frac{2x}{x^2+1}.dx = x^2 - ln|x^2+1| + c$$

الإجابة الصحيحة هي (d)

$$226 - \int \frac{1-\sqrt{x}}{1+\sqrt[4]{x}}dx, u = \sqrt[4]{x} = x^{\frac{1}{4}} \Rightarrow du = \frac{1}{4}x^{-\frac{3}{4}}.dx \Rightarrow dx = \frac{4du}{x^{-\frac{3}{4}}} = 4x^{\frac{3}{4}}.du$$

$$, u = \sqrt[4]{x} \Rightarrow \sqrt{x} = u^2, x^{\frac{3}{4}} = u^3$$

$$\therefore \int \frac{1-\sqrt{x}}{1+\sqrt[4]{x}}dx = \int \frac{1-\sqrt{x}}{1+\sqrt[4]{x}}.4x^{\frac{3}{4}}.du = \int \frac{1-u^2}{1+u}.4u^3.du = \int \frac{(1-u)(1+u)}{1+u}.4u^3.du$$

$$= 4\int (u^3-u^4).du = 4\left(\int u^3.du - \int u^4.du\right) = 4\left(\frac{u^4}{4} - \frac{u^5}{5}\right) = u^4 - \frac{4u^5}{5} = \left(\sqrt[4]{x}\right)^4 - \frac{4\left(\sqrt[4]{x}\right)^5}{5}$$

$$= x - \frac{4\left(\sqrt[4]{x}\right)^5}{5} = x - \frac{4\sqrt[4]{x^5}}{5} + c, x^{\frac{m}{n}} = \sqrt[n]{x^m} = \left(\sqrt[n]{x}\right)^m$$

الإجابة الصحيحة هي (a)

$$227-\int \frac{dx}{\sqrt[3]{x}-x}dx \,, u=\sqrt[3]{x}=x^{\frac{1}{3}} \Rightarrow du=\frac{1}{3}x^{-\frac{2}{3}}.dx \Rightarrow dx=\frac{3du}{x^{-\frac{2}{3}}}=3x^{\frac{2}{3}}.du$$

$$,u=\sqrt[3]{x} \Rightarrow x=u^3, u^2=x^{\frac{2}{3}}$$

$$\therefore \int \frac{dx}{\sqrt[3]{x}-x}=\int \frac{3x^{\frac{2}{3}}.du}{\sqrt[3]{x}-x}=\int \frac{3u^2.du}{u-u^3}=3\int \frac{u^2}{u-u^3}.du=3\int \frac{u^2}{u(1-u^2)}.du=3\int \frac{u}{1-u^2}.du$$

$$=\frac{3}{-2}\int \frac{-2u}{1-u^2}.du=-\frac{3}{2}(ln|1-u^2|=-\frac{3}{2}(ln|1-(\sqrt[3]{x})^2|=-\frac{3}{2}(ln\left|1-\sqrt[3]{x^2}\right|+c$$

$$,x^{\frac{m}{n}}=\sqrt[n]{x^m}=\left(\sqrt[n]{x}\right)^m, \int \frac{g`(x)}{g(x)}.dx=ln|g(x)|+c$$

الإجابة الصحيحة هي (d)

$$228-\int \frac{e^{2x}}{e^{2x}+3e^x+2}dx \,, u=e^x \Rightarrow du=e^x.dx \Rightarrow dx=\frac{du}{e^x}, u^2=e^{2x}$$

$$\therefore \int \frac{e^{2x}}{e^{2x}+3e^x+2}.dx=\int \frac{e^{2x}}{e^{2x}+3e^x+2}.\frac{du}{e^x}=\int \frac{u}{u^2+3u+2}.du=\int \frac{u}{(u+1)(u+2)}.du$$

$$\Rightarrow \frac{u}{(u+1)(u+2)}=\frac{A}{u+1}+\frac{B}{u+2} \Rightarrow \frac{u}{(u+1)(u+2)}=\frac{A(u+2)}{u+1}+\frac{B(u+1)}{u+2}$$

$$\Rightarrow u=A(u+2)+B(u+1), \Rightarrow u=-1 \Rightarrow A+0=-1 \Rightarrow A=-1$$

$$,\Rightarrow u=1 \Rightarrow -3+2B=1 \Rightarrow B=2$$

$$\therefore \int (\frac{u}{(u+1)(u+2)})\,du=\int \frac{-1}{u+1}.dx+\int \frac{2}{u+2}.du=-\int \frac{1}{u+1}.dx+2\int \frac{1}{u+2}.du$$

$$=-ln|u+1|+2ln|u+2|=-ln|e^x+1|+2ln|e^x+2|+c, \int \frac{g`(x)}{g(x)}.dx=ln|g(x)|+c$$

الإجابة الصحيحة هي (a)

$$229-\int \frac{csc^2(x)}{cot^2(x)-1}dx \,, u=cot(x) \Rightarrow du=-csc^2(x).dx \Rightarrow dx=\frac{du}{-csc^2(x)}, u^2=cot^2(x)$$

$$\therefore \int \frac{csc^2(x)}{cot^2(x)-1}.dx=\int \frac{csc^2(x)}{cot^2(x)-1}.\frac{du}{-csc^2(x)}=\int \frac{-1}{u^2-1}.du=\int \frac{-1}{(u+1)(u-1)}.du$$

$$\Rightarrow \frac{-1}{(u+1)(u-1)}=\frac{A}{u+1}+\frac{B}{u-1} \Rightarrow \frac{-1}{(u+1)(u-1)}=\frac{A(u-1)}{u+1}+\frac{B(u+1)}{u-1}$$

$$\Rightarrow -1=A(u-1)+B(u+1), \Rightarrow u=-1 \Rightarrow -2A+0=-1 \Rightarrow A=\frac{1}{2}$$

$$,\Rightarrow u=1 \Rightarrow 0+2B=-1 \Rightarrow B=-\frac{1}{2}$$

$$\therefore \int (\frac{u}{(u+1)(u-1)})\,du=\int \frac{\frac{1}{2}}{u+1}.dx+\int \frac{-\frac{1}{2}}{u-1}.du=\frac{1}{2}\int \frac{1}{u+1}.dx-\frac{1}{2}\int \frac{1}{u-1}.du$$

$$=\frac{1}{2}ln|u+1|-\frac{1}{2}ln|u-1|=\frac{1}{2}ln|cot(x)+1|-\frac{1}{2}ln|cot(x)-1|+c, \int \frac{g`(x)}{g(x)}.dx=ln|g(x)|+c$$

الإجابة الصحيحة هي (b)

$$230-\int_0^{\frac{\pi}{4}} (4x\cos(4x))\,dx \Rightarrow, \int f(x).dx=\int u.dv=u.v-\int v.du$$

$$\Rightarrow u=4x \Rightarrow du=4.dx, dv=cos(4x) \Rightarrow v=\int cos(4x).dx=\frac{1}{4}sin(4x)$$

$$\therefore \int_0^{\frac{\pi}{4}} (4x\cos(4x))\,dx=4x.\frac{1}{4}sin(4x)-\int 4\times\frac{1}{4}sin(4x).dx=x.sin(4x)\Big|_0^{\frac{\pi}{4}}+\frac{1}{4}cos(4x)\Big|_0^{\frac{\pi}{4}}$$

$$= \left(\frac{\pi}{4}\sin(\pi) + \frac{1}{4}\cos(\pi)\right) - \left(0 \times \sin(0) + \frac{1}{4}\cos(0)\right) = \left(0 - \frac{1}{4}\right) - \left(0 + \frac{1}{4}\right) = -\frac{1}{4} - \frac{1}{4} = -\frac{1}{2}$$

الإجابة الصحيحة هي (b)

$$231 - \int_0^{\ln 2} (4xe^{2x})\, dx \Rightarrow, \int f(x).dx = \int u.dv = u.v - \int v.du$$

$$\Rightarrow u = 4x \Rightarrow du = 4.dx, dv = e^{2x} \Rightarrow v = \int e^{2x}.dx = \frac{1}{2}e^{2x}$$

$$\therefore \int_0^{\ln 2} (4xe^{2x})\, dx = 4x.\frac{1}{2}e^{2x} - \int 4 \times \frac{1}{2}e^{2x}.dx = 2x.e^{2x}\Big|_0^{\ln 2} - e^{2x}\Big|_0^{\ln 2}$$

$$= (2\ln 2 . e^{2\ln 2} - e^{2\ln 2}) - (2 \times 0 \times e^{2\times 0} - e^{2\times 0}) = (2\ln 2 . e^{\ln 4} - e^{\ln 4}) - (2 \times 0 \times e^0 - e^0)$$

$$= (2\ln 2 . 4 - 4) + (1) = 8\ln 2 - 3 , e^{\ln(x)} = x, \ln(x^n) = n \ln x$$

الإجابة الصحيحة هي (a)

$$232 - \int_{\sqrt{e}}^{e} (16x^3 \ln(x))\, dx \Rightarrow, \int f(x).dx = \int u.dv = u.v - \int v.du$$

$$\Rightarrow u = \ln(x) \Rightarrow du = \frac{1}{x}.dx, dv = 16x^3 \Rightarrow v = \int 16x^3.dx = 4x^4$$

$$\therefore \int_{\sqrt{e}}^{e} (16x^3 \ln(x))\, dx = 4x^4.\ln(x) - \int \frac{1}{x} \times 4x^4 .dx = 4x^4.\ln(x)\Big|_{\sqrt{e}}^{e} - x^4\Big|_{\sqrt{e}}^{e}$$

$$= (4e^4.\ln(e) - e^4) - (4(\sqrt{e})^4.\ln(\sqrt{e}) - (\sqrt{e})^4) = (4e^4 - e^4) - (4(e^{\frac{1}{2}})^4.\ln(e^{\frac{1}{2}}) - (e^{\frac{1}{2}})^4)$$

$$= (3e^4) - (4e^2.\frac{1}{2}\ln(e) - e^2) = (3e^4) - (2e^2 - e^2) = (3e^4) - (e^2) = e^2(3e^2 - 1)$$

$$, \ln(e) = 1, \ln(x^n) = n \ln x$$

الإجابة الصحيحة هي (d)

$$233 - \int (x^3 e^{x^2})\, dx , u = x^2 \Rightarrow du = 2x.dx \Rightarrow dx = \frac{du}{2x}$$

$$\therefore \int (x^3 e^{x^2})\, dx = \int (x^3 e^u).\frac{du}{2x} = \int (x^2 e^u).du = \frac{1}{2}\int (u.e^u).du$$

$$\Rightarrow \int f(x).dx = \int w.dv = w.v - \int v.dw$$

$$\Rightarrow w = u \Rightarrow dw = du, dv = e^u \Rightarrow v = \int e^u.du = e^u$$

$$\therefore \int (x^3 e^{x^2})\, dx = \frac{1}{2}\int (u.e^u).du = \frac{1}{2}(u.e^u - \int e^u.du) = \frac{1}{2}(u.e^u - e^u) = \frac{1}{2}\left(x^2.e^{x^2} - e^{x^2}\right)$$

$$= \frac{1}{2}e^{x^2}(x^2 - 1) + c$$

الإجابة الصحيحة هي (d)

$$234 - \int_1^2 (\ln(\sqrt{xe^x}))\, dx = \int_1^2 \left(\ln\left((xe^x)^{\frac{1}{2}}\right)\right) dx = \frac{1}{2}\int_1^2 (\ln(xe^x))\, dx , \int f(x).dx = \int u.dv = u.v - \int v.du$$

$$\Rightarrow u = \ln(xe^x) \Rightarrow du = \frac{e^x + xe^x}{xe^x}.dx = (\frac{1}{x} + 1).dx, dv = dx \Rightarrow v = \int dx = x$$

$$\therefore \int_1^2 (\ln(\sqrt{xe^x}))\, dx = \frac{1}{2}(x.\ln(xe^x) - \int x\left(\frac{1}{x} + 1\right).dx) = \frac{1}{2}(x.\ln(xe^x) - \int (1 + x).dx)$$

$$= \frac{1}{2}(x.\ln(xe^x) - (\int 1.dx + \int x.dx)) = \frac{1}{2}(x.\ln(xe^x)\Big|_1^2 - (x + \frac{1}{2}x^2)\Big|_1^2)$$

$$= \frac{1}{2}((2 \times \ln(2e^2) - (2 + \frac{2^2}{2})) - (1 \times \ln(e^1) - (1 + \frac{1^2}{2})))$$

$$= \frac{1}{2}((4\,ln(2e) - (4)) - (ln(e) - (\frac{3}{2}))) = \frac{1}{2}((4(ln(2) + ln(e))) - (4)) - (ln(e) - (\frac{3}{2})))$$

$$= \frac{1}{2}((4(ln(2) + 1)) - (4)) - (-\frac{1}{2}))) = \frac{1}{2}(4\,ln(2) + 4 - 4 + \frac{1}{2}) = \frac{1}{2}(4\,ln(2) + \frac{1}{2}) = 2\,ln(2) + \frac{1}{4}$$

$, ln(e) = 1, ln(x^n) = n\,ln\,x$

الإجابة الصحيحة هي (a)

$$235 - \int (e^{\sqrt{x}})\,dx\,, u = \sqrt{x} \Rightarrow du = \frac{1}{2\sqrt{x}}.dx \Rightarrow dx = 2\sqrt{x}.du$$

$$\therefore \int (e^{\sqrt{x}})\,dx = \int (e^{\sqrt{x}}).2\sqrt{x}.du = \int (e^{u}).2u.du = 2\int ue^{u}.du$$

$$\Rightarrow \int f(x).dx = \int w.dv = w.v - \int v.dw$$

$$\Rightarrow w = u \Rightarrow dw = du, dv = e^{u} \Rightarrow v = \int e^{u}.du = e^{u}$$

$$\therefore \int (e^{\sqrt{x}})\,dx = 2\int ue^{u}.du = 2(u.e^{u} - \int e^{u}.du) = 2(u.e^{u} - e^{u}) = 2(\sqrt{x}.e^{\sqrt{x}} - e^{\sqrt{x}})$$

$$= 2e^{\sqrt{x}}(\sqrt{x} - 1) + c$$

الإجابة الصحيحة هي (b)

$$236 - \int x^2e^x\,dx \Rightarrow \int f(x).dx = \int u.dv = u.v - \int v.du$$

$$\Rightarrow u = x^2 \Rightarrow du = 2x.dx, dv = e^x \Rightarrow v = \int e^x.dx = e^x$$

$$\therefore \int x^2e^x\,dx = x^2e^x - \int e^x.2x.dx = x^2e^x - 2\int xe^x.dx\,, \int f(x).dx = \int u.dv = u.v - \int v.du$$

$$\Rightarrow u = x \Rightarrow du = dx, dv = e^x \Rightarrow v = \int e^x.dx = e^x$$

$$\therefore \int x^2e^x\,dx = x^2e^x - 2(xe^x - \int e^x.dx) = x^2e^x - 2xe^x + 2e^x = e^x(x^2 - 2x + 2) + c$$

