السؤال الأول: ضع دائرة حول رمز الاجابه الصحيحة فيما يلي:

1- يُبسط المقدار المثلثي $[3\cos^2\theta + 3\sin^2\theta]$

a) 3 b) 1 c) $3\sec^2\theta$ d) $1-\tan^2\theta$

2- المقدار المكافئ للمقدار المثلثي $[\cos^2\theta + \tan^2\theta\cos^2\theta]$ هو

a) $\cos\theta$ b) 1 c) $\sec^2\theta$ d) $1-\tan^2\theta$

3- احد الاتية يكافئ المقدار المثلثي $[\csc\theta]$ هو

a) $\sin\theta\cot\theta$ b) $\cos\theta\cot\theta$ c) $\sec\theta\cot\theta$ d) $\cos\theta\tan\theta$

4- تبسط المقادير المثلثية الاتية بحيث تساوي (1) عدا

a) $\sec^2 x(1+\cot^2 x)$ b) $\frac{\cos\theta}{\sec\theta}+\frac{\sin\theta}{\csc\theta}$ c) $\cos^2 x(1+\tan^2 x)$ d) $\sin^2 x(1+\cot^2 x)$

5- المقدار المكافئ للمقدار المثلثي $[(\cos\frac{\pi}{2}-x)\tan x-\sec x]$ هو

a) $\sec x$ b) $\cos x$ c) $-\sec x$ d) $-\cos x$

6- يُبسط المقدار المثلثي $[\frac{\sin^2(-\theta)-\cos^2(-\theta)}{\sin(-\theta)-\cos(-\theta)}]$

a) $\sin\theta-\cos\theta$ b) $\cos\theta-\sin\theta$ c) $\sin\theta+\cos\theta$ d) $-\sin\theta-\cos\theta$

7- المقدار المكافئ للمقدار المثلثي $[\frac{(1-\cos\theta)(1+\cos\theta)}{\cos^2\theta}]$ هو

a) $\tan^2\theta$ b) 1 c) $\cot^2\theta$ d) 0

8- يُبسط المقدار المثلثي $[\frac{\sin x}{1+\cos x}+\frac{1+\cos x}{\sin x}]$

a) $\csc x$ b) $\sec x$ c) $2\sec x$ d) $2\csc x$

9- احد الاتية يكافئ المقدار المثلثي $[(\cos x+\sin -x)^2]$ هو

a) $1+\sin 2x$ b) $1-\sin 2x$ c) $1+2\sin x$ d) $1-2\sin x$

10- يمكن كتابة المقدار المثلثي $[\frac{1}{1-\cos x}]$ بحيث لا يحوي كسراً

a) $\sec^2 x-\tan x\sec x$ b) $\sec^2 x+\tan x\sec x$ c) $\csc^2 x-\cot x\csc x$ d) $\csc^2 x+\cot x\csc x$

☒ اذا كان $[\cos\left(x-\frac{\pi}{2}\right)=\frac{3}{5}]$ ، $\frac{\pi}{2}<x<\pi$ اجب عن الفقرات (11،12،13)

11- قيمة $[\sin(x-\frac{\pi}{2})]$ هي

a) $\frac{-3}{5}$ b) $\frac{-4}{5}$ c) $\frac{3}{5}$ d) $\frac{4}{5}$

12- قيمة $[\sec x]$ هي

a) $\frac{5}{3}$ b) $\frac{5}{4}$ c) $\frac{-5}{3}$ d) $\frac{-5}{4}$

13- قيمة $[\cot -x]$ هي

a) $\frac{-3}{4}$ b) $\frac{-4}{3}$ c) $\frac{3}{4}$ d) $\frac{4}{3}$

14- قيمة المقدار $[\ln|\sec x+\tan x|+\ln|\sec x-\tan x|]$ هي

a) 0 b) 1 c) -1 d) غير معرف

15- قيمة $[\cot 165]$ هي

a) $2-\sqrt{3}$ b) $2+\sqrt{3}$ c) $\sqrt{3}-2$ d) $-\sqrt{3}-2$

☒ اذا كان $[\sin\alpha=\frac{4}{5}]$ ، $0<\alpha<\frac{\pi}{2}$ و $[\cos\beta=\frac{-12}{13}]$ ، $\pi<\beta<\frac{3\pi}{2}$ اجب عن الفقرتين (16،17)

16- قيمة $[\cos(\alpha-\beta)]$ هي

a) $\frac{-63}{65}$ b) $\frac{16}{65}$ c) $\frac{56}{65}$ d) $\frac{-56}{65}$

17- قيمة $[\cot(\alpha-\beta)]$ هي

a) $\frac{63}{16}$ b) $\frac{16}{63}$ c) $\frac{56}{33}$ d) $\frac{33}{65}$

☒ اذا كان $[\sin x = \frac{-1}{\sqrt{2}}]$ ، $[\pi < x < \frac{3\pi}{2}]$ اجب عن الفقرتين (18،19)

18- قيمة $[\cos 2x]$ هي

a) 1 b) 0 c) -1 d) $\frac{-1}{\sqrt{2}}$

19- قيمة $[\csc 2x]$ هي

a) 1 b) 0 c) -1 d) $\sqrt{2}$

☒ اذا كان $[\cos\theta = \frac{-3}{5}]$ ، $[\pi < \theta < \frac{3\pi}{2}]$ اجب عن الفقرات (20،21،22،23،24،25)

20- قيمة $[\sin 2\theta]$ هي

a) $\frac{24}{25}$ b) $\frac{-24}{25}$ c) $\frac{-7}{25}$ d) $\frac{7}{25}$

21- قيمة $[\tan 2\theta]$ هي

a) $\frac{-27}{40}$ b) $\frac{-24}{5}$ c) $\frac{-40}{27}$ d) $\frac{-5}{24}$

22- قيمة $[\sin\frac{\theta}{2}]$ هي

a) $\frac{-\sqrt{5}}{5}$ b) $\frac{\sqrt{5}}{5}$ c) $\frac{-2\sqrt{5}}{5}$ d) $\frac{2\sqrt{5}}{5}$