الإجابة الصحيحة هي (a)

$$237 - \int \left(\frac{sin(x) - x\,cos(x)}{x^2}\right)dx = \int ((x^{-2}\,sin(x)) - (x^{-1}\,cos(x)).dx$$

$$\Rightarrow \int f(x).dx = \int u.dv = u.v - \int v.du$$

$$\Rightarrow \int ((x^{-2}\,sin(x)) - (x^{-1}\,cos(x)).dx = \int x^{-2}\,sin(x).dx - \int x^{-1}\,cos(x).dx$$

$$\because \int x^{-2}\,sin(x).dx \Rightarrow u = x^{-2} \Rightarrow du = -2x^{-3}.dx, dv = sin(x) \Rightarrow v = \int sin(x).dx = -cos(x)$$

$$\because \int x^{-1}\,cos(x).dx \Rightarrow u = x^{-1} \Rightarrow du = -x^{-2}.dx, dv = cos(x) \Rightarrow v = \int cos(x).dx = sin(x)$$

$$\therefore \int x^{-2}\,sin(x).dx - \int x^{-1}\,cos(x).dx =$$

$$= \left(-x^{-2}\,cos(x) - \int -cos(x).-2x^{-3}.dx\right) - \left(x^{-1}\,sin(x) - \int sin(x).-x^{-2}.dx\right)$$

$$= \left(-x^{-2}\,cos(x) - 2\int x^{-3}\,cos(x).dx\right) - \left(x^{-1}\,sin(x) + \int x^{-2}sin(x).dx\right)$$

$$= \left(-x^{-2}\,cos(x) - 2\int x^{-3}\,cos(x).dx\right) - \left(x^{-1}\,sin(x) + \left(-x^{-2}\,cos(x) - 2\int x^{-3}\,cos(x).dx\right)\right)$$

$$= -x^{-2}\,cos(x) - 2\int x^{-3}\,cos(x).dx - x^{-1}\,sin(x) + x^{-2}\,cos(x) + 2\int x^{-3}\,cos(x).dx$$

$$= -x^{-1}\sin(x) = -\frac{1}{x}\sin(x) + c$$

الإجابة الصحيحة هي (c)

$$238 - \int \left(\frac{\ln(x) - 1}{x^2}\right) dx = \int ((x^{-2}\ln(x)) - x^{-2}).dx$$

$$\Rightarrow \int f(x).dx = \int u.dv = u.v - \int v.du$$

$$\Rightarrow \int ((x^{-2}\ln(x)) - x^{-2}).dx = \int x^{-2}\ln(x).dx - \int x^{-2}.dx$$

$$\because \int x^{-2}\ln(x).dx \Rightarrow u = \ln(x) \Rightarrow du = \frac{1}{x}.dx, dv = x^{-2} \Rightarrow v = \int x^{-2}.dx = -x^{-1}$$

$$\therefore \int x^{-2}\ln(x).dx - \int x^{-2}.dx = \left(-x^{-1}\ln(x) - \int -x^{-1}.\frac{1}{x}.dx\right) - \int x^{-2}.dx$$

$$\Rightarrow \int x^{-2}\ln(x).dx - \int x^{-2}.dx = -x^{-1}\ln(x) + \int x^{-2}.dx - \int x^{-2}.dx$$

$$\Rightarrow \int x^{-2}\ln(x).dx - \int x^{-2}.dx = -x^{-1}\ln(x) = -x^{-1}\ln(x) = -\frac{1}{x}\ln(x) + c$$

الإجابة الصحيحة هي (d)

$$239 - \int (xe^{\frac{x}{2}})\,dx \Rightarrow \int f(x).dx = \int u.dv = u.v - \int v.du$$

$$\because \int xe^{\frac{x}{2}}.dx \Rightarrow u = x \Rightarrow du = dx, dv = e^{\frac{x}{2}} \Rightarrow v = \int e^{\frac{x}{2}}.dx = 2e^{\frac{x}{2}}$$

$$\therefore \int xe^{\frac{x}{2}}.dx = 2xe^{\frac{x}{2}} - \int 2e^{\frac{x}{2}}.dx = 2xe^{\frac{x}{2}} - 2\int e^{\frac{x}{2}}.dx = 2xe^{\frac{x}{2}} - 4e^{\frac{x}{2}} = 2e^{\frac{x}{2}}(x-2) + c$$

الإجابة الصحيحة هي (a)

$$240 - \int (xe^{2x})\,dx \Rightarrow \int f(x).dx = \int u.dv = u.v - \int v.du$$

$$\because \int xe^{2x}.dx \Rightarrow u = x \Rightarrow du = dx, dv = e^{2x} \Rightarrow v = \int e^{2x}.dx = \frac{1}{2}e^{2x}$$

$$\therefore \int xe^{2x}.dx = \frac{1}{2}xe^{2x} - \int \frac{1}{2}e^{2x}.dx = \frac{1}{2}xe^{2x} - \frac{1}{2}\int e^{2x}.dx = \frac{1}{2}xe^{2x} - \frac{1}{4}e^{2x} = \frac{1}{4}e^{2x}(2x-1) + c$$

الإجابة الصحيحة هي (a)

$$241 - \int_0^{\frac{\pi}{3}} (2\sin(x)\ln(\sec(x)))\,dx \Rightarrow \int f(x).dx = \int u.dv = u.v - \int v.du$$

$$\because \int_0^{\frac{\pi}{3}} (2\sin(x)\ln(\sec(x)))\,dx \Rightarrow u = \ln(\sec(x)) \Rightarrow du = \frac{\sec(x)\tan(x)}{\sec(x)}dx = \tan(x).dx$$

$$, dv = 2\sin(x) \Rightarrow v = \int 2\sin(x).dx = -2\cos(x)$$

$$\therefore \int_0^{\frac{\pi}{3}} (2\sin(x)\ln(\sec(x)))\,dx = -2\cos(x)\ln(\sec(x)) - \int -2\cos(x).\tan(x).dx$$

$$= -2\cos(x)\ln(\sec(x)) - \int -2\sin(x).dx = -2\cos(x)\ln(\sec(x))\Big|_0^{\frac{\pi}{3}} - 2\cos(x)\Big|_0^{\frac{\pi}{3}}$$

$$= (-2\cos(\frac{\pi}{3})\ln(\sec(\frac{\pi}{3})) - 2\cos(\frac{\pi}{3})) - (-2\cos(0)\ln(\sec(0)) - 2\cos(0))$$

$$= (-2 \times \frac{1}{2} ln(2) - 2 \times \frac{1}{2}) - (-2ln(1) - 2) = -ln(2) - 1 + 2 = -ln(2) + 1$$

الإجابة الصحيحة هي (a)

$$242 - \int (e^{-x} cos(2x))\, dx \Rightarrow \int f(x).dx = \int u.dv = u.v - \int v.du$$

$$\because \int (e^{-x} cos(2x))\, dx \Rightarrow u = e^{-x} \Rightarrow du = -e^{-x}dx, dv = cos(2x) \Rightarrow v = \int cos(2x).dx = \frac{1}{2} sin(2x)$$

$$\therefore \int (e^{-x} cos(2x))\, dx = \frac{1}{2} e^{-x} sin(2x) - \int -e^{-x} \frac{1}{2} sin(2x).dx$$

$$\therefore \int (e^{-x} cos(2x))\, dx = \frac{1}{2} e^{-x} sin(2x) + \int e^{-x} \frac{1}{2} sin(2x).dx$$

$$\because \int e^{-x} \frac{1}{2} sin(2x).dx \Rightarrow \int f(x).dx = \int u.dv = u.v - \int v.du$$

$$\because \int e^{-x} \frac{1}{2} sin(2x).dx \Rightarrow u = e^{-x} \Rightarrow du = -e^{-x}dx, dv = \frac{1}{2} sin(2x)$$

$$\Rightarrow v = \int \frac{1}{2} sin(2x).dx = -\frac{1}{4} cos(2x)$$

$$\therefore \int (e^{-x} cos(2x))\, dx = \frac{1}{2} e^{-x} sin(2x) + (-\frac{1}{4} e^{-x} cos(2x) - \int \frac{1}{4} e^{-x} cos(2x)\, dx)$$

$$\therefore \int (e^{-x} cos(2x))\, dx = \frac{1}{2} e^{-x} sin(2x) - \frac{1}{4} e^{-x} cos(2x) - \frac{1}{4} \int e^{-x} cos(2x)\, dx$$

$$\therefore \int (e^{-x} cos(2x))\, dx + \frac{1}{4} \int e^{-x} cos(2x)\, dx = \frac{1}{2} e^{-x} sin(2x) - \frac{1}{4} e^{-x} cos(2x)$$

$$\therefore \frac{5}{4} \int (e^{-x} cos(2x))\, dx = \frac{1}{2} e^{-x} sin(2x) - \frac{1}{4} e^{-x} cos(2x)$$

$$\therefore \int (e^{-x} cos(2x))\, dx = \frac{4}{5} (\frac{1}{2} e^{-x} sin(2x) - \frac{1}{4} e^{-x} cos(2x))$$

$$\therefore \int (e^{-x} cos(2x))\, dx = \frac{1}{5} e^{-x} (2\, sin(2x) - cos(2x))$$

الإجابة الصحيحة هي (b)

$$243 - m = \int \left(\frac{e^x}{sec(x)}\right) dx = \int (e^x cos(x))\, dx \Rightarrow \int f(x).dx = \int u.dv = u.v - \int v.du$$

$$\because \int (e^x cos(x))\, dx \Rightarrow u = e^x \Rightarrow du = e^x dx, dv = cos(x) \Rightarrow v = \int cos(x).dx = sin(x)$$

$$\therefore \int (e^x cos(x))\, dx = e^x sin(x) - \int e^x sin(x).dx$$

$$\because \int e^x sin(x).dx \Rightarrow \int f(x).dx = \int u.dv = u.v - \int v.du$$

$$\because \int e^x sin(x).dx \Rightarrow u = e^x \Rightarrow du = e^x dx, dv = sin(x) \Rightarrow v = \int sin(x).dx = -cos(x)$$

$$\therefore \int (e^x cos(x))\, dx = e^x sin(x) - (-e^x cos(x) - \int -e^x cos(x)\, dx)$$

$$\therefore \int (e^x cos(x))\, dx = e^x sin(x) + e^x cos(x) - \int e^x cos(x)\, dx)$$

$$\therefore 2 \int (e^x cos(x))\, dx = e^x sin(x) + e^x cos(x)$$

$$\therefore \int (e^x cos(x))\, dx = \frac{1}{2} e^x sin(x) + \frac{1}{2} e^x cos(x) + c$$

$$\because n = \int (\frac{e^x}{csc(x)})\, dx = \int e^x sin(x).dx \Rightarrow \int f(x).dx = \int u.dv = u.v - \int v.du$$

$$\because \int e^x sin(x).dx \Rightarrow u = e^x \Rightarrow du = e^x dx, dv = sin(x) \Rightarrow v = \int sin(x).dx = -cos(x)$$

$$\therefore \int e^x \sin(x)\,.dx = -e^x\cos(x) - \int -e^x \cos(x)\,dx = -e^x\cos(x) + \int e^x \cos(x)\,dx$$

$$\because \int (e^x \cos(x))\,dx \Rightarrow \int f(x).dx = \int u.dv = u.v - \int v.du$$

$$\therefore \int (e^x \cos(x))\,dx \Rightarrow u = e^x \Rightarrow du = e^x dx, dv = \cos(x) \Rightarrow v = \int \cos(x)\,.dx = \sin(x)$$

$$\therefore \int e^x \sin(x)\,.dx = -e^x\cos(x) + (e^x \sin(x) - \int e^x \sin(x)\,.dx$$

$$\therefore 2\int e^x \sin(x)\,.dx = -e^x\cos(x) + e^x \sin(x)$$

$$\therefore \int e^x \sin(x)\,.dx = \frac{1}{2}e^x\sin(x) - \frac{1}{2}e^x\cos(x) + c$$

$$\therefore (m-n) = \frac{1}{2}e^x\sin(x) + \frac{1}{2}e^x\cos(x) + c - (\frac{1}{2}e^x\sin(x) - \frac{1}{2}e^x\cos(x) + c)$$

$$\therefore (m-n) = \frac{1}{2}e^x\sin(x) + \frac{1}{2}e^x\cos(x) + c - \frac{1}{2}e^x\sin(x) + \frac{1}{2}e^x\cos(x) - c = e^x\cos(x)$$