23- قيمة $[\cos\frac{\theta}{2}]$ هي

a) $\frac{-\sqrt{5}}{5}$ b) $\frac{\sqrt{5}}{5}$ c) $\frac{-2\sqrt{5}}{5}$ d) $\frac{2\sqrt{5}}{5}$

24- قيمة $[\tan\frac{\theta}{2}]$ هي

a) $\frac{-1}{2}$ b) $\frac{1}{2}$ c) -2 d) 2

25- قيمة $[\sin 2\theta \tan\frac{\theta}{2}]$ هي

a) 1 b) -1 c) $\frac{48}{25}$ d) $\frac{-48}{25}$

26- يمكن كتابة المقدار المثلثي $[\cos 3\theta]$ على صورة $[\cos\theta]$ على النحو التالي

a) $4\cos^3\theta + 3\cos\theta$ b) $2\cos^3\theta - 3\cos\theta$ c) $4\cos^3\theta - 3\cos\theta$ d) $4\cos^3\theta - \cos\theta$

27- يمكن كتابة المقدار المثلثي $[\sin^4 x \cos^2 x]$ بدلالة القوة الأولى لجيب التمام على النحو التالي

a) $\frac{1}{16}(1 - \cos 2x - \cos 4x + 2\cos x \cos 2x)$ b) $\frac{1}{16}(\cos 2x - \cos 4x + \cos 2x \cos 4x)$

c) $\frac{1}{16}(1 - \cos 2x - \cos 4x + \cos 2x \cos 4x)$ d) $\frac{1}{16}(1 - \cos 2x - \cos 2x \cos 4x)$

28- قيمة $[\tan 22.5°]$ هي

a) $\sqrt{\frac{1-\sqrt{2}}{1+\sqrt{2}}}$ b) $-\sqrt{\frac{\sqrt{2}-1}{\sqrt{2}+1}}$ c) $-\sqrt{\frac{1-\sqrt{2}}{1+\sqrt{2}}}$ d) $\sqrt{\frac{\sqrt{2}-1}{\sqrt{2}+1}}$

29- يمكن كتابة المقدار المثلثي $[\cos 3x \sin 5x]$ في صورة مجموع او فرق على النحو التالي

a) $-\frac{1}{2}[\cos 2x + \cos 8x]$ b) $-\frac{1}{2}[-\sin 2x - \sin 8x]$

c) $\frac{1}{2}[\cos 2x - \cos 8x]$ d) $\frac{1}{2}[\sin 8x - \sin 2x$

30- يمكن كتابة المقدار المثلثي $[\sin 2x - \sin 6x]$ في صورة ضرب على النحو التالي

a) $-2\cos(2x)\sin(4x)$ b) $2\sin(4x)\cos(2x)$

c) $-2\cos(4x)\sin(2x)$ d) $2\sin(2x)\cos(4x)$

31- المقدار المكافئ للمقدار المثلثي $[\frac{\cos x+\sin x}{\cos x-\sin x} - \frac{\cos x-\sin x}{\cos x+\sin x}]$ هو

a) $\tan^2 x$ b) $2\tan 2x$ c) $\tan 2x$ d) $2\tan x$

32- حل المعادلة المثلثية $[\cos x = -\frac{1}{\sqrt{2}}]$ في الفترة $[0, 2\pi]$ هو

a) $\frac{3\pi}{4}, \frac{7\pi}{4}$ b) $\frac{3\pi}{4} + 2k\pi, \frac{7\pi}{4} + 2k\pi$ c) $\frac{3\pi}{4} + 2k\pi, \frac{5\pi}{4} + 2k\pi$ d) $\frac{3\pi}{4}, \frac{5\pi}{4}$

33- اذا كان $[\tan^{-1} -2 = -1.1]$ فإن حل المعادلة المثلثية $[\tan x = -2]$ هو

a) $1.1 + k\pi$ b) $1.1 + 2k\pi$ c) $-1.1 + k\pi$ d) $-1.1 + 2k\pi$

34- اذا كان $[\sin^{-1}\frac{1}{3} = 0.34 , \pi = 3.14]$ فإن حل المعادلة المثلثية $[4\sin\beta + 1 = \sin\beta]$ في الفترة $[0,2\pi]$ هو

a) $\beta = 2.80$ او 5.94 b) $\beta = 3.34$ او 5.94 c) $\beta = 2.80$ او 3.34 d) $\beta = 2.80$ او 6.62

35- حل المعادلة المثلثية $[2\cos x \sin x = -\sin x]$ في الفترة $[0,2\pi]$ هو

a) $(0,\pi,\frac{\pi}{3},\frac{2\pi}{3},2\pi)$ b) $(0,\pi,\frac{2\pi}{3},\frac{4\pi}{3},2\pi)$ c) $(0,\pi,\frac{\pi}{3},\frac{4\pi}{3},2\pi)$ d) $(\pi,\frac{\pi}{3},\frac{2\pi}{3},\frac{4\pi}{3},2\pi)$

36- احد الاتية يعد حل للمعادلة المثلثية $[\cos x \sin x = \frac{1}{2}]$ في الفترة $[0,2\pi]$ هو

a) $(\frac{\pi}{4})$ b) $(\frac{5\pi}{4})$ c) $(\frac{\pi}{2})$ d) $(\frac{3\pi}{4})$

37- عدد حلول المعادلة المثلثية $[\cos x - 1 = \sin x]$ في الفترة $[0,2\pi]$ هو

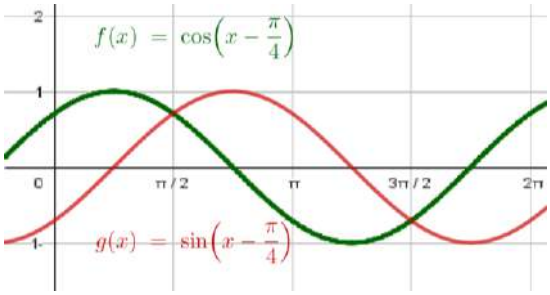
a) 1 b) 2 c) 3 d) 4

38- حل المعادلة المثلثية $[\frac{1+\cos x}{1-\cos x} = 3]$ في الفترة $[0,2\pi]$ هو

a) $(\frac{\pi}{3},\frac{2\pi}{3})$ b) $(\frac{2\pi}{3},\frac{4\pi}{3})$ c) $(\frac{\pi}{3},\frac{5\pi}{3})$ d) $(\frac{2\pi}{3},\frac{5\pi}{3})$

39- حل المعادلة المثلثية $[2\cos\frac{x}{2} - 1 = 0]$ في الفترة $[0,2\pi]$ هو

a) $(\frac{\pi}{3},\frac{5\pi}{3})$ b) $(\frac{5\pi}{3},\frac{10\pi}{3})$ c) $(\frac{\pi}{3},\frac{10\pi}{3})$ d) $(\frac{2\pi}{3},\frac{10\pi}{3})$



40- بالاستعانة بالشكل المجاور حل المعادلة المثلثية $[\cos(x - \frac{\pi}{4}) - \sin(x - \frac{\pi}{4}) = 0]$ في الفترة $[0,2\pi]$ هو

a) $(0,\pi,2\pi)$ b) $(\frac{\pi}{2},\frac{3\pi}{2})$

c) $(0,\pi,\frac{\pi}{2},\frac{3\pi}{2})$ d) $(\pi,\frac{\pi}{2},\frac{3\pi}{2})$

السؤال الثاني: اثبت صحة المتطابقات الآتية:

1- $\ln|\cos x| = \frac{1}{2}(\ln|1 + \cos 2x| - \ln|2|)$

2- $4(\sin^6 x + \cos^6 x) = 4 - 3\sin^2 2x$

3- $\tan^2 x \cos x = \tan x \sin x$

4- $\cos\theta = \frac{1-\tan^2\frac{\theta}{2}}{1+\tan^2\frac{\theta}{2}}$

السؤال الثالث: حل المعادلات المثلثية الآتية في الفترة $[0,2\pi]$:

1- $\sin x + 2\sin x \cos x = 0$

2- $\tan\frac{x}{2} - \sin x = 0$

3- $\sin(\cos x) = 0$

4- $4\sin^2 x - 8\sin x + 3 = 0$

5- $\cos^2 x + 3\cos x + 2 = 0$

6- $2\cos^2 x + 3\sin x = 0$

مع تحيات لؤي مرعي

الإجابات

السؤال الأول

1- $3\cos^2\theta + 3\sin^2\theta$

$= 3(\cos^2\theta + \sin^2\theta)$ $\quad (\cos^2\theta + \sin^2\theta = 1)$

$= 3(1) = 3$ $\quad$ (a)

2- $\cos^2\theta + \tan^2\theta\cos^2\theta$

$= \cos^2\theta(1 + \tan^2\theta)$

$= \cos^2\theta\ \sec^2\theta$

$= \cos^2\theta \frac{1}{\cos^2\theta}$ $\quad , \sec\theta = \frac{1}{\cos\theta}$

$= 1$ $\quad$ (b)

3-

$$\sec\theta\cot\theta = \frac{1}{\cos\theta} \times \frac{\cos\theta}{\sin\theta} = \frac{1}{\sin\theta} = \csc\theta$$

نجرب الخيارات واحدا تلو الآخر للوصول الى الخيار الصحيح وهو (c)

4-

$$\sec^2 x(1 + \cot^2 x) = \sec^2 x \csc^2 x$$

$$= \frac{1}{\cos^2 x} \times \frac{1}{\sin^2 x} = \frac{1}{\cos^2 x \sin^2 x} \neq 1$$

نجرب الخيارات واحدا تلو الآخر للوصول الى الخيار الصحيح وهو (a)

5-

$$(\cos\frac{\pi}{2} - x)\tan x - \sec x = \sin x \tan x - \sec x = \sin x \frac{\sin x}{\cos x} - \sec x$$

$$= \frac{\sin^2 x}{\cos x} - \frac{1}{\cos x} =$$

$$= \frac{\sin^2 x - 1}{\cos x}$$

$$= -\frac{\cos^2 x}{\cos x} = -\cos x \quad (d)$$

6- $\frac{\sin^2(-\theta) - \cos^2(-\theta)}{\sin(-\theta) - \cos(-\theta)}$

$$\frac{\sin^2(-\theta) - \cos^2(-\theta)}{\sin(-\theta) - \cos(-\theta)} = \frac{(-\sin(\theta))^2 - \cos^2(\theta)}{-\sin(\theta) - \cos(\theta)} = \frac{\sin^2(\theta) - \cos^2(\theta)}{-\sin(\theta) - \cos(\theta)}$$

$$= \frac{(\sin x - \cos x)(\sin x + \cos x)}{-(\sin x + \cos x)}$$

$$= -(\sin x + \cos x) = \cos x - \sin x \quad (b)$$

7- $\frac{(1-\cos\theta)(1+\cos\theta)}{\cos^2\theta} = \frac{(1-\cos^2\theta)}{\cos^2\theta} = \frac{\sin^2\theta}{\cos^2\theta} = \tan^2\theta$ $\quad$ (a)

8- $\frac{\sin x}{1+\cos x} + \frac{1+\cos x}{\sin x}$ $\quad$ توحيد المقامات

$= \frac{\sin^2 x+(1+\cos x)^2}{\sin x(1+\cos x)}$

$= \frac{\sin^2 x+1+2\cos x+\cos^2 x}{\sin x(1+\cos x)}$

$= \frac{(\sin^2 x+\cos^2 x)+1+2\cos x}{\sin x(1+\cos x)} = \frac{2+2\cos x}{\sin x(1+\cos x)} = \frac{2(1+\cos x)}{\sin x(1+\cos x)} = \frac{2}{\sin x} = 2\csc x$ (d)

9– $(\cos x + \sin -x)^2 = (\cos x - \sin x)^2 = (\cos^2 x + \sin^2 x) - 2\sin x \cos x$

$= 1 - 2\sin x \cos x = 1 - \sin 2x$ (b)