الإجابة الصحيحة هي (d)

$$244 - m = \int_0^\pi (e^x \cos(x))\,dx \Rightarrow \int f(x).dx = \int u.dv = u.v - \int v.du$$

$$\because \int_0^\pi (e^x \cos(x))\,dx \Rightarrow u = e^x \Rightarrow du = e^x dx, dv = \cos(x) \Rightarrow v = \int \cos(x)\,.dx = \sin(x)$$

$$\therefore \int_0^\pi (e^x \cos(x))\,dx = e^x\sin(x)\Big|_0^\pi - \int_0^\pi e^x \sin(x)\,.dx$$

$$\because \int_0^\pi e^x \sin(x)\,.dx \Rightarrow \int f(x).dx = \int u.dv = u.v - \int v.du$$

$$\because \int_0^\pi e^x \sin(x)\,.dx \Rightarrow u = e^x \Rightarrow du = e^x dx, dv = \sin(x) \Rightarrow v = \int \sin(x)\,.dx = -\cos(x)$$

$$\therefore \int_0^\pi e^x \sin(x)\,.dx = e^x\sin(x)\Big|_0^\pi - (-e^x\cos(x)\Big|_0^\pi - \int_0^\pi -(e^x \cos(x))\,dx)$$

$$\therefore \int_0^\pi e^x \sin(x)\,.dx = e^x\sin(x)\Big|_0^\pi + e^x\cos(x)\Big|_0^\pi - \int_0^\pi (e^x \cos(x))\,dx)$$

$$\therefore 2\int_0^\pi (e^x \cos(x))\,dx = e^x\sin(x)\Big|_0^\pi + e^x\cos(x)\Big|_0^\pi$$

$$\therefore \int_0^\pi (e^x \cos(x))\,dx = \frac{1}{2}e^x\sin(x)\Big|_0^\pi + \frac{1}{2}e^x\cos(x)\Big|_0^\pi$$

$$= \frac{1}{2}e^\pi\sin(\pi) + \frac{1}{2}e^\pi\cos(\pi) - (\frac{1}{2}e^0\sin(0) + \frac{1}{2}e^0\cos(0))$$

$$= 0 - \frac{1}{2}e^\pi - 0 - \frac{1}{2} = -\frac{1}{2}e^\pi - \frac{1}{2} = \frac{-e^\pi - 1}{2}$$

$$\because n = \int_0^\pi (e^x \sin(x))\,dx \Rightarrow \int f(x).dx = \int u.dv = u.v - \int v.du$$

$$\because \int_0^\pi (e^x \sin(x))\,dx \Rightarrow u = e^x \Rightarrow du = e^x dx, dv = \sin(x) \Rightarrow v = \int \sin(x)\,.dx = -\cos(x)$$

$$\therefore \int_0^{\pi} (e^x \sin(x))\, dx = -e^x \cos(x)\Big|_0^{\pi} - \int -e^x \cos(x)\, dx = -e^x \cos(x)\Big|_0^{\pi} + \int_0^{\pi} (e^x \cos(x))\, dx$$

$$\because \int_0^{\pi} (e^x \cos(x))\, dx \Rightarrow \int f(x).dx = \int u.dv = u.v - \int v.du$$

$$\therefore \int_0^{\pi} (e^x \cos(x))\, dx \Rightarrow u = e^x \Rightarrow du = e^x dx, dv = \cos(x) \Rightarrow v = \int \cos(x).dx = \sin(x)$$

$$\therefore \int_0^{\pi} (e^x \sin(x))\, dx = -e^x \cos(x)\Big|_0^{\pi} + (e^x \sin(x)\Big|_0^{\pi} - \int_0^{\pi} (e^x \sin(x))\, dx)$$

$$\therefore 2\int_0^{\pi} (e^x \sin(x))\, dx = -e^x \cos(x)\Big|_0^{\pi} + e^x \sin(x)\Big|_0^{\pi}$$

$$\therefore \int_0^{\pi} (e^x \sin(x))\, dx = \frac{1}{2}e^x \sin(x)\Big|_0^{\pi} - \frac{1}{2}e^x \cos(x)\Big|_0^{\pi}$$

$$= \frac{1}{2}e^{\pi}\sin(\pi) - \frac{1}{2}e^{\pi}\cos(\pi) - (\frac{1}{2}e^0 \sin(0) - \frac{1}{2}e^0 \cos(0))$$

$$= 0 + \frac{1}{2}e^{\pi} - 0 + \frac{1}{2} = \frac{1}{2}e^{\pi} + \frac{1}{2} = \frac{e^{\pi}+1}{2}$$

$$\therefore (m, n) = (-\frac{1+e^{\pi}}{2}, \frac{1+e^{\pi}}{2})$$

الإجابة الصحيحة هي (a)

$$245 - m = \int_0^{\pi} (x \sin(x))\, dx \Rightarrow \int f(x).dx = \int u.dv = u.v - \int v.du$$

$$\because \int_0^{\pi} (x \sin(x))\, dx \Rightarrow u = x \Rightarrow du = dx, dv = \sin(x) \Rightarrow v = \int \sin(x).dx = -\cos(x)$$

$$\therefore \int_0^{\pi} (x \sin(x))\, dx = -x\cos(x)\Big|_0^{\pi} - \int_0^{\pi} (-\cos(x))\, dx = -x\cos(x)\Big|_0^{\pi} + \int_0^{\pi} (\cos(x))\, dx$$

$$\therefore \int_0^{\pi} (x \sin(x))\, dx = -x\cos(x)\Big|_0^{\pi} + \sin(x)\Big|_0^{\pi} =$$

$$= -\pi\cos(\pi) + \sin(\pi) - (-0 \times \cos(0) + \sin(0)) = \pi + 0 - 0 + 0 = \pi$$

$$\because n = \int_0^{\pi} (x \cos(x))\, dx \Rightarrow \int f(x).dx = \int u.dv = u.v - \int v.du$$

$$\therefore \int_0^{\pi} (x \cos(x))\, dx \Rightarrow u = x \Rightarrow du = dx, dv = \cos(x) \Rightarrow v = \int \cos(x).dx = \sin(x)$$

$$\therefore \int_0^{\pi} (x \cos(x))\, dx = -x\sin(x)\Big|_0^{\pi} - \int_0^{\pi} (\sin(x))\, dx) = -x\sin(x)\Big|_0^{\pi} + \cos(x)\Big|_0^{\pi}$$

$$\therefore \int_0^{\pi} (x \cos(x))\, dx = -\pi \sin(\pi) + \cos(\pi) - (-0 \times \sin(0) + \cos(0))$$

$$\therefore \int_0^{\pi} (x \cos(x))\, dx = 0 - 1 + 0 - 1 = -2$$

$$\therefore m - n = \pi - (-2) = \pi + 2$$

الإجابة الصحيحة هي (c)

$$246-\int_0^3 (\ln(2x+3))\,dx \Rightarrow \int f(x).dx = \int u.dv = u.v - \int v.du$$

$$\because \int_0^3 (\ln(2x+3))\,dx \Rightarrow u = \ln(2x+3) \Rightarrow du = \frac{2dx}{2x+3}, dv = dx \Rightarrow v = \int dx = x$$

$$\therefore \int_0^3 (\ln(2x+3))\,dx = x\ln(2x+3)\Big|_0^3 - \int_0^3 x.\frac{2dx}{2x+3} = x\ln(2x+3)\Big|_0^3 - \int_0^3 \frac{2xdx}{2x+3}$$

$$\because \int_0^3 \frac{2xdx}{2x+3}, u = 2x+3 \Rightarrow du = 2.dx \Rightarrow dx = \frac{du}{2}, u = 2x+3 \Rightarrow x = \frac{u-3}{2}$$

$$\because x = 0 \Rightarrow u = 3, \because x = 3 \Rightarrow u = 9$$

$$\therefore \int_3^9 \frac{2xdx}{2x+3} = \int_3^9 \frac{2x}{2x+3}.\frac{du}{2} = \int_3^9 \frac{\frac{u-3}{2}}{u}.du = \int_3^9 \frac{u-3}{2u}.du = \int_3^9 \frac{1}{2}.du - \int_3^9 \frac{3}{2u}.du$$

$$= \int_3^9 \frac{1}{2}.du - \frac{3}{2}\int_3^9 \frac{1}{u}.du = \frac{1}{2}u\Big|_3^9 - \frac{3}{2}\ln(u)\Big|_3^9 = \frac{1}{2}(9) - \frac{3}{2}\ln(9) - (\frac{1}{2}(3) - \frac{3}{2}\ln(3))$$

$$= \frac{9}{2} - \frac{3}{2}\ln(9) - \frac{3}{2} + \frac{3}{2}\ln(3)$$

حل آخر

$$\because \int_0^3 \frac{2xdx}{2x+3}, \int_0^3 \frac{2xdx}{2x+3} = \int_0^3 dx - \int_0^3 \frac{3dx}{2x+3}$$ ، بالقسمة الطويلة

$$\therefore \int_0^3 \frac{2xdx}{2x+3} = \int_0^3 dx - \int_0^3 \frac{3dx}{2x+3} = \int_0^3 dx - \frac{3}{2}\int_0^3 \frac{2dx}{2x+3} = x\Big|_0^3 - \frac{3}{2}\ln(2x+3)\Big|_0^3$$

$$= 3 - \frac{3}{2}\ln(9) - 0 + \frac{3}{2}\ln(3) = 3 - 3\ln(3) + \frac{3}{2}\ln(3) = 3 - \frac{3}{2}\ln(3)$$

$$\therefore \int_0^3 (\ln(2x+3))\,dx = x\ln(2x+3)\Big|_0^3 - \int_0^3 x.\frac{2dx}{2x+3} = x\ln(2x+3)\Big|_0^3 - (\frac{9}{2} - \frac{3}{2}\ln(9) - \frac{3}{2} + \frac{3}{2}\ln(3))$$

$$\therefore \int_0^3 (\ln(2x+3))\,dx = 3\ln(9) - \left(\frac{9}{2} - \frac{3}{2}\ln(9) - \frac{3}{2} + \frac{3}{2}\ln(3)\right) = 3\ln(9) - \frac{9}{2} + \frac{3}{2}\ln(9) + \frac{3}{2} - \frac{3}{2}\ln(3))$$

$$\therefore \int_0^3 (\ln(2x+3))\,dx = \frac{9}{2}\ln(9) - \frac{6}{2} - \frac{3}{2}\ln(3) = 9\ln(3) - 3 - \frac{3}{2}\ln(3) = \frac{15}{2}\ln(3) - 3$$

$$, \int \frac{g`(x)}{g(x)}.dx = \ln|g(x)| + c, \ln(x^n) = n\ln x$$

الإجابة الصحيحة هي (c)

$$247-\int_0^2 (xe^{-x})\,dx \Rightarrow \int f(x).dx = \int u.dv = u.v - \int v.du$$

$$\because \int_0^2 (xe^{-x})\,dx \Rightarrow u = x \Rightarrow du = dx, dv = e^{-x} \Rightarrow v = \int e^{-x}.dx = -e^{-x}$$

$$\therefore \int_0^2 (xe^{-x})\,dx = -xe^{-x}\Big|_0^2 - \int_0^2 -e^{-x}.dx = -xe^{-x}\Big|_0^2 - e^{-x}\Big|_0^2$$

$$\therefore \int_0^2 (xe^{-x})\,dx = -2e^{-2} - e^{-2} - (-0 \times e^0 - e^0) = -3e^{-2} + 1$$

الإجابة الصحيحة هي (d)

$$248 - \int (x\,sec^2(x))\,dx \Rightarrow \int f(x).dx = \int u.dv = u.v - \int v.du$$

$$\because \int (x\,sec^2(x))\,dx \Rightarrow u = x \Rightarrow du = dx, dv = sec^2(x) \Rightarrow v = \int sec^2(x).dx = tan(x)$$

$$\therefore \int (x\,sec^2(x))\,dx = x\,tan(x) - \int tan(x).dx = x\,tan(x) - \int \frac{sin(x)}{cos(x)}.dx$$

$$\therefore \int (x\,sec^2(x))\,dx = x\,tan(x) + \int \frac{-sin(x)}{cos(x)}.dx = x\,tan(x) + ln|cos(x)| + c$$

$$, \int \frac{g`(x)}{g(x)}.dx = ln|g(x)| + c$$

الإجابة الصحيحة هي (b)

$$249 - \int (3x\,ln(x))\,dx \Rightarrow \int f(x).dx = \int u.dv = u.v - \int v.du$$

$$\because \int (3x\,ln(x))\,dx \Rightarrow u = ln(x) \Rightarrow du = \frac{1}{x}dx, dv = 3x \Rightarrow v = \int 3x.dx = \frac{3}{2}x^2$$

$$\therefore \int (3x\,ln(x))\,dx = \frac{3}{2}x^2\,ln(x) - \int \frac{3}{2}x^2.\frac{1}{x}dx = \frac{3}{2}x^2\,ln(x) - \frac{3}{2}\int x.dx = \frac{3}{2}x^2\,ln(x) - \frac{3}{4}x^2$$

$$\therefore \int (3x\,ln(x))\,dx = \frac{3}{4}x^2(2ln(x) - 1) + c$$

الإجابة الصحيحة هي (a)

$$250 - \int \left(e^{cos(x)}\,sin(2x)\right)dx = \int \left(2e^{cos(x)}\,sin(x)\,cos(x)\right)dx, u = cos(x)$$

$$\Rightarrow du = -\,sin(x)dx \Rightarrow dx = \frac{du}{-\,sin(x)}$$

$$\therefore \int \left(2e^{cos(x)}\,sin(x)\,cos(x)\right)dx = \int \left(2e^{cos(x)}\,sin(x)\,cos(x)\right).\frac{du}{-\,sin(x)} = \int (-2ue^u).du$$

$$\therefore \int (-2ue^u).du \Rightarrow \int f(x).dx = \int u.dv = u.v - \int v.du$$

$$\because \int (-2ue^u).du \Rightarrow w = -\,2u \Rightarrow dw = -2du, dv = e^u \Rightarrow v = \int e^u.du = e^u$$

$$\therefore \int (-2ue^u).du = -2ue^u - \int -2e^u du = -2ue^u + 2e^u = -2\,cos(x)\,e^{cos(x)} + 2e^{cos(x)} + c$$

الإجابة الصحيحة هي (c)

$$251 - \int ((ln(x))^2)\,dx,\ u = ln(x) \Rightarrow du = \frac{1}{x}dx \Rightarrow dx = xdu, u = ln(x) \Rightarrow x = e^u$$

$$\therefore \int ((ln(x))^2)\,dx = \int ((ln(x))^2)\,xdu = \int (u^2)\,xdu = \int (u^2)e^u\,du$$

$$\therefore \int (u^2)e^u\,du \Rightarrow \int f(x).dx = \int u.dv = u.v - \int v.du$$

$$\because \int (u^2)e^u\,du \Rightarrow w = u^2 \Rightarrow dw = 2u.du, dv = e^u \Rightarrow v = \int e^u.du = e^u$$

$$\therefore \int (u^2)e^u\,du = u^2e^u - \int 2ue^u\,du = u^2e^u - 2\int ue^u\,du, \Rightarrow \int f(x).dx = \int u.dv = u.v - \int v.du$$

$$\because \int ue^u\,du \Rightarrow z = u \Rightarrow dz = du, dv = e^u \Rightarrow v = \int e^u.du = e^u$$

$$\therefore \int ((ln(x))^2)\,dx = u^2e^u - 2\int ue^u\,du = u^2e^u - 2(\,ue^u - \int e^u\,du) = u^2e^u - 2ue^u + 2e^u$$

$\therefore \int ((ln(x))^2)\,dx = ln(x)^2\, e^{ln(x)} - 2\,ln(x)\, e^{ln(x)} + 2e^{ln(x)}, e^{ln(x)} = x, ln(x) = y \Rightarrow x = e^y$