10– $\frac{1}{1-\cos x}$ بالضرب بالمرافق

$= \frac{1}{1-\cos x} \times \frac{(1+\cos x)}{(1+\cos x)} = \frac{(1+\cos x)}{(1-\cos^2 x)}$, $1 - \cos^2 x = \sin^2 x$

$\frac{(1+\cos x)}{(\sin^2 x)} = \frac{1}{(\sin^2 x)} + \frac{\cos x}{(\sin^2 x)} = \frac{1}{(\sin^2 x)} + (\frac{\cos x}{(\sin x)} \times \frac{1}{\sin x})$

$= \csc^2 x + \cot x \csc x$ (d)

☒ $\cos\left(x - \frac{\pi}{2}\right) = \frac{3}{5}\ \frac{\pi}{2} < x < \pi, \therefore \cos\left(x - \frac{\pi}{2}\right) = \cos\left(-(\frac{\pi}{2} - x)\right) = \cos\left(\frac{\pi}{2} - x\right) = \sin x = \frac{3}{5}$

$\therefore \sin^2 x + \cos^2 x = 1 \Rightarrow \cos^2 x = 1 - \sin^2 x \Rightarrow \cos^2 x = 1 - (\frac{3}{5})^2 \Rightarrow \cos^2 x = \frac{25-9}{25}$

$\therefore \cos^2 x = \frac{16}{25} \Rightarrow \cos x = \pm\frac{4}{5} \Rightarrow \cos x = -\frac{4}{5}, \frac{\pi}{2} < x < \pi$ (الزاوية بالربع الثاني)

11– $\sin\left(x - \frac{\pi}{2}\right) = \sin\left(-(\frac{\pi}{2} - x)\right) = -\sin\left(\frac{\pi}{2} - x\right) = -\cos x = -\left(\frac{-4}{5}\right) = \frac{4}{5}$ (d)

12– $\sec x = \frac{1}{\cos x} = \frac{1}{\frac{-4}{5}} = 1 \times \frac{-5}{4} = \frac{-5}{4}$ (d)

13– $\cot -x = -\cot x = -\frac{\cos x}{\sin x} = -(\frac{\frac{-4}{5}}{\frac{3}{5}}) = -(\frac{-4}{5} \times \frac{5}{3}) = \frac{4}{3}$ (d)

14– $\ln|\sec x + \tan x| + \ln|\sec x - \tan x| = \ln|(\sec x + \tan x)(\sec x - \tan x)| = \ln|\sec^2 x - \tan^2 x|$

$= \ln|1| = 0$, $\sec^2 x = 1 + \tan^2 x$ (a)

15– $\cot 165 = -\cot 15 = -(\frac{\cos 15}{\sin 15})$, $15° = 60° - 45°$

$\therefore \cos(a - b) = (\cos a \cos b + \sin a \sin b) = \cos 60° \cos 45° + \sin 60° \sin 45° = \left(\frac{1}{2} \times \frac{1}{\sqrt{2}}\right) + \left(\frac{\sqrt{3}}{2} \times \frac{1}{\sqrt{2}}\right)$

$= \frac{1}{2\sqrt{2}} + \frac{\sqrt{3}}{2\sqrt{2}} = \frac{1+\sqrt{3}}{2\sqrt{2}}$

$\therefore \sin(a - b) = (\sin a \cos b - \cos a \sin b) = \sin 60° \cos 45° - \cos 60° \sin 45° = \left(\frac{\sqrt{3}}{2} \times \frac{1}{\sqrt{2}}\right) - \left(\frac{1}{2} \times \frac{1}{\sqrt{2}}\right)$

$= \frac{\sqrt{3}}{2\sqrt{2}} - \frac{1}{2\sqrt{2}} = \frac{\sqrt{3}-1}{2\sqrt{2}}$

$\therefore \cot 165 = -\cot 15 = -(\frac{\cos 15}{\sin 15}) = -(\frac{\frac{1+\sqrt{3}}{2\sqrt{2}}}{\frac{\sqrt{3}-1}{2\sqrt{2}}}) = -(\frac{1+\sqrt{3}}{2\sqrt{2}} \times \frac{2\sqrt{2}}{\sqrt{3}-1}) = -(\frac{1+\sqrt{3}}{\sqrt{3}-1}) = -(\frac{1+\sqrt{3}}{\sqrt{3}-1} \times \frac{\sqrt{3}+1}{\sqrt{3}+1}) = -(\frac{1+3+2\sqrt{3}}{3-1})$

$= -\left(\frac{4+2\sqrt{3}}{2}\right) = -\left(\frac{4}{2} + \frac{2\sqrt{3}}{2}\right) = -(2 + \sqrt{3}) = -2 - \sqrt{3}$ (d)

☒ $\sin\alpha = \frac{4}{5}\ ,\ 0 < \alpha < \frac{\pi}{2}$

$\cos\beta = \frac{-12}{13}, \pi < \beta < \frac{3\pi}{2}$

$\Rightarrow\therefore \cos^2\alpha = 1 - \sin^2\alpha = 1 - \left(\frac{4}{5}\right)^2 = \frac{25-16}{25} = \frac{9}{25}$

$\therefore \cos\alpha = \pm\frac{3}{5}\ \ , \therefore \cos\alpha = \frac{3}{5}\ ,\ 0 < \alpha < \frac{\pi}{2}$ (الزاوية بالربع الاول)

$\Rightarrow\therefore \sin^2\beta = 1 - \cos^2\beta = 1 - \left(\frac{-12}{13}\right)^2 = \frac{169-144}{169} = \frac{25}{169}$

$\therefore \sin\beta = \pm\frac{5}{13}\ , \therefore \sin\beta = \frac{-5}{13}\ , \pi < \beta < \frac{3\pi}{2}$ (الزاوية بالربع الثالث)

$\Rightarrow\therefore \tan\alpha = \frac{\sin\alpha}{\cos\alpha} = \left(\frac{\frac{4}{5}}{\frac{3}{5}}\right) = \frac{4}{5}\times\frac{5}{3} = \frac{4}{3}$

$\Rightarrow\therefore \tan\beta = \frac{\sin\beta}{\cos\beta} = \left(\frac{\frac{-5}{13}}{\frac{-12}{13}}\right) = \frac{-5}{13}\times\frac{13}{-12} = \frac{5}{12}$

16– $\cos(\alpha-\beta) = (\cos\alpha\cos\beta + \sin\alpha\sin\beta) = \left(\frac{3}{5}\times\frac{-12}{13}\right) + \left(\frac{4}{5}\times\frac{-5}{13}\right) = \left(\frac{-36}{65}\right) + \left(\frac{-20}{65}\right) = \frac{-36-20}{65}$

$\cos(\alpha-\beta) = \frac{-56}{65}$ (d)

17- $\cot(\alpha-\beta) = \frac{1}{\tan(\alpha-\beta)}$

$\therefore \tan(\alpha-\beta) = \frac{\tan\alpha-\tan\beta}{1+\tan\alpha\tan\beta} = \frac{\frac{4}{3}-\frac{5}{12}}{1+(\frac{4}{3}\times\frac{5}{12})} = \frac{\frac{16-5}{12}}{1+\frac{20}{36}} = \frac{\frac{11}{12}}{\frac{36+20}{36}} = \frac{\frac{11}{12}}{\frac{56}{36}} = \frac{11}{12}\times\frac{36}{56} = \frac{33}{56}$

$\therefore \cot(\alpha-\beta) = \frac{1}{\tan(\alpha-\beta)} = \frac{1}{\frac{33}{56}} = \frac{56}{33}$ (c)

☒ $\sin x = \frac{-1}{\sqrt{2}}\ ,\ \pi < x < \frac{3\pi}{2}$,(الزاوية بالربع الثالث)

$\therefore \sin^2 x + \cos^2 x = 1 \Rightarrow \cos^2 x = 1 - \sin^2 x \Rightarrow \cos^2 x = 1 - \left(\frac{-1}{\sqrt{2}}\right)^2 = 1 - \frac{1}{2} = \frac{1}{2}$

$\therefore \cos x = \pm\frac{1}{\sqrt{2}}\ \ , \therefore \cos x = -\frac{1}{\sqrt{2}}\ \ , \pi < x < \frac{3\pi}{2}$,(الزاوية بالربع الثالث)

18– $\cos 2x = 1 - 2\sin^2 x = 1 - \left(2\times\frac{1}{2}\right) = 1 - 1 = 0$ (b)

19– $\csc 2x = \frac{1}{\sin 2x} \Rightarrow\therefore \sin 2x = 2\sin x\cos x = 2\times\frac{-1}{\sqrt{2}}\times\frac{-1}{\sqrt{2}} = 1$

$\therefore \csc 2x = \frac{1}{\sin 2x} = \frac{1}{1} = 1$ (a)

☒ $\cos\theta = \frac{-3}{5}\ ,\ \pi < \theta < \frac{3\pi}{2}$,(الزاوية بالربع الثالث)

$\Rightarrow\therefore \sin^2\theta = 1 - \cos^2\theta = 1 - \left(\frac{-3}{5}\right)^2 = \frac{25-9}{25} = \frac{16}{25}$

$\therefore \sin\theta = \pm\frac{4}{5} \Rightarrow \sin\theta = \frac{-4}{5}\ \ , \pi < \theta < \frac{3\pi}{2}$,(الزاوية بالربع الثالث)

20– $\sin 2\theta = 2\sin\theta\cos\theta = 2\times\frac{-4}{5}\times\frac{-3}{5} = \frac{24}{25}$ (a)

21– $\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta}$

$\Rightarrow\therefore \tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{\frac{-4}{5}}{\frac{-3}{5}} = \frac{-4}{5}\times\frac{5}{-3} = \frac{4}{3}$

$\Rightarrow\therefore \tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta} = \frac{2\times\frac{4}{3}}{1-(\frac{4}{3})^2} = \frac{\frac{8}{3}}{1-\frac{16}{9}} = \frac{\frac{8}{3}}{\frac{9-16}{9}} = \frac{\frac{8}{3}}{\frac{-5}{9}} = \frac{8}{3}\times\frac{9}{-5} = \frac{-72}{15} = \frac{-24}{5}$ (b)

22–$\sin\frac{\theta}{2} = \pm\sqrt{\frac{1-\cos\theta}{2}}$, $\pi < \theta < \frac{3\pi}{2}$,(الزاوية بالربع الثالث) , $\frac{\pi}{2} < \frac{\theta}{2} < \frac{3\pi}{4}$, الزاوية $(\frac{\theta}{2})$ بالربع الثاني

$\therefore \sin\frac{\theta}{2} = +\sqrt{\frac{1-\cos\theta}{2}} = \sqrt{\frac{1-\frac{-3}{5}}{2}} = \sqrt{\frac{\frac{5-(-3)}{5}}{2}} = \sqrt{\frac{\frac{5+3}{5}}{2}} = \sqrt{\frac{\frac{8}{5}}{2}} = \sqrt{\frac{8}{5}\times\frac{1}{2}} = \sqrt{\frac{8}{10}} = \sqrt{\frac{4}{5}} = \frac{\sqrt{4}}{\sqrt{5}} = \frac{2}{\sqrt{5}} = \frac{2\sqrt{5}}{5}$ (d)

23- $\cos\frac{\theta}{2} = \pm\sqrt{\frac{1+\cos\theta}{2}}$, $\pi < \theta < \frac{3\pi}{2}$ (الزاوية بالربع الثالث), $\frac{\pi}{2} < \frac{\theta}{2} < \frac{3\pi}{4}$, الزاوية $(\frac{\theta}{2})$ بالربع الثاني

$\therefore \cos\frac{\theta}{2} = -\sqrt{\frac{1+\cos\theta}{2}} = -\sqrt{\frac{1+\frac{-3}{5}}{2}} = -\sqrt{\frac{\frac{5+(-3)}{5}}{2}} = -\sqrt{\frac{\frac{5-3}{5}}{2}} = -\sqrt{\frac{\frac{2}{5}}{2}} = -\sqrt{\frac{2}{5}\times\frac{1}{2}} = -\sqrt{\frac{1}{5}} = -\frac{\sqrt{1}}{\sqrt{5}} = -\frac{1}{\sqrt{5}} = -\frac{\sqrt{5}}{5}$ (a)