$\therefore \int ((ln(x))^2)\,dx = ln(x)^2\, x - 2\,ln(x)\, x + 2x = x(ln(x))^2 - 2x\,ln(x) + 2x + c$

الإجابة الصحيحة هي (a)

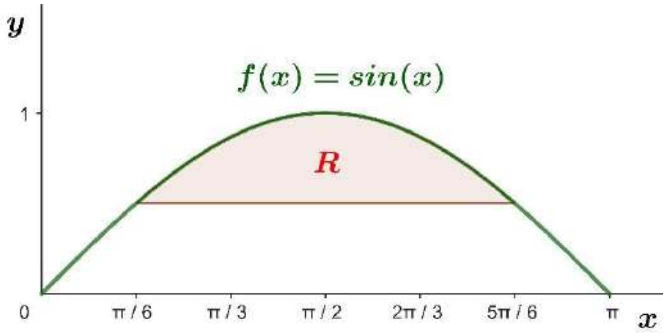
$252 - \because f(x) = sin(x) \Rightarrow f\left(\frac{\pi}{6}\right) = sin\left(\frac{\pi}{6}\right) = \frac{1}{2}$

$\therefore R = \int_{\frac{\pi}{6}}^{\frac{5\pi}{6}} \left(f(x) - \frac{1}{2}\right).dx = \int_{\frac{\pi}{6}}^{\frac{5\pi}{6}} \left(sin(x) - \frac{1}{2}\right).dx = \int_{\frac{\pi}{6}}^{\frac{5\pi}{6}} sin(x)\,.dx - \int_{\frac{\pi}{6}}^{\frac{5\pi}{6}} \frac{1}{2}.dx = \left(-cos(x) - \frac{1}{2}x\right)\Big|_{\frac{\pi}{6}}^{\frac{5\pi}{6}}$

$\therefore R = \left(-cos\left(\frac{5\pi}{6}\right) - \frac{1}{2} \times \frac{5\pi}{6}\right) - \left(-cos\left(\frac{\pi}{6}\right) - \frac{1}{2} \times \frac{\pi}{6}\right)$

$\therefore R = \frac{\sqrt{3}}{2} - \frac{5\pi}{12} + \frac{\sqrt{3}}{2} + \frac{\pi}{12} = \frac{2\sqrt{3}}{2} - \frac{4\pi}{12} = \sqrt{3} - \frac{4\pi}{12}$

$\therefore R = \sqrt{3} - \frac{\pi}{3} u^2$



الإجابة الصحيحة هي (a)

$253 - \because y = 2, f(x) = sec^2(x), g(x) = 1 - x^2$

$\therefore R = \int_{\frac{\pi}{4}}^{0} (2 - f(x)).dx + \int_{0}^{1} (2 - g(x)).dx$

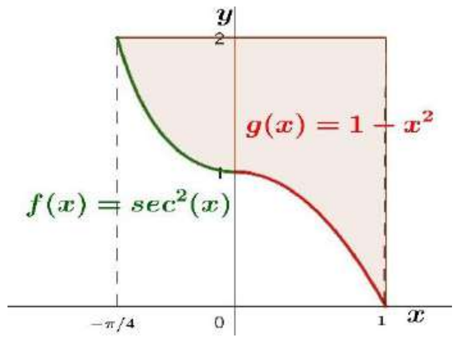
$\therefore R = \left(\int_{\frac{\pi}{4}}^{0} 2.dx - \int_{\frac{\pi}{4}}^{0} f(x).dx\right) + \left(\int_{0}^{1} 2.dx - \int_{0}^{1} g(x).dx\right)$

$\therefore R = (2x - tan(x))\Big|_{-\frac{\pi}{4}}^{0} + \left(2x - \left(x - \frac{x^3}{3}\right)\right)\Big|_{0}^{1}$

$\therefore R = ((2 \times 0 - tan(0)) - (-\frac{\pi}{2} - (tan(-\frac{\pi}{4})) + ((2 \times 1 - (1 - \frac{1}{3}) - (2 \times 0 - (0 - \frac{0}{3}))$

$\therefore R = ((0 - 0) - (-\frac{\pi}{2} + (tan(\frac{\pi}{4})) + ((2 - (\frac{2}{3}) - (0 - 0)$

$\therefore R = \left(\frac{\pi}{2} - 1\right) + \left(\frac{4}{3}\right) = \frac{\pi}{2} + \frac{1}{3} u^2, tan(-x) = -tan(x)$



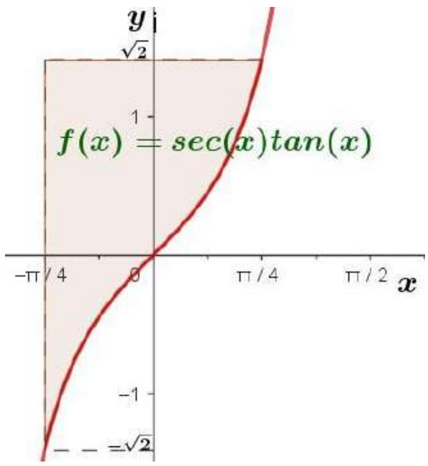
الإجابة الصحيحة هي (b)

$254 - \because f(x) = sec(x)\,tan(x)\,, y = \sqrt{2}$

$\therefore R = \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \left(\sqrt{2} - f(x)\right).dx = \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} (\sqrt{2} - sec(x)\,tan(x)).dx$

$\therefore R = \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \sqrt{2}.dx - \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} sec(x)\,tan(x)\,.dx$

$\therefore R = \sqrt{2}x\Big|_{-\frac{\pi}{4}}^{\frac{\pi}{4}} - sec(x)\Big|_{-\frac{\pi}{4}}^{\frac{\pi}{4}}$

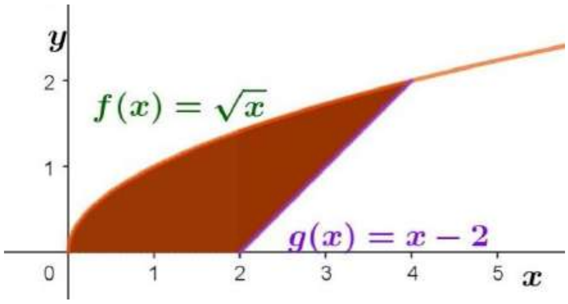


$\therefore R = (\sqrt{2} \times \frac{\pi}{4} - \sqrt{2} \times -\frac{\pi}{4}) - (sec(\frac{\pi}{4}) - sec(-\frac{\pi}{4})) = (\frac{\sqrt{2}\pi}{4} + \frac{\sqrt{2}\pi}{4}) - (\sqrt{2} - \sqrt{2})$

$\therefore R = \frac{2\sqrt{2}\pi}{4} = \frac{\sqrt{2}\pi}{2} = \frac{\pi}{\sqrt{2}} u^2, cos(-x) = cos(x), \frac{1}{cos(x)} = sec(x)$

الإجابة الصحيحة هي (a)

$255 - \because f(x) = \sqrt{x}, g(x) = x - 2$



$\therefore R = \int_0^2 (f(x)).dx + \int_2^4 (f(x) - g(x)).dx$

$\therefore R = \int_0^2 \sqrt{x}.dx + \int_2^4 (\sqrt{x} - (x-2)).dx$

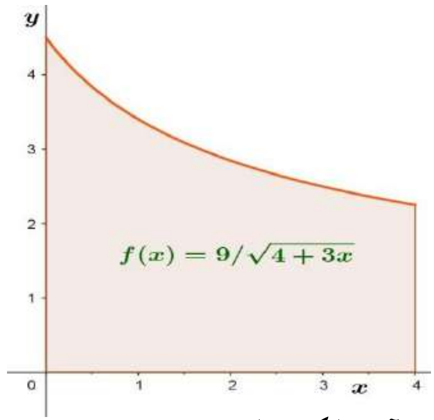
$\therefore R = \frac{2}{3}\sqrt{x^3}\Big|_0^2 + (\frac{2}{3}\sqrt{x^3} - \frac{x^2}{2} + 2x)\Big|_2^4$

$\therefore R = (\frac{2}{3} \times \sqrt{8} - \frac{2}{3} \times \sqrt{0}) + (\frac{2}{3}\sqrt{64} - \frac{16}{2} + 2 \times 4) - (\frac{2}{3}\sqrt{8} - \frac{4}{2} + 2 \times 2))$

$\therefore R = (\frac{2\sqrt{8}}{3}) + (\frac{16}{3} - 8 + 8) - (\frac{2\sqrt{8}}{3} - 2 + 4)) = \frac{2\sqrt{8}}{3} + \frac{16}{3} - \frac{2\sqrt{8}}{3} - 2 = \frac{16}{3} - \frac{6}{3} = \frac{10}{3} u^2$

الإجابة الصحيحة هي (c)

$256 - \because f(x) = \frac{9}{\sqrt{4+3x}} = 9(4+3x)^{-\frac{1}{2}}$



$\therefore R = \int_0^4 (f(x)).dx = \int_0^4 \left(\frac{9}{\sqrt{4+3x}}\right).dx$

$\therefore R \int_0^4 \left(\frac{9}{\sqrt{4+3x}}\right).dx = \int_0^4 9(4+3x)^{-\frac{1}{2}}.dx = \frac{9(4+3x)^{\frac{1}{2}}}{3 \times \frac{1}{2}}\Big|_0^4$

$= 6(4+3x)^{\frac{1}{2}}\Big|_0^4 = 6\sqrt{4+3x}\Big|_0^4 = 24 - 12 = 12 = 12u^2$

حل آخر بالتكامل بالتعويض

$\because f(x) = \frac{9}{\sqrt{4+3x}}, u = 4 + 3x \Rightarrow du = 3.dx \Rightarrow dx = \frac{du}{3}$

$\because x = 0 \Rightarrow u = 4, x = 4 \Rightarrow u = 16$

$\therefore R \int_0^4 \left(\frac{9}{\sqrt{4+3x}}\right).dx = \int_0^4 \left(\frac{9}{\sqrt{u}}\right).\frac{du}{3} = \int_0^4 3u^{-\frac{1}{2}}.du = \frac{3u^{\frac{1}{2}}}{\frac{1}{2}} = 6\sqrt{u}\Big|_4^{16} = 6\sqrt{16} - 6\sqrt{4} = 24 - 12$

$= 12u^2$

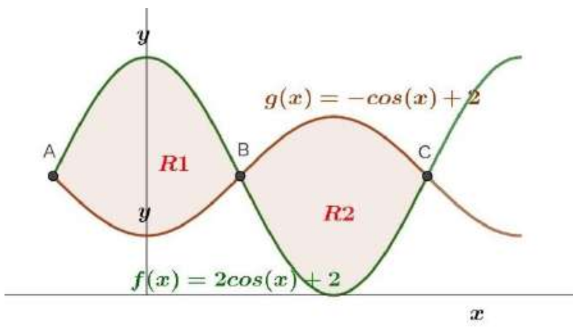
الإجابة الصحيحة هي (a)

$257 - \because f(x) = 2\cos(x) + 2, g(x) = -\cos(x) + 2$

$\therefore f(x) = g(x) \Rightarrow 2\cos(x) + 2 = -\cos(x) + 2$

$\Rightarrow 3\cos(x) + 2 = 0 \Rightarrow \cos(x) = 0 \Rightarrow x^+ = \frac{\pi}{2}, \frac{3\pi}{2}$

$x^- = -\frac{\pi}{2}, -\frac{3\pi}{2} \Rightarrow A = -\frac{\pi}{2}, B = \frac{\pi}{2}, C = \frac{3\pi}{2}$



$\therefore R = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} (f(x) - g(x)).dx + \int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} (g(x) - f(x)).dx$

$$\therefore R = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} (2\cos(x) + 2 - (-\cos(x) + 2)).dx + \int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} (-\cos(x) + 2 - (2\cos(x) + 2)).dx$$

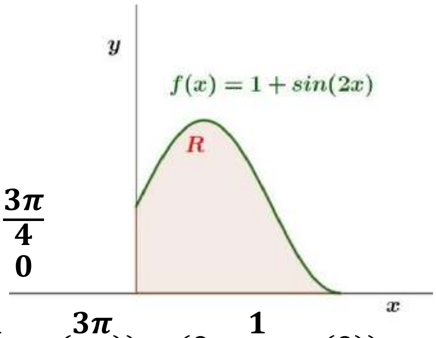
$$\therefore R = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} (2\cos(x) + 2 + \cos(x) - 2).dx + \int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} (-\cos(x) + 2 - 2\cos(x) - 2)).dx$$

$$\therefore R = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} 3\cos(x).dx + \int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} -3\cos(x).dx = 3\sin(x)\Big|_{-\frac{\pi}{2}}^{\frac{\pi}{2}} - 3\sin(x)\Big|_{\frac{\pi}{2}}^{\frac{3\pi}{2}}$$

$$\therefore R = (3\sin\left(\frac{\pi}{2}\right) - 3\sin\left(-\frac{\pi}{2}\right)) - (3\sin\left(\frac{3\pi}{2}\right) - 3\sin\left(\frac{\pi}{2}\right)) = 3 + 3 + 3 + 3 = 12\,u^2$$

$, \sin(-x) = \sin(x)$

الإجابة الصحيحة هي (a)



$258 - \because f(x) = 1 + \sin(2x)$

$\therefore f(x) = 0 \Longrightarrow 1 + \sin(2x) = 0$

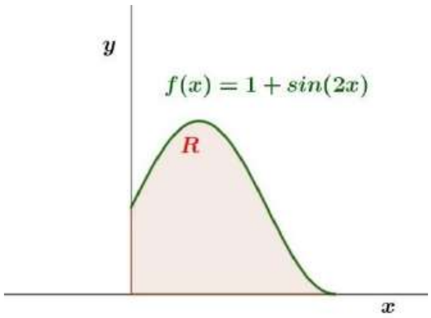
$\Longrightarrow \sin(2x) = -1 \Longrightarrow 2x = \frac{3\pi}{2} \Longrightarrow 4x = 3\pi \Longrightarrow x = \frac{3\pi}{4}$

$$\therefore R = \int_{0}^{\frac{3\pi}{4}} f(x).dx = \int_{0}^{\frac{3\pi}{4}} (1 + \sin(2x)).dx = (x - \frac{1}{2}\cos(2x))\Big|_{0}^{\frac{3\pi}{4}}$$

$$\therefore R = (\frac{3\pi}{4} - \frac{1}{2}\cos(2 \times \frac{3\pi}{4})) - (0 - \frac{1}{2}\cos(2 \times 0)) = (\frac{3\pi}{4} - \frac{1}{2}\cos(\frac{3\pi}{2})) - (0 - \frac{1}{2}\cos(0))$$

$$\therefore R = \frac{3\pi}{4} - 0 + \frac{1}{2} = \frac{3\pi}{4} + \frac{1}{2}u^2$$

الإجابة الصحيحة هي (a)



$259 - \because f(x) = 1 + \sin(2x)$

$\therefore f(x) = 0 \Longrightarrow 1 + \sin(2x) = 0$

$\Longrightarrow \sin(2x) = -1 \Longrightarrow 2x = \frac{3\pi}{2} \Longrightarrow 4x = 3\pi \Longrightarrow x = \frac{3\pi}{4}$

$$\therefore v = \int_{0}^{\frac{3\pi}{4}} \pi\left(f(x)\right)^2.dx = \int_{0}^{\frac{3\pi}{4}} \pi(1 + \sin(2x))^2.dx$$

$$\therefore v = \pi\int_{0}^{\frac{3\pi}{4}} (1 + (\sin(2x))^2 + 2\sin(2x)).dx = \pi\int_{0}^{\frac{3\pi}{4}} (1 + (\frac{1}{2} - \frac{1}{2}\cos(4x)) + 2\sin(2x)).dx$$

$$\therefore v = \pi\int_{0}^{\frac{3\pi}{4}} \frac{3}{2}.dx - \frac{\pi}{2}\int_{0}^{\frac{3\pi}{4}} \cos(4x).dx + 2\pi\int_{0}^{\frac{3\pi}{4}} \sin(2x).dx$$

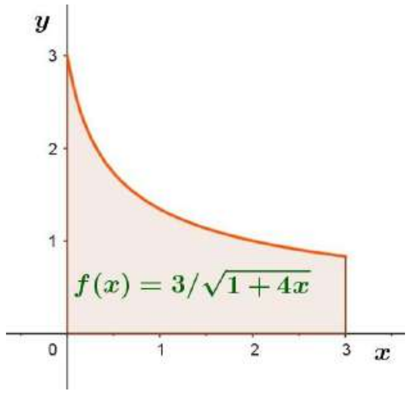
$$\therefore v = \frac{3\pi}{2}x\Big|_{0}^{\frac{3\pi}{4}} - \frac{\pi}{8}\sin(4x)\Big|_{0}^{\frac{3\pi}{4}} - \pi\cos(4x)\Big|_{0}^{\frac{3\pi}{4}}$$

$$\therefore v = (\frac{3\pi}{2} \times \frac{3\pi}{4}) - (\frac{\pi}{8}\sin(4 \times \frac{3\pi}{4}) - \frac{\pi}{8}\sin(0)) - (\pi\cos(4 \times \frac{3\pi}{4}) - \pi\cos(0))$$

$$\therefore v = \left(\frac{9\pi^2}{8}\right) - \left(\frac{\pi}{8}sin(3\pi) - \frac{\pi}{8}sin(0)\right) - (\pi\, cos(3\pi) - \pi\, cos(0))$$

$$\therefore v = \left(\frac{9\pi^2}{8}\right) - (0) - (-\pi - \pi) = \frac{9\pi^2}{8} + 2\pi u^3$$

الإجابة الصحيحة هي (b)



$$260 - \because f(x) = \frac{3}{\sqrt{1+4x}} = 3(1+4x)^{-\frac{1}{2}}$$

$$\therefore R = \int_0^3 (f(x)).dx = \int_0^3 \left(\frac{3}{\sqrt{1+4x}}\right).dx$$

$$\therefore R \int_0^3 \left(\frac{3}{\sqrt{1+4x}}\right).dx = \int_0^3 3(1+4x)^{-\frac{1}{2}}.dx = \frac{3(1+4x)^{\frac{1}{2}}}{4 \times \frac{1}{2}}\Big|_0^3$$

$$= \frac{3}{2}(1+4x)^{\frac{1}{2}}\Big|_0^3 = \frac{3}{2}\sqrt{1+4x}\,\Big|_0^3 = \frac{3}{2}\sqrt{13} - \frac{3}{2}u^2$$

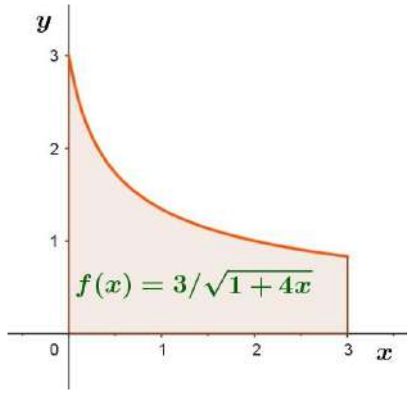
حل آخر بالتكامل بالتعويض

$$\because f(x) = \frac{3}{\sqrt{1+4x}}, u = 1+4x \Rightarrow du = 4.dx \Rightarrow dx = \frac{du}{4}$$

$$\because x = 0 \Rightarrow u = 1, x = 3 \Rightarrow u = 13$$

$$\therefore R \int_0^3 \left(\frac{3}{\sqrt{1+4x}}\right).dx = \int_1^{13} \left(\frac{3}{\sqrt{u}}\right).\frac{du}{4} = \int_1^{13} \frac{3}{4}u^{-\frac{1}{2}}.du = \frac{3u^{\frac{1}{2}}}{2} = \frac{3}{2}\sqrt{u}\,\Big|_1^{13} = \frac{3}{2}\sqrt{13} - \frac{3}{2}u^2$$

الإجابة الصحيحة هي (d)



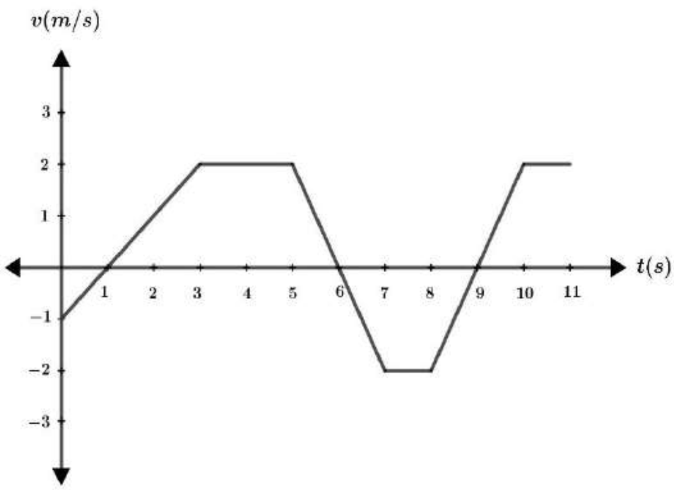
$$261 - \because f(x) = \frac{3}{\sqrt{1+4x}}$$

$$\therefore v = \int_0^3 \pi(f(x))^2.dx = \int_0^3 \pi\left(\frac{3}{\sqrt{1+4x}}\right)^2.dx$$

$$\therefore v = \pi\int_0^3 \frac{9}{1+4x}.dx = \frac{9\pi}{4}\int_0^3 \frac{4}{1+4x}.dx = \frac{9\pi}{4}ln|1+4x|\,\Big|_0^3$$

$$\therefore v = \frac{9\pi}{4}ln|13| - \frac{9\pi}{4}ln|1| = \frac{9\pi}{4}ln|13| = \frac{9\pi}{4}ln\,13\,u^3$$

الإجابة الصحيحة هي (c)



$$262 - \because s = s(t_2) - s(t_1) = \int_{t_1}^{t_2} v(t).dt$$

$$\therefore A_1 = \int_0^1 v(t).dt = \frac{1}{2} \times 1 \times 1 = \frac{1}{2}m^2$$

$$\therefore A_2 = \int_1^6 v(t).dt = \frac{1}{2} \times (2+5) \times 2 = 7m^2$$

$$\therefore A_3 = \int_6^9 v(t).dt = \frac{1}{2} \times (3+1) \times 2 = 4m^2$$

$$\therefore A_4 = \int_9^{11} v(t).dt = \frac{1}{2} \times (2+1) \times 2 = 3m^2$$

$$\therefore s = s(11) - s(0) = \int_0^{11} v(t).dt = \int_0^{1} v(t).dt + \int_1^{6} v(t).dt + \int_6^{9} v(t).dt + \int_9^{11} v(t).dt$$

$= -\frac{1}{2} + 7 - 4 + 3 = 5.5m = 5.5m$ نحو اليمين

الإجابة الصحيحة هي (a)

$$263 - \because d = \int_{t_1}^{t_2} |v(t)|.dt = \sum A \Rightarrow d = \int_0^{11} |v(t)|.dt = A_1 + A_2 + A_3 + A_4 = \frac{1}{2} + 7 + 4 + 3 = 14.5m$$

الإجابة الصحيحة هي (c)

$$264 - \because s(t_2) - s(t_1) = \int_{t_1}^{t_2} v(t).dt \Rightarrow s(11) - s(0) = \int_0^{11} v(t).dt \Rightarrow s(11) - 9 = 5.5$$

$$\Rightarrow s(11) = 14.5m$$

الإجابة الصحيحة هي (d)

$$265 - \because \frac{dv}{dt} = -\frac{v^2}{100}, t \geq 0, v_0 = 20m/s, t_2 = 20s$$

$$\therefore \frac{dv}{dt} = -\frac{v^2}{100} \Rightarrow \frac{dv}{v^2} = -\frac{1}{100}.dt \Rightarrow -v^{-2}.dv = \frac{1}{100}.dt \Rightarrow -\int v^{-2}.dv = \int \frac{1}{100}.dt$$

$$\Rightarrow v^{-1} = \frac{t}{100} \Rightarrow \frac{1}{v} = \frac{t}{100} + c \Rightarrow v_0 = \frac{1}{20} = \frac{0}{100} + c \Rightarrow \frac{1}{20} = 0 + c \Rightarrow c = \frac{1}{20}$$

$$\therefore \frac{1}{v} = \frac{t}{100} + \frac{1}{20} = \frac{t+5}{100} \Rightarrow v = \frac{100}{t+5} \Rightarrow v(20) = \frac{100}{25} = 4m/s$$

الإجابة الصحيحة هي (a)

$$266 - \because \frac{dx}{dt} = -\lambda x, \lambda > 0, m_0 = x(0) = a, x > 0$$

$$\therefore \frac{dx}{dt} = -\lambda x \Rightarrow \frac{dx}{x} = -\lambda.dt \Rightarrow \frac{1}{x}.dx = -\lambda.dt \Rightarrow \int \frac{1}{x}.dx = \int -\lambda.dt$$

$$\Rightarrow ln|x| = -\lambda t + c \Rightarrow ln(x) = -\lambda t + c \Rightarrow x = e^{-\lambda t + c} \Rightarrow a = x(0) = e^{-\lambda \times 0 + c} = e^c \Rightarrow x = ae^{-\lambda t}$$

$$\therefore \frac{1}{2}a = ae^{-\lambda t} \Rightarrow \frac{1}{2} = e^{-\lambda t} \Rightarrow ln\left(\frac{1}{2}\right) = ln(e^{-\lambda t}) \Rightarrow ln(2^{-1}) = ln(e^{-\lambda t}) \Rightarrow -ln(2) = -\lambda t$$

$$\Rightarrow ln(2) = \lambda t \Rightarrow t = \frac{ln(2)}{\lambda}, ln(x) = y \Rightarrow x = e^y, ln(x^n) = n\, ln\, x, ln(e^{h(x)}) = h(x)$$

الإجابة الصحيحة هي (b)

$$267 - \because d = \int_{t_1}^{t_2} |v(t)|.dt, v(t) = \frac{t}{9} - \frac{1}{\sqrt{t+6}}$$

$$\therefore d = \int_1^{10} \left|\frac{t}{9} - \frac{1}{\sqrt{t+6}}\right|.dt \Rightarrow \frac{t}{9} - \frac{1}{\sqrt{t+6}} = 0 \Rightarrow 9 = t\sqrt{t+6} \Rightarrow 81 = t^2(t+6) \Rightarrow 81 = t^3 + 6t^2$$

$\Rightarrow t^3 + 6t^2 - 81 = 0 \Rightarrow (t-3)(t^2 - 9t + 81) \Rightarrow t = 3s, t^2 - 9t + 81$ لا تحلل

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$$\therefore \left|\frac{t}{9} - \frac{1}{\sqrt{t+6}}\right| = \begin{cases} \frac{1}{\sqrt{t+6}} - \frac{t}{9}, 1 \leq t < 3 \\ \frac{t}{9} - \frac{1}{\sqrt{t+6}}, 3 < t \leq 10 \end{cases}$$

$$\therefore d = \int_1^{10} \left|\frac{t}{9} - \frac{1}{\sqrt{t+6}}\right|.dt = -\int_1^{3} \frac{t}{9} - \frac{1}{\sqrt{t+6}}.dt + \int_3^{10} \frac{t}{9} - \frac{1}{\sqrt{t+6}}.dt$$

$$\therefore d=\int_{1}^{10}\left|\frac{t}{9}-\frac{1}{\sqrt{t+6}}\right|.dt=\int_{1}^{3}\frac{1}{\sqrt{t+6}}-\frac{t}{9}.dt+\int_{3}^{10}\frac{t}{9}-\frac{1}{\sqrt{t+6}}.dt$$

$$\therefore d=\int_{1}^{10}\left|\frac{t}{9}-\frac{1}{\sqrt{t+6}}\right|.dt=(\int_{1}^{3}\frac{1}{\sqrt{t+6}}.dt-\int_{1}^{3}\frac{t}{9}.dt)+(\int_{3}^{10}\frac{t}{9}.dt-\int_{3}^{10}\frac{1}{\sqrt{t+6}}.dt)$$

$$\therefore d=\int_{1}^{10}\left|\frac{t}{9}-\frac{1}{\sqrt{t+6}}\right|.dt=(\int_{1}^{3}\frac{1}{\sqrt{t+6}}.dt-(\frac{t^2}{18}\Big|_{1}^{3}))+((\frac{t^2}{18}\Big|_{3}^{10})-\int_{3}^{10}\frac{1}{\sqrt{t+6}}.dt)$$

$$\because f(x)=\frac{1}{\sqrt{t+6}}=(t+6)^{-\frac{1}{2}}$$

$$\therefore \int\frac{1}{\sqrt{t+6}}dt=\int(t+6)^{-\frac{1}{2}}dt=\frac{(t+6)^{\frac{1}{2}}}{\frac{1}{2}}=2(t+6)^{\frac{1}{2}}=2\sqrt{t+6}$$

$$\Rightarrow\therefore \int_{1}^{3}\frac{1}{\sqrt{t+6}}.dt=2\sqrt{t+6}\Big|_{1}^{3}=6-2\sqrt{7}$$

$$\Rightarrow\therefore \int_{3}^{10}\frac{1}{\sqrt{t+6}}.dt=2\sqrt{t+6}\Big|_{3}^{10}=8-6=2$$