24- $\tan\frac{\theta}{2} = \pm\sqrt{\frac{1-\cos\theta}{1+\cos\theta}}$, $\pi < \theta < \frac{3\pi}{2}$ (الزاوية بالربع الثالث), $\frac{\pi}{2} < \frac{\theta}{2} < \frac{3\pi}{4}$, الزاوية $(\frac{\theta}{2})$ بالربع الثاني

$\therefore \tan\frac{\theta}{2} = -\sqrt{\frac{1-\cos\theta}{1+\cos\theta}} = -\sqrt{\frac{1-\frac{-3}{5}}{1+\frac{-3}{5}}} = -\sqrt{\frac{\frac{5-(-3)}{5}}{\frac{5+(-3)}{5}}} = -\sqrt{\frac{\frac{5+3}{5}}{\frac{5-3}{5}}} = -\sqrt{\frac{\frac{8}{5}}{\frac{2}{5}}} = -\sqrt{\frac{8}{5}\times\frac{5}{2}} = -\sqrt{\frac{8}{2}} == -\sqrt{4} = -2$ (c)

25- $\sin 2\theta \tan\frac{\theta}{2} = \frac{24}{25}\times -2 = \frac{-48}{25}$ (d)

26- $\cos 3\theta = \cos(2\theta+\theta) = \cos 2\theta\cos\theta - \sin 2\theta\sin\theta$

$= (2\cos^2\theta - 1)\cos\theta - (2\sin\theta\cos\theta)\sin\theta$, $\cos 2\theta = 2\cos^2\theta - 1$, $\sin 2\theta = 2\sin\theta\cos\theta$

$= 2\cos^3\theta - \cos\theta - 2\cos\theta\sin^2\theta = 2\cos^3\theta - \cos\theta - 2\cos\theta(1-\cos^2\theta)$, $\sin^2\theta = 1-\cos^2\theta$

$= 2\cos^3\theta - \cos\theta - 2\cos\theta + \cos^3\theta) = 4\cos^3\theta - 3\cos\theta$ (c)

27- $\sin^4 x\cos^2 x = \sin^2 x(\sin^2 x\ \cos^2 x)$

$= \sin^2 x\ \sin^2 x\ \cos^2 x = (\frac{1-\cos 2x}{2})(\frac{1-\cos 2x}{2})(\frac{1+\cos 2x}{2})$, $\sin^2 x = \frac{1-\cos 2x}{2}$, $\cos^2 x = \frac{1+\cos 2x}{2}$

$= \left(\frac{1-\cos 2x}{2}\right)\left(\frac{1-\cos^2 2x}{4}\right) = \left(\frac{1-\cos 2x}{2}\right)\left(\frac{1}{4} - \frac{\cos^2 2x}{4}\right) = \left(\frac{1-\cos 2x}{2}\right)\left(\frac{1}{4} - \frac{1}{4}\cos^2 2x\right)$

$= \left(\frac{1-\cos 2x}{2}\right)\left(\frac{1}{4} - \frac{1}{4}(\frac{1+\cos 4x}{2})\right)$, $\cos^2 x = \frac{1+\cos 2x}{2}$

$= \left(\frac{1-\cos 2x}{2}\right)\left(\frac{1}{4} - \frac{1}{8} - (\frac{\cos 4x}{8})\right) = \left(\frac{1-\cos 2x}{2}\right)\left(\frac{2-1}{8} - (\frac{\cos 4x}{8})\right) = \left(\frac{1-\cos 2x}{2}\right)\left(\frac{1}{8} - (\frac{\cos 4x}{8})\right)$

$= \left(\frac{1-\cos 2x}{2}\right)\left(\frac{1}{8}(1-\cos 4x)\right) = \frac{1}{16}((1-\cos 2x)(1-\cos 4x)$

$= \frac{1}{16}(1 - \cos 2x - \cos 4x + \cos 2x\cos 4x)$ (c)

28- $\tan 22.5° = \tan\frac{45°}{2} = \sqrt{\frac{1-\cos\theta}{1+\cos\theta}}$, $0 < \frac{\theta}{2} < \frac{\pi}{2}$ الزاوية $(\frac{\theta}{2})$ بالربع الاول,

$= \tan\frac{45°}{2} = \sqrt{\frac{1-\cos 45°}{1+\cos 45°}} = \sqrt{\frac{1-\frac{1}{\sqrt{2}}}{1+\frac{1}{\sqrt{2}}}} = \sqrt{\frac{\frac{\sqrt{2}-1}{\sqrt{2}}}{\frac{\sqrt{2}+1}{\sqrt{2}}}} = \sqrt{\frac{\sqrt{2}-1}{\sqrt{2}}\times\frac{\sqrt{2}}{\sqrt{2}+1}} = \sqrt{\frac{\sqrt{2}-1}{\sqrt{2}+1}}$ (d)

29- $\cos 3x\sin 5x = -\frac{1}{2}[\sin(\alpha-\beta) - \sin(\alpha+\beta)] = -\frac{1}{2}[\sin(3x-5x) - \sin(3x+5x)]$

$= -\frac{1}{2}[\sin(-2x) - \sin(8x)]$ (b)

30- $\sin 2x - \sin 6x = 2\cos(\frac{\alpha+\beta}{2})\sin(\frac{\alpha-\beta}{2}) = 2\cos(\frac{2x+6x}{2})\sin(\frac{2x-6x}{2}) = 2\cos(\frac{8x}{2})\sin(\frac{-4x}{2})$

$= 2\cos 4x\sin -2x = -2\cos 4x\sin 2x$ (c)

31- $\frac{\cos x+\sin x}{\cos x-\sin x} - \frac{\cos x-\sin x}{\cos x+\sin x} = \frac{(\cos x+\sin x)^2-(\cos x-\sin x)^2}{\cos^2 x-\sin^2 x} = \frac{(\cos^2 x+\sin^2 x+2\cos x\sin x)-(\cos^2 x+\sin^2 x-2\cos x\sin x)}{\cos^2 x-\sin^2 x}$