حل آخر بالتكامل بالتعويض

$$\because f(x)=\frac{1}{\sqrt{t+6}},u=t+6\Rightarrow du=dt\Rightarrow dt=du$$

$$\because t=1\Rightarrow u=7,t=3\Rightarrow u=9,t=10\Rightarrow u=16$$

$$\therefore \int\frac{1}{\sqrt{t+6}}dt=\int\frac{1}{\sqrt{u}}du=\int u^{-\frac{1}{2}}du=\frac{u^{\frac{1}{2}}}{\frac{1}{2}}=2u^{\frac{1}{2}}=2\sqrt{u}$$

$$\Rightarrow\therefore \int_{1}^{3}\frac{1}{\sqrt{t+6}}.dt=\int_{7}^{9}u^{-\frac{1}{2}}du=2\sqrt{u}\Big|_{7}^{9}=6-2\sqrt{7}$$

$$\Rightarrow\therefore \int_{3}^{10}\frac{1}{\sqrt{t+6}}.dt=\int_{9}^{16}u^{-\frac{1}{2}}du=2\sqrt{t+6}\Big|_{3}^{10}=8-6=2$$

$$\therefore d=\int_{1}^{10}\left|\frac{t}{9}-\frac{1}{\sqrt{t+6}}\right|.dt=(\int_{1}^{3}\frac{1}{\sqrt{t+6}}.dt-(\frac{t^2}{18}\Big|_{1}^{3}))+((\frac{t^2}{18}\Big|_{3}^{10})-\int_{3}^{10}\frac{1}{\sqrt{t+6}}.dt)$$

$$\therefore d=\int_{1}^{10}\left|\frac{t}{9}-\frac{1}{\sqrt{t+6}}\right|.dt=(6-2\sqrt{7}-(\frac{9}{18}-\frac{1}{18}))+((\frac{100}{18}-\frac{9}{18})-2)$$

$$\therefore d=\int_{1}^{10}\left|\frac{t}{9}-\frac{1}{\sqrt{t+6}}\right|.dt=\left(6-2\sqrt{7}-\frac{8}{18}\right)+\left(\frac{91}{18}-2\right)=\frac{108}{18}-2\sqrt{7}-\frac{8}{18}+\frac{91}{18}-\frac{36}{18}=\frac{155}{18}-2\sqrt{7}m$$

الإجابة الصحيحة هي (d)

268$-\because y` = 3y$

$\because f(x)=ke^{-3x}, k\in\mathcal{R}\Rightarrow f`(x)=y`=-3ke^{-3x}\Rightarrow -3ke^{-3x}\neq 3ke^{-3x}$, الاقتران ليس حلا للمعادلة التفاضلية

$\because f(x)=ke^{3x}, k\in\mathcal{R}\Rightarrow f`(x)=y`=3ke^{3x}\Rightarrow 3ke^{3x}=3ke^{-3x}$, الاقتران يعد حلا للمعادلة التفاضلية

الإجابة الصحيحة هي (b)

$269-\because y` + y = x + 1$

$\because f(x) = e^{-x} + 1 \Rightarrow f`(x) = y` = -e^{-x} \Rightarrow e^{-x} + 1 + (-e^{-x}) \neq x + 1$, الاقتران ليس حلا للمعادلة التفاضلية

$\because f(x) = e^{-x} + x \Rightarrow f`(x) = y` = -e^{-x} + 1 \Rightarrow e^{-x} + x + (-e^{-x} + 1) = x + 1$, الاقتران يعد حلا للمعادلة التفاضلية

الإجابة الصحيحة هي (b)

$270-\because y`` + 4y = 0$

$\because f(x) = ke^{4x}, k \in \mathcal{R} \Rightarrow f`(x) = y` = 4ke^{4x} \Rightarrow y`` = 16ke^{4x} \Rightarrow ke^{4x} + 16ke^{4x} \neq 0$

, الاقتران ليس حلا للمعادلة التفاضلية

$\because f(x) = a\cos(2x) + b\sin(2x), a, b \in \mathcal{R} \Rightarrow f`(x) = y` = -2a\sin(2x) + 2b\cos(2x)$

$\Rightarrow y`` = -4a\cos(2x) - 4b\sin(2x) \Rightarrow -4a\cos(2x) - 4b\sin(2x) + 4(a\cos(2x) + b\sin(2x)) = 0$

$\Rightarrow -4a\cos(2x) - 4b\sin(2x) + 4a\cos(2x) + 4b\sin(2x) = 0$, الاقتران يعد حلا للمعادلة التفاضلية

الإجابة الصحيحة هي (b)

$271-\because y` = y, f(1) = 2,$

$$\because y` = y \Rightarrow \frac{dy}{dx} = y \Rightarrow \frac{dy}{y} = dx \Rightarrow \int \frac{dy}{y} = \int dx \Rightarrow \ln|y| = x + c \Rightarrow e^{x+c} = y \Rightarrow f(x) = 2e^{x-1}$$

الإجابة الصحيحة هي (c)

$272-\because y` + 2y = 6, f(0) = 2,$ عند التعويض جميع الإجابات صحيحة لذا نلجا الى حل المعادلة التفاضلية

$\because f(x) = -e^{-2x} + 3 \Rightarrow f`(x) = y` = 2e^{-2x} \Rightarrow 2e^{-2x} + 2(-e^{-2x} + 3) = 6 \Rightarrow 2e^{-2x} - 2e^{-2x} + 6 = 6$

, الاقتران يعد حلا للمعادلة التفاضلية

الإجابة الصحيحة هي (a)

$273-\because (y + 1)y` = 3x, (0, 1)$

$$\because (y + 1)y` = 3x \Rightarrow (y + 1)\frac{dy}{dx} = 3x \Rightarrow (y + 1)dy = 3xdx \Rightarrow \int (y + 1)dy = \int 3xdx$$

$$\Rightarrow \int ydy + \int dy = \int 3xdx \Rightarrow \frac{y^2}{2} + y = \frac{3x^2}{2} + c \Rightarrow \frac{1}{2} + 1 = 0 + c \Rightarrow c = \frac{3}{2}$$

الحل الخاص للمعادلة التفاضلية

$$\frac{y^2}{2} + y = \frac{3x^2}{2} + \frac{3}{2} = y^2 + y = 3x^2 + 3$$

الإجابة الصحيحة هي (b)

$$274-\because y` = \frac{2x}{y}e^{y-x}$$

$$\because y` = \frac{2x}{y}e^{y-x} \Rightarrow \frac{dy}{dx} = \frac{2xe^{y}}{ye^{x}} \Rightarrow \frac{ydy}{e^{y}} = \frac{2x}{e^{x}}dx \Rightarrow ye^{-y}dy = 2xe^{-x}dx \Rightarrow \int ye^{-y}dy = \int 2xe^{-x}dx$$

$$\because \int ye^{-y}dy \Rightarrow \int f(x).dx = \int u.dv = u.v - \int v.du$$

$$\because \int ye^{-y}dy \Rightarrow u = y \Rightarrow du = dy, dv = e^{-y} \Rightarrow v = \int e^{-y}.du = -e^{-y}$$

$$\because \int ye^{-y}dy = -ye^{-y} - \int -e^{-y}\,dy = -ye^{-y} - e^{-y} = e^{-y}(-y - 1)$$

$$\because \int 2xe^{-x}dx \Rightarrow \int f(x).dx = \int u.dv = u.v - \int v.du$$

$$\because \int 2xe^{-x}dx \Rightarrow u = 2x \Rightarrow du = 2dx, dv = e^{-x} \Rightarrow v = \int e^{-x}.du = -e^{-x}$$

$$\because \int 2xe^{-x}dx = -2xe^{-x} - \int -2e^{-x}\,dx = -2xe^{-x} - 2e^{-x} = e^{-x}(-2x - 2) + c$$

الحل للمعادلة التفاضلية

$$e^{-y}(-y - 1) = e^{-x}(-2x - 2) + c$$

الإجابة الصحيحة هي (c)

$275 - \because y` = \dfrac{cos(x)}{sin(y)}$

$\because y` = \dfrac{cos(x)}{sin(y)} \Rightarrow sin(y)\dfrac{dy}{dx} = cos(x) \Rightarrow sin(y)\,dy = cos(x)\,dx \Rightarrow \int sin(y)\,dy = \int cos(x)\,dx$

$\therefore -cos(y) = sin(x) \Rightarrow cos(y) = -sin(x) + c$

الحل للمعادلة التفاضلية

$cos(y) = -sin(x) + c$

الإجابة الصحيحة هي (a)

$276 - \because y` = x\,cos^2(y)\,,\left(1,\dfrac{\pi}{4}\right), \dfrac{1}{cos^2(x)} = sec^2(x)$

$\because y` = x\,cos^2(y) \Rightarrow \dfrac{dy}{cos^2(y)\,dx} = x \Rightarrow \dfrac{dy}{cos^2(y)} = xdx \Rightarrow sec^2(y)dy = xdx$

$\therefore \Rightarrow \int sec^2(y)dy = \int xdx = tan(y) = \dfrac{x^2}{2} \Rightarrow 2\,tan(y) = x^2 + c \Rightarrow (1,\dfrac{\pi}{4})$

$2\,tan\left(\dfrac{\pi}{4}\right) = 1 + c \Rightarrow 2 = 1 + c \Rightarrow c = 1$

الحل الخاص للمعادلة التفاضلية

$2\,tan(y) = x^2 + 1$

الإجابة الصحيحة هي (b)

$277 - \because \dfrac{dy}{dx} = \dfrac{3}{y\,cos^2(x)}, \left(x = \dfrac{\pi}{4}\right)$ عند $(y = 2)$, عندما , $\dfrac{1}{cos^2(x)} = sec^2(x)$

$\because \dfrac{dy}{dx} = \dfrac{3}{y\,cos^2(x)} \Rightarrow ydy = \dfrac{3dx}{cos^2(x)} \Rightarrow ydy = 3sec^2(x)dx$

$\therefore \Rightarrow \int ydy = \int 3sec^2(x)dx \Rightarrow \dfrac{y^2}{2} \Rightarrow 3\,tan(x) \Rightarrow y^2 = 6\,tan(x) + c \Rightarrow 4 = 6\,tan\left(\dfrac{\pi}{4}\right) + c$

$4 = 6 + c \Rightarrow c = -2$

الحل الخاص للمعادلة التفاضلية

$y^2 = 6\,tan(x) - 2 = \dfrac{1}{2}y^2 = 3\,tan(x) - 1$

الإجابة الصحيحة هي (a)

$278 - \because \dfrac{dy}{dx} = y\,cos(x)\,,(0,1)$

$\because \dfrac{dy}{dx} = y\,cos(x) \Rightarrow \dfrac{dy}{y} = cos(x)\,dx \Rightarrow \int \dfrac{dy}{y} = \int cos(x)\,dx \Rightarrow ln|y| = sin(x) + c$

$\therefore ln|1| = sin(0) + c \Rightarrow 0 = 0 + c \Rightarrow c = 0$

الحل للمعادلة التفاضلية

$ln|y| = sin(x)$

الإجابة الصحيحة هي (c)

$279 - \because \dfrac{dy}{dx} = -\dfrac{2x}{3y}, (5,4)$

$\because \dfrac{dy}{dx} = -\dfrac{2x}{3y} \Rightarrow 3ydy = -2xdx \Rightarrow \int 3ydy = \int -2xdx \Rightarrow \dfrac{3}{2}y^2 = -x^2 + c$

$\therefore \dfrac{3}{2} \times 16 = -25 + c \Rightarrow 24 = -25 + c \Rightarrow c = 49$

الحل للمعادلة التفاضلية

$\dfrac{3}{2}y^2 = -x^2 + 49$

الإجابة الصحيحة هي (a)

$$280-\because \frac{dy}{dx}=\frac{\sqrt{y}}{x},(1,4)$$

$$\because \frac{dy}{dx}=\frac{\sqrt{y}}{x}\Rightarrow y^{-\frac{1}{2}}dy=\frac{dx}{x}\Rightarrow \int y^{-\frac{1}{2}}dy=\int \frac{dx}{x}\Rightarrow 2y^{\frac{1}{2}}=ln|x|+c\Rightarrow 2\sqrt{y}=ln|x|+c$$

$$\therefore 4\Rightarrow ln|1|+c\Rightarrow 4=0+c\Rightarrow c=4$$

الحل للمعادلة التفاضلية

$$2\sqrt{y}=ln|x|+4$$

الإجابة الصحيحة هي (b)

السؤال الثاني:

1– جد قيمة التكامل الآتي:

$$\int_0^1 5x(1-x^2)^{\frac{3}{2}}.dx, u=1-x^2\Rightarrow du=-2x.dx\Rightarrow dx=\frac{du}{-2x}$$

$$\therefore x=0\Rightarrow u=1, x=1\Rightarrow u=0$$

$$\therefore \int_0^1 5x(1-x^2)^{\frac{3}{2}}.dx=\int_1^0 5x(u)^{\frac{3}{2}}.\frac{du}{-2x}=-\frac{5}{2}\int_1^0 (u)^{\frac{3}{2}}.du=-\frac{5}{2}\left(\frac{2u^{\frac{5}{2}}}{5}\right)=-\sqrt{u^5}\Big|_1^0=-(\sqrt{0}-\sqrt{1})$$

$$=-(-1)=1$$

–2

$$1-\int cos(x)\sqrt{4-sin(x)}.dx, u=4-sin(x)\Rightarrow du=-cos(x).dx\Rightarrow dx=\frac{du}{-cos(x)}$$

$$\therefore \int cos(x)\sqrt{4-sin(x)}.dx=\int cos(x)\sqrt{u}.\frac{du}{-cos(x)}=-\int \sqrt{u}.du=-\int u^{\frac{1}{2}}.du$$

$$=-\frac{2}{3}u^{\frac{3}{2}}=-\frac{2}{3}(\sqrt{4-sin(x)})^{\frac{3}{2}}=-\frac{2}{3}\sqrt{(4-sin(x))^3}+c$$

$$2-\int \frac{4x^3-7x}{x^4-3x^2+4}.dx=\int \frac{4x^3-7x}{(x^2-4)(x^2+1)}.dx=\int \frac{4x^3-7x}{(x-2)(x+2)(x^2+1)}.dx$$

$$\Rightarrow \frac{4x^3-7x}{(x-2)(x+2)(x^2+1)}=\frac{A}{x-2}+\frac{B}{x+2}+\frac{C}{x^2+1}$$

$$\Rightarrow \frac{4x^3-7x}{(x-2)(x+2)(x^2+1)}=\frac{A\left((x+2)(x^2+1)\right)}{x-2}+\frac{B\left((x+2)(x^2+1)\right)}{x+2}+\frac{C(x^2-4)}{x^2+1}$$

$$\therefore 4x^3-7x=A\left((x+2)(x^2+1)\right)+B\left((x-2)(x^2+1)\right)+C(x^2-4)$$

$$\therefore x=2\Rightarrow 20A+0+0=18\Rightarrow A=\frac{18}{20}=\frac{9}{10}$$

$$\therefore x=-2\Rightarrow 0-20B+0=-18\Rightarrow B=\frac{-18}{-20}=\frac{9}{10}$$

$$\therefore x=0\Rightarrow \frac{18}{20}-\frac{18}{20}-4C=0\Rightarrow C=0$$

$$\Rightarrow \frac{4x^3-7x}{(x-2)(x+2)(x^2+1)}=\frac{\frac{9}{10}}{x-2}+\frac{\frac{9}{10}}{x+2}+\frac{0}{x^2+1}$$

$$\Rightarrow \int \frac{4x^3-7x}{x^4-3x^2+4}.dx=\int \frac{\frac{9}{10}}{x-2}.dx+\int \frac{\frac{9}{10}}{x+2}.dx=\frac{9}{10}\int \frac{1}{x-2}.dx+\frac{9}{10}\int \frac{1}{x+2}.dx$$

$$=\frac{9}{10}ln|x-2|+\frac{9}{10}ln|x+2|=\frac{9}{10}ln|x^2-4|+c$$

$$3-\int \frac{x^4+3x^2-3x+2}{x^3-x^2-2x}.dx$$, بالقسمة الطويلة

$$\Rightarrow \int \frac{x^4+3x^2-3x+2}{x^3-x^2-2x}.dx=\int (x+1+\frac{6x^2-x+2}{x^3-x^2-2x}).dx=\int (x+1+\frac{6x^2-x+2}{x(x^2-x-2)}).dx$$

$$\Rightarrow \int \frac{x^4+3x^2-3x+2}{x^3-x^2-2x}.dx=\int (x+1+\frac{6x^2-x+2}{x(x^2-x-2)}).dx=\int (x+1+\frac{6x^2-x+2}{x(x-2)(x+1)}).dx$$

$$\Rightarrow \frac{6x^2-x+2}{x(x-2)(x+1)}=\frac{A}{x}+\frac{B}{x-2}+\frac{C}{x+1}$$

$$\Rightarrow \frac{6x^2-x+2}{x(x-2)(x+1)}=\frac{A(x^2-x-2)}{x}+\frac{B(x^2+x)}{x-2}+\frac{C(x^2-2x)}{x+1}$$

$$\therefore 6x^2-x+2=A(x^2-x-2)+B(x^2+x)+C(x^2-2x)$$

$$\therefore x=2\Rightarrow 0+6B+0=24\Rightarrow B=\frac{24}{6}=4$$

$$\therefore x=0\Rightarrow -2A+0+0=2\Rightarrow A=\frac{2}{-2}=-1$$

$$\therefore x=1\Rightarrow 2+8-C=7\Rightarrow C=3$$

$$\Rightarrow \frac{6x^2-x+2}{x(x-2)(x+1)}=\frac{-1}{x}+\frac{4}{x-2}+\frac{3}{x+1}$$

$$\Rightarrow \int \frac{x^4+3x^2-3x+2}{x^3-x^2-2x}.dx=\int (x+1+\frac{-1}{x}+\frac{4}{x-2}+\frac{3}{x+1}).dx$$

$$=\int x.dx+\int 1.dx+\int \frac{-1}{x}.dx+\int \frac{4}{x-2}.dx+\int \frac{3}{x+1}.dx$$

$$=\int x.dx+\int 1.dx-\int \frac{1}{x}.dx+4\int \frac{1}{x-2}.dx+3\int \frac{1}{x+1}.dx$$

$$=\frac{1}{2}x^2+x-ln|x|+4\,ln|x-2|+3\,ln|x+1+c|$$

$$4-\int \frac{e^{2x}+1}{e^x+1}.dx$$, بالقسمة الطويلة

$$\Rightarrow \int \frac{e^{2x}+1}{e^x+1}.dx=\int (e^x-1+\frac{2}{e^x+1}).dx=\int e^x.dx-\int 1.dx+2\int \frac{1}{e^x+1}.dx$$

$$=\int e^x.dx-\int 1.dx+\int \frac{2\times e^{-x}}{e^xe^{-x}+e^{-x}}.dx=\int e^x.dx-\int 1.dx+2\int \frac{e^{-x}}{e^0+e^{-x}}.dx$$

$$=\int e^x.dx-\int 1.dx-2\int \frac{-e^{-x}}{1+e^{-x}}.dx=e^x+x-2\,ln|1+e^{-x}|+c$$

$$5-\int \frac{dx}{e^{2x}+2e^x+1}=\int \frac{dx}{(e^x+1)(e^x+1)}=\int \frac{dx}{(e^x+1)^2}=\int \frac{e^x\,dx}{e^x(e^x+1)^2}$$

$$\Rightarrow \int \frac{e^x\,dx}{e^x(e^x+1)^2}\ ,u=e^x\Rightarrow du=e^x.dx\Rightarrow dx=\frac{du}{e^x}$$

$$\Rightarrow \int \frac{e^x\,dx}{e^x(e^x+1)^2}=\int \frac{du}{u(u+1)^2}\Rightarrow \frac{1}{u(u+1)^2}=\frac{A}{u}+\frac{B}{u+1}+\frac{C}{(u+1)^2}$$

$$\Rightarrow \frac{1}{u(u+1)^2}=\frac{A((u+1)^2)}{u}+\frac{B(u^2+u)}{u+1}+\frac{C(u)}{(u+1)^2}$$

$$\therefore 1=A((u+1)^2)+B(u^2+u)+C(u)$$

$$\therefore u=0\Rightarrow A+0+0=1\Rightarrow A=1$$

$$\therefore u=1\Rightarrow 4+2B+C=1\Rightarrow 2B+C=-3\qquad (1)$$

$$\therefore u=-1\Rightarrow 0+2B-C=1\Rightarrow 2B-C=1\qquad (2)$$

بجمع المعادلتين ينتج

$$\Rightarrow 4B=-2\Rightarrow B=-\frac{1}{2}\ ,C=-2$$

$$\Rightarrow \frac{1}{u(u+1)^2}=\frac{1}{u}+\frac{-\frac{1}{2}}{u+1}+\frac{-2}{(u+1)^2}$$

$$\Rightarrow \int \frac{e^x\,dx}{e^x(e^x+1)^2}=\int \frac{du}{u(u+1)^2}\Rightarrow \int \frac{1}{u(u+1)^2}.du=\int \frac{1}{u}.du+\int \frac{-\frac{1}{2}}{u+1}.du+\int \frac{-2}{(u+1)^2}.du$$

$$=\int \frac{1}{u}.du-\frac{1}{2}\int \frac{1}{u+1}.du-2\int (u+1)^{-2}.du=ln|u|-\frac{1}{2}ln|u+1|+2(u+1)^{-1}$$

$$=ln|u|-\frac{1}{2}ln|u+1|+\frac{2}{u+1}=ln|e^x|-\frac{1}{2}ln|e^x+1|+\frac{2}{e^x+1}+c$$

$$6-\int \frac{dx}{1+\sqrt[3]{x}}=u=\sqrt[3]{x}=x^{\frac{1}{3}}\Rightarrow du=\frac{1}{3}x^{-\frac{2}{3}}.dx=\frac{1}{3x^{\frac{2}{3}}},dx=\frac{1}{3\sqrt[3]{x^2}}.dx=\frac{1}{3(\sqrt[3]{x})^2}.dx$$

$$\Rightarrow dx=3(\sqrt[3]{x})^2du$$

$$\Rightarrow \int \frac{dx}{1+\sqrt[3]{x}}=\int \frac{1}{1+\sqrt[3]{x}}.3(\sqrt[3]{x})^2du=\int \frac{3u^2}{1+u}.du=3\int \frac{u^2}{1+u}.du,$$ بالقسمة الطويلة

$$\Rightarrow \frac{u^2}{1+u}=u-\frac{1}{1+u}\Rightarrow 3\int \frac{u^2}{1+u}.du=3(\int u.du-\int \frac{1}{1+u}.du)$$

$$\Rightarrow 3\int \frac{u^2}{1+u}.du=3\left(\frac{u^2}{2}-ln|u|\right)=\frac{3}{2}u^2-3\,ln|u|=\frac{3}{2}(\sqrt[3]{x})^2-3\,ln|\sqrt[3]{x}|=\frac{3}{2}\sqrt[3]{x^2}-3\,ln|\sqrt[3]{x}|+c$$

$$7-\int \frac{2x-3}{(x^2-3x+2)(x-1)}dx=\int \frac{2x-3}{(x-2)(x-1)(x-1)}dx=\int \frac{2x-3}{(x-2)(x-1)^2}.dx$$

$$\Rightarrow \frac{2x-3}{(x^2-3x+2)(x-1)}=\frac{2x-3}{(x-2)(x-1)^2}=\frac{A}{x-2}+\frac{B}{x-1}+\frac{C}{(x-1)^2}$$

$$\Rightarrow \frac{2x-3}{(x^2-3x+2)(x-1)}=\frac{2x-3}{(x-2)(x-1)^2}=\frac{A((x-1)^2)}{x-2}+\frac{B(x^2-3x+2)}{x-1}+\frac{C(x-2)}{(x-1)^2}$$

$$\therefore 2x-3=A((x-1)^2+B(x^2-3x+2)+C(x-2)$$

$$\therefore x=1\Rightarrow 0+0-C=-1\Rightarrow C=1$$

$$\therefore x=2\Rightarrow A+0+0=1\Rightarrow A=1$$

$$\therefore x=0\Rightarrow 1+28-2=-3\Rightarrow B=-1$$

$$\Rightarrow \frac{2x-3}{(x^2-3x+2)(x-1)}=\frac{2x-3}{(x-2)(x-1)^2}=\frac{1}{x-2}+\frac{-1}{x-1}+\frac{1}{(x-1)^2}$$

$$\Rightarrow \int \frac{2x-3}{(x^2-3x+2)(x-1)}.dx=\int \frac{1}{x-2}.dx-\int \frac{1}{x-1}.dx+\int \frac{1}{(x-1)^2}.dx$$

$$\Rightarrow \int \frac{2x-3}{(x^2-3x+2)(x-1)}.dx=\int \frac{1}{x-2}.dx-\int \frac{1}{x-1}.dx+\int (x-1)^{-2}.dx$$

$$\Rightarrow \int (x-1)^{-2}.dx\,,u=x-1\Rightarrow du=dx\Rightarrow \int (u)^{-2}.du=-(u)^{-1}=-\frac{1}{x-1}$$

$$\Rightarrow \int \frac{2x-3}{(x^2-3x+2)(x-1)}.dx=ln|x-2|-ln|x-1|-\frac{1}{x-1}+c$$

3– جد ناتج كلا من التكاملات الآتية:

$$1-\int x^2\,sin(2x).dx\Rightarrow \int f(x).dx=\int u.dv=u.v-\int v.du$$

$$\Rightarrow u=x^2\Rightarrow du=2x.dx,dv=sin(2x)\Rightarrow v=\int sin(2x).dx=-\frac{1}{2}cos(2x)$$

$$\therefore \int x^2\,sin(2x).dx=-\frac{1}{2}x^2\,cos(2x)-\int -\frac{1}{2}cos(2x).2x.dx=-\frac{1}{2}x^2\,cos(2x)+\int x\,cos(2x).dx$$

$$\Rightarrow \int xcos(2x).dx\,,\int f(x).dx=\int u.dv=u.v-\int v.du$$

$$\Rightarrow u=x\Rightarrow du=dx,dv=cos(2x)\Rightarrow v=\int cos(2x).dx=\frac{1}{2}sin(2x)$$

$$\therefore \int x^2 \sin(2x).dx = -\frac{1}{2}x^2\cos(2x) + \left(\frac{1}{2}x\sin(2x) - \int \frac{1}{2}\sin(2x)\,dx\right)$$

$$\therefore \int x^2 \sin(2x).dx = -\frac{1}{2}x^2\cos(2x) + \frac{1}{2}x\sin(2x) + \frac{1}{4}\cos(2x) + c$$

$$2 - \int x^2\cos(x).dx \Rightarrow, \int f(x).dx = \int u.dv = u.v - \int v.du$$

$$\Rightarrow u = x^2 \Rightarrow du = 2xdx, dv = \cos(x) \Rightarrow v = \int \cos(x).dx = \sin(x)$$

$$\therefore \int x^2\cos(x).dx = x^2\sin(x) - \int 2x\sin(x)\,dx, \int f(x).dx = \int u.u = u.v - \int v.du$$

$$\Rightarrow \int 2x\sin(x)\,dx, u = 2x \Rightarrow du = 2dx, dv = \sin(x) \Rightarrow v = \int \sin(x)\,dx = -\cos(x)$$

$$\therefore \int x^2\cos(x).dx = x^2\sin(x) - \left(-2x\cos(x) - \int -2\cos(x).dx\right)$$

$$\therefore \int x^2\cos(x).dx = x^2\sin(x) + 2x\cos(x) - 2\sin(x) + c$$

$$3 - \int x^5\cos(x).dx$$

اشتقاق $(x^5 = u)$ بصورة متكررة	إشارة الضرب	تكامل $(\cos(x) = dv)$ بصورة متكررة
x^5	$+$	$\cos(x)$
$5x^4$	$-$	$\sin(x)$
$20x^3$	$+$	$-\cos(x)$
$60x^2$	$-$	$-\sin(x)$
$120x$	$+$	$\cos(x)$
120	$-$	$\sin(x)$
0	$+\int$	$-\cos(x)$