$= \frac{\cos^2 x+\sin^2 x+2\cos x\sin x-\cos^2 x-\sin^2 x+2\cos x\sin x)}{\cos^2 x-\sin^2 x} = \frac{4\cos x\sin x}{\cos 2x}$, $\cos 2x = \cos^2 x - \sin^2 x$

$= \frac{2(2\cos x\sin x)}{\cos 2x}$, $\sin 2x = 2\sin x\cos x$

$= \frac{2\sin 2x}{\cos 2x} = 2\tan 2x$ (b)

32- $\cos x = -\frac{1}{\sqrt{2}}$, $[0,2\pi]$

$\therefore \cos x = \left|-\frac{1}{\sqrt{2}}\right| = \frac{1}{\sqrt{2}} \Rightarrow x = 45° = \frac{\pi}{4}$, سالب بالربع الثاني والثالث $(\cos x)$

$\Rightarrow x = \pi - \frac{\pi}{4} = \frac{4\pi-\pi}{4} = \frac{3\pi}{4}$ الربع الثاني,

$\Rightarrow x = \pi + \frac{\pi}{4} = \frac{4\pi+\pi}{4} = \frac{5\pi}{4}$ الربع الثالث, $\Rightarrow \therefore x = \frac{3\pi}{4}, x = \frac{5\pi}{4}$ (d)

33- $\tan x = -2 \Rightarrow x = \tan^{-1} -2 = -1.1$, من السؤال

$\because \tan x \in [-\frac{\pi}{2}, \frac{\pi}{2}], [-\frac{\pi}{2}, 0]$ سالب بالفترة $(\tan x)$

$\Rightarrow \therefore x = -1,1 + k\pi$ (c)

34- $4\sin\beta + 1 = \sin\beta$, $\sin^{-1}\frac{1}{3} = 0.34, \pi = 3.14, [0,2\pi]$

$4\sin\beta + 1 = \sin\beta \Rightarrow 4\sin\beta - \sin\beta = -1 \Rightarrow 3\sin\beta = -1 \Rightarrow \sin\beta = \frac{-1}{3}$

$\therefore \sin\beta = \left|\frac{-1}{3}\right| = \frac{1}{3} \Rightarrow \beta = \sin^{-1}\frac{1}{3} = 0.34$, سالب بالربع الثالث والرابع $(\sin\beta)$

$\Rightarrow \beta = \pi + 0,34 = 3,48$, $\in [0,2\pi]$ الربع الثالث

$\Rightarrow \beta = 2\pi - 0,34 = 5,94$, $\in [0,2\pi]$ الربع الرابع $\Rightarrow \beta = 3.48, \beta = 5.94$ (b)

35- $2\cos x \sin x = -\sin x$, $[0,2\pi]$

$2\cos x \sin x + \sin x = 0 \Rightarrow \sin x(2\cos x + 1) = 0$

$\therefore \sin x = 0 \Rightarrow x = 0 \; or \; \pi \; or \; 2\pi \in [0,2\pi]$

or $\therefore 2\cos x + 1 = 0 \Rightarrow \cos x = \frac{-1}{2} \Rightarrow \therefore \cos x = \left|-\frac{1}{2}\right| = \frac{1}{2} \Rightarrow x = 60° = \frac{\pi}{3}$, سالب بالربع الثاني والثالث $(\cos x)$

$\Rightarrow x = \pi - \frac{\pi}{3} = \frac{3\pi-\pi}{3} = \frac{2\pi}{3}$ الربع الثاني,

$\Rightarrow x = \pi + \frac{\pi}{3} = \frac{3\pi+\pi}{3} = \frac{4\pi}{3}$ الربع الثالث, $\Rightarrow \therefore x = \frac{2\pi}{3}, x = \frac{4\pi}{3} \in [0,2\pi]$

$\therefore x = 0, \frac{2\pi}{3}, \pi, \frac{4\pi}{3}, 2\pi \in [0,2\pi]$ (b)

36- $\cos x \sin x = \frac{1}{2}$, $[0,2\pi]$

$\therefore \cos x \sin x = \frac{1}{2} \Rightarrow 2\cos x \sin x = 1 \Rightarrow \sin 2x = 1$, $\sin 2x = 2\sin x \cos x$

$\therefore 2x = \frac{\pi}{2} \Rightarrow x = \frac{\pi}{4}$, $\in [0,2\pi]$ موجب بالربع الأول والثاني , $(\sin 2x)$ (a)

37- $\cos x - 1 = \sin x$, $[0,2\pi]$

$(\cos x - 1)^2 = (\sin x)^2$ بتربيع الطرفين

$\Rightarrow \cos^2 x - 2\cos x + 1 = \sin^2 x \Rightarrow \cos^2 x - \sin^2 x - 2\cos x + 1 = 0$

$\Rightarrow \cos^2 x - (1 - \cos^2 x) - 2\cos x + 1 = 0$, $\sin^2 x = 1 - \cos^2 x$

$\Rightarrow \cos^2 x - 1 + \cos^2 x - 2\cos x + 1 = 0 \Rightarrow 2\cos^2 x - 2\cos x = 0 \Rightarrow 2\cos x(\cos x - 1) = 0$

$\therefore 2\cos x = 0 \Rightarrow \cos x = 0 \Rightarrow \therefore x = 90° = \frac{\pi}{2}, 270° = \frac{3\pi}{2}$ نعوض قيم (x) في المعادلة المثلثية الرئيسية

$\therefore \cos\frac{\pi}{2} - 1 = \sin\frac{\pi}{2} \Rightarrow -1 \neq 1 \; \therefore x = \frac{\pi}{2}$ مرفوضة لا تحقق المعادلة

$\therefore \cos\frac{3\pi}{2} - 1 = \sin\frac{3\pi}{2} \Rightarrow -1 = -1 \; \therefore x = \frac{3\pi}{2}$ تحقق المعادلة

or $\therefore \cos x - 1 = 0 \Rightarrow \cos x = 1 \Rightarrow \therefore \cos x = 1 \Rightarrow x = 0° = 0, 360° = 2\pi$ نعوض قيم (x) في المعادلة المثلثية الرئيسية