$$\therefore \int x^5\cos(x).dx = x^5\sin(x) + 5x^4\cos(x) - 20x^3\sin(x) - 60x^2\cos(x) + 120x\sin(x) + 120\cos(x) + c$$

$$4 - \int e^{-x}\sin(2x).dx \Rightarrow \int f(x).dx = \int u.dv = u.v - \int v.du$$

$$\Rightarrow u = e^{-x} \Rightarrow du = -e^{-x}.dx, dv = \sin(2x) \Rightarrow v = \int \sin(2x).dx = -\frac{1}{2}\cos(2x)$$

$$\therefore \int e^{-x}\sin(2x).dx = -\frac{1}{2}e^{-x}\cos(2x) - \int -\frac{1}{2}\cos(2x).-e^{-x}.dx$$

$$\therefore \int e^{-x}\sin(2x).dx = -\frac{1}{2}e^{-x}\cos(2x) - \int \frac{1}{2}e^{-x}\cos(2x).dx$$

$$\Rightarrow \int \frac{1}{2}e^{-x}\cos(2x).dx, \int f(x).dx = \int u.dv = u.v - \int v.du$$

$$\Rightarrow u = \frac{1}{2}e^{-x} \Rightarrow du = -\frac{1}{2}e^{-x}dx, dv = \cos(2x) \Rightarrow v = \int \cos(2x).dx = \frac{1}{2}\sin(2x)$$

$$\therefore \int e^{-x}\sin(2x).dx = -\frac{1}{2}e^{-x}\cos(2x) - \left(\frac{1}{4}e^{-x}\sin(2x) - \int \frac{1}{2}\sin(2x) \times -\frac{1}{2}e^{-x}dx\right)$$

$$\therefore \int e^{-x}\sin(2x).dx = -\frac{1}{2}e^{-x}\cos(2x) - \left(\frac{1}{4}e^{-x}\sin(2x) + \frac{1}{4}\int e^{-x}\sin(2x)\,dx\right)$$

$$\therefore \int e^{-x}\sin(2x).dx + \frac{1}{4}\int e^{-x}\sin(2x)\,dx = -\frac{1}{2}e^{-x}\cos(2x) - \frac{1}{4}e^{-x}\sin(2x)$$

$$\therefore \frac{5}{4}\int e^{-x}\sin(2x)\,dx = -\frac{1}{2}e^{-x}\cos(2x) - \frac{1}{4}e^{-x}\sin(2x)$$

$$\therefore \int e^{-x}\sin(2x)\,dx = \frac{4}{5}\left(-\frac{1}{2}e^{-x}\cos(2x) - \frac{1}{4}e^{-x}\sin(2x)\right)$$
$$\therefore \int e^{-x}\sin(2x)\,dx = -\frac{2}{5}e^{-x}\cos(2x) - \frac{1}{5}e^{-x}\sin(2x) + c$$
$$5 - \int \cos(\ln(x)).dx \Rightarrow \int f(x).dx = \int u.dv = u.v - \int v.du$$
$$\Rightarrow u = \cos(\ln(x)) \Rightarrow du = -\frac{1}{x}\sin(\ln(x))\,dx, dv = dx \Rightarrow v = \int dx = x$$
$$\therefore \int \cos(\ln(x)).dx = x\cos(\ln(x)) - \int x \times -\frac{1}{x}\sin(\ln(x))\,dx$$
$$\therefore \int \cos(\ln(x)).dx = x\cos(\ln(x)) + \int \sin(\ln(x))\,dx, \int f(x).dx = \int u.dv = u.v - \int v.du$$
$$\Rightarrow u = \sin(\ln(x)) \Rightarrow du = \frac{1}{x}\cos(\ln(x))\,dx, dv = dx \Rightarrow v = \int dx = x$$
$$\therefore \int \cos(\ln(x)).dx = x\cos(\ln(x)) + \left(x\sin(\ln(x)) - \int x \times \frac{1}{x}\cos(\ln(x)).dx\right)$$
$$\therefore \int \cos(\ln(x)).dx = x\cos(\ln(x)) + x\sin(\ln(x)) - \int \cos(\ln(x)).dx$$
$$\therefore \int \cos(\ln(x)).dx + \int \cos(\ln(x)).dx = x\cos(\ln(x)) + x\sin(\ln(x))$$
$$\therefore 2\int \cos(\ln(x)).dx = x\cos(\ln(x)) + x\sin(\ln(x))$$
$$\therefore \int \cos(\ln(x)).dx = \frac{1}{2}(x\cos(\ln(x)) + x\sin(\ln(x)))$$
$$\therefore \int \cos(\ln(x)).dx = \frac{x}{2}(\cos(\ln(x)) + \sin(\ln(x))) + c$$

السؤال الثالث:

1– اوجد الحل الخاص للمعادلة التفاضلية عندما $(y = 4)$ عند $(x = 1)$

$$\frac{dy}{dx} = \frac{3(y-1)}{(2x+1)(x+2)} \Rightarrow \frac{dy}{3(y-1)} = \frac{dx}{(2x+1)(x+2)}$$
$$\Rightarrow \int \frac{dy}{3(y-1)} = \int \frac{dx}{(2x+1)(x+2)} \Rightarrow \frac{1}{3}\int \frac{1}{(y-1)}.dy = \int \frac{1}{(2x+1)(x+2)}.dx$$
$$\because \frac{1}{(2x+1)(x+2)} = \frac{A}{2x+1} + \frac{B}{x+2} \Rightarrow \frac{1}{(2x+1)(x+2)} = \frac{A(x+2)}{2x+1} + \frac{B(2x+1)}{x+2}$$
$$\therefore 1 = A(x+2) + B(2x+1), \because x = -2 \Rightarrow 0 - 3B = 1 \Rightarrow B = -\frac{1}{3}$$
$$, \because x = 0 \Rightarrow 2A - \frac{1}{3} = 1 \Rightarrow A = \frac{2}{3} \Rightarrow \frac{1}{(2x+1)(x+2)} = \frac{\frac{2}{3}}{2x+1} + \frac{-\frac{1}{3}}{x+2}$$
$$\Rightarrow \frac{1}{3}\int \frac{1}{(y-1)}.dy = \int \frac{1}{(2x+1)(x+2)}.dx = \int \frac{\frac{2}{3}}{2x+1}.dx + \int \frac{-\frac{1}{3}}{x+2}.dx$$
$$\Rightarrow \frac{1}{3}\int \frac{1}{(y-1)}.dy = \frac{1}{3}\int \frac{2}{2x+1}.dx - \frac{1}{3}\int \frac{-1}{x+2}.dx$$
$$\Rightarrow \frac{1}{3}\ln|y-1| = \frac{1}{3}\int \ln|2x+1| - \frac{1}{3}\ln|x+2| + c$$
$$\Rightarrow \frac{1}{3}\ln|3| = \frac{1}{3}\int \ln 3 - \frac{1}{3}\ln|3| + c = 0 + c \Rightarrow c = \frac{1}{3}\ln|3|$$
$$\therefore \frac{1}{3}\ln|y-1| = \frac{1}{3}\int \ln|2x+1| - \frac{1}{3}\ln|x+2| + \frac{1}{3}\ln|3|$$

2- اوجد الحل العام للمعادلات التفاضلية الاتية:

$$1-(1+x^2)\frac{dy}{dx}=x\tan(y)\Rightarrow\frac{dy}{\tan(y)}=\frac{x.dx}{1+x^2}\Rightarrow\int\frac{dy}{\tan(y)}=\int\frac{x.dx}{1+x^2}$$

$$\Rightarrow\int\cot(y).dy=\frac{1}{2}\int\frac{2x}{1+x^2}.dx\Rightarrow\int\frac{\cos(y)}{\sin(y)}.dy=\frac{1}{2}\int\frac{2x}{1+x^2}.dx\Rightarrow\ln|\sin(y)|=\frac{1}{2}\ln|1+x^2|+c$$

$$2-\frac{dy}{dx}+e^xy=e^xy^2\Rightarrow\frac{dy}{dx}=e^xy^2-e^xy\Rightarrow\frac{dy}{dx}=e^x(y^2-y)\Rightarrow\frac{dy}{y^2-y}=e^x.dx$$

$$\Rightarrow\int\frac{1}{y^2-y}.dy=\int e^x.dx\Rightarrow\frac{1}{y^2-y}=\frac{1}{y(y-1)}=\frac{A}{y}+\frac{B}{y-1}\Rightarrow\frac{1}{y(y-1)}=\frac{A(y-1)}{y}+\frac{B(y)}{y-1}$$

$$\therefore 1=A(y-1)+B(y)\Rightarrow\because y=0\Rightarrow -A+0=1\Rightarrow A=-1, y=1\Rightarrow 0+B=0\Rightarrow B=1$$

$$\Rightarrow\frac{1}{y(y-1)}=\frac{-1}{y}+\frac{1}{y-1}$$

$$\therefore\int\frac{1}{y^2-y}.dy=\int e^x.dx\Rightarrow\int\frac{-1}{y}.dy+\int\frac{1}{y-1}.dy=\int e^x.dx$$

$$\Rightarrow-\int\frac{1}{y}.dy+\int\frac{1}{y-1}.dy=\int e^x.dx\Rightarrow-\int\frac{1}{y}.dy+\int\frac{1}{y-1}.dy=\int e^x.dx$$

$$\therefore -\ln|y|+\ln|y-1|=e^x\Rightarrow\ln\left|\frac{y-1}{y}\right|=e^x+c$$

$$3-\frac{dy}{dx}=xy+y\Rightarrow\frac{dy}{dx}=y(x+1)\Rightarrow\frac{dy}{y}=(x+1).dx\Rightarrow\int\frac{dy}{y}=\int(x+1).dx$$

$$\Rightarrow\ln|y|=\frac{1}{2}x^2+x+c$$

3- اوجد المساحة للمنطقة المحصورة و حجم الجسم الناتج من دورانها في الربع الاول حول محور (x) في الحالتين الأتيتين:

1- $f(x)=e^x, g(x)=x, x=0, x=2$

$\because e^x\neq x, e^x>x,$ الاقترانان لا يتقاطعان

$$\therefore A=\int_a^b\big(f(x)-g(x)\big).dx$$

$$\Rightarrow A=\int_0^2(e^x-x).dx=\int_0^2(e^x).dx-\int_0^2(x).dx=(e^x-\frac{1}{2}x^2)\Big|_0^2$$

$$\therefore A=(e^2-2)-(e^0-0)=e^2-2-1=(e^2-3)\ u^2$$

$$\therefore V=\pi\int_a^b((f(x))^2-(g(x))^2).dx$$

$$\Rightarrow V=\pi\int_0^2(e^{2x}-x^2).dx=\pi(\int_0^2(e^{2x}).dx-\int_0^2(x^2).dx)$$

$$=\pi\left(e^{2x}-\frac{1}{3}x^3\right)\Big|_0^2=\pi\left((e^4-\frac{8}{3})-(e^0-0)\right)=\pi\left(e^4-\frac{8}{3}-1\right)=\pi(e^4-\frac{5}{3})\ u^3$$

2- $f(x)=x^3, g(x)=x, x=0, x=2$

$$\because x^3=x\Rightarrow x^3-x=0\Rightarrow x(x^2-1)=0\Rightarrow x(x-1)(x+1)=0\Rightarrow x=0,1,-1$$

$\Rightarrow x=0,1$ في الربع الاول

$$\because x^3>x, x\in[1,2], x>x^3, x\in[0,1]$$

$$\therefore A=\int_a^b\big(f(x)-g(x)\big).dx$$

$$\Rightarrow A=\int_0^1(x-x^3).dx+\int_1^2(x^3-x).dx=\left(\frac{1}{2}x^2-\frac{1}{4}x^4\right)\Big|_0^1+(\frac{1}{4}x^4-\frac{1}{2}x^2)\Big|_1^2=\frac{1}{4}+\left(2-\frac{1}{4}\right)=2\ u^2$$

$$\therefore V=\pi\int_a^b\left((f(x))^2-(g(x))^2\right).dx$$

$$\Rightarrow V=\pi\int_0^1(x^2-x^6).dx+\pi\int_1^2(x^6-x^2).dx=\pi\left(\frac{1}{3}x^3-\frac{1}{7}x^7\right)\Big|_0^1+\pi(\frac{1}{7}x^7-\frac{1}{3}x^3)\Big|_1^2$$

$$=\frac{\pi}{3}+\pi(\left(\frac{128}{7}-\frac{8}{3}\right)-\left(\frac{1}{7}-\frac{1}{3}\right)=\frac{\pi}{3}+\pi(\left(\frac{384-8}{21}\right)-\left(\frac{3-7}{21}\right)=\frac{\pi}{3}+\pi(\left(\frac{376}{21}\right)-\left(\frac{-4}{21}\right)$$

$$=\frac{\pi}{3}+\pi\frac{380}{21}=\frac{7\pi}{21}+\frac{380\pi}{21}=\frac{387\pi}{21}u^3$$

4− اوجد حجم المجسم الناتج من دوران المنطقة المحصورة في الربع الأول بين منحنى الاقتران $(f(x)=\sqrt{xe^{-x}})$ والمستقيمين $(x=1\ ,x=2)$ حول محور (x) .

$$\therefore V=\pi\int_a^b((f(x))^2).dx=\pi\int_1^2\left(\sqrt{xe^{-x}}\right)^2.dx=\pi\int_1^2 xe^{-x}.dx\Rightarrow\int f(x).dx=\int u.dv=u.v-\int v.du$$

$$\because\int xe^{-x}.dx\Rightarrow u=x\Rightarrow du=dx,dv=e^{-x}\Rightarrow v=\int e^{-x}.dx=-e^{-x}$$

$$\therefore\int xe^{-x}.dx=-xe^{-x}-\int -e^{-x}.dx=-xe^{-x}-e^{-x}$$

$$\therefore V=\pi\int_1^2 xe^{-x}.dx=\pi(-xe^{-x}-e^{-x})\Big|_1^2=\pi\left((-2e^{-2}-e^{-2})-(-e^{-1}-e^{-1})\right)$$

$$=\pi(-3e^{-2}+2e^{-1})=\pi\left(\frac{2}{e}-\frac{3}{e^2}\right)=\pi(\frac{2e-3}{e^2})u^3$$