$\therefore \cos 0 - 1 = \sin 0 \Rightarrow 1 - 1 = 0 \; \therefore x = 0$ تحقق المعادلة

$\therefore \cos 2\pi - 1 = \sin 2\pi \Rightarrow 1 - 1 = 0 \; \therefore x = 2\pi$ تحقق المعادلة ∴

(c) عدد الحلول هو (3)

38- $\frac{1+\cos x}{1-\cos x} = 3$, $[0,2\pi]$

$1 + \cos x = 3(1 - \cos x) \Rightarrow 1 + \cos x = 3 - 3\cos x \Rightarrow \cos x + 3\cos x = 3 - 1 \Rightarrow 4\cos x = 2$

$\Rightarrow \cos x = \frac{2}{4} = \frac{1}{2} \Rightarrow x = 60° = \frac{\pi}{3}$, موجب بالربع الاول والرابع $(\cos x)$

$\Rightarrow x = 2\pi - \frac{\pi}{3} = \frac{6\pi-\pi}{3} = \frac{5\pi}{3}$ الربع الرابع, $\Rightarrow \therefore x = \frac{\pi}{3}, x = \frac{5\pi}{3} \in [0,2\pi]$ (c)

39- $2\cos\frac{x}{2} - 1 = 0$, $[0,2\pi]$

$2\cos\frac{x}{2} - 1 = 0 \Rightarrow 2\cos\frac{x}{2} = 1 \Rightarrow \cos\frac{x}{2} = \frac{1}{2}$

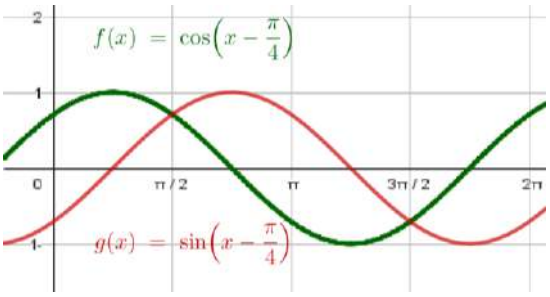
$\Rightarrow \therefore \frac{x}{2} = \frac{\pi}{3} \Rightarrow x = \frac{\pi}{3} \times 2 = \frac{2\pi}{3}$

$\Rightarrow \therefore x = \left(2\pi - \frac{\pi}{3}\right) \times 2 = \left(\frac{6\pi-\pi}{3}\right) \times 2 = \frac{10\pi}{3}$, $\therefore x = \frac{2\pi}{3}, \frac{10\pi}{3} \in [0, 2\pi]$, موجب بالربع الأول والرابع ($\cos\frac{x}{2}$) (d)

40- $\cos(x - \frac{\pi}{4}) - \sin(x - \frac{\pi}{4}) = 0$, $[0,2\pi]$

$\cos(x - \frac{\pi}{4}) = \sin(x - \frac{\pi}{4})$, $(x = \frac{\pi}{2}, \frac{3\pi}{2})$ مجموعة الحل هي نقاط تقاطع المنحنيين معا وهي

(b)



السؤال الثاني:

1- $\ln|\cos x| = \frac{1}{2}(\ln|1 + \cos 2x| - \ln|2|)$

$\Rightarrow \frac{1}{2}(\ln|1 + \cos 2x| - \ln|2|) = \frac{1}{2}(ln\left|\frac{1+cos\,2x}{2}\right|) = \frac{1}{2}\ln|\cos^2 x| = \frac{1}{2} \times 2\ln|\cos x| = \ln|\cos x|$

2- $4(\sin^6 x + \cos^6 x) = 4 - 3\sin^2 2x$

$4(\sin^6 x + \cos^6 x) = 4(\sin^2 x + \cos^2 x)(\sin^4 x - \sin^2 x \cos^2 x + \cos^4 x)$, $\sin^2 x + \cos^2 x = 1$

$= 4(1)(\sin^4 x - \sin^2 x \cos^2 x + \cos^4 x) = 4((\sin^2 x)^2 - \sin^2 x \cos^2 x + (\cos^2 x)^2)$

$= 4((\frac{1-\cos 2x}{2})^2 - ((\frac{1-\cos 2x}{2})(\frac{1+\cos 2x}{2})) + (\frac{1+\cos 2x}{2})^2)$, $\sin^2 x = \frac{1-\cos 2x}{2}$, $\cos^2 x = \frac{1+\cos 2x}{2}$

$= 4\left(\left(\frac{1 - 2\cos 2x + \cos^2 2x}{4}\right) - \left(\frac{1 - \cos^2 2x}{4}\right) + \left(\frac{1 + 2\cos 2x + \cos^2 2x}{4}\right)\right)$

$= 4\left(\frac{1 - 2\cos 2x + \cos^2 2x - 1 + \cos^2 2x + 1 + 2\cos 2x + \cos^2 2x}{4}\right)$

$= 4\left(\frac{\cos^2 2x + \cos^2 2x + 1 + \cos^2 2x}{4}\right) = 4\left(\frac{3\cos^2 2x + 1}{4}\right) = 4\left(\frac{1}{4} + \frac{3\cos^2 2x}{4}\right)$

$= 4\left(\frac{1}{4} + \frac{3}{4}(\cos^2 2x)\right) = 4\left(\frac{1}{4} + \frac{3}{4}(1 - \sin^2 2x)\right)$, $\cos^2 2x = 1 - \sin^2 2x$

$= 4\left(\frac{1}{4} + \frac{3}{4} - \frac{3}{4}\sin^2 2x)\right) = 4\left(1 - \frac{3}{4}\sin^2 2x\right) = 4 - \frac{12}{4}\sin^2 2x = 4 - 3\sin^2 2x$

3- $\tan^2 x \cos x = \tan x \sin x$

$\tan^2 x \cos x = \frac{\sin^2 x}{\cos^2 x} \cos x = \frac{\sin^2 x}{\cos x} = \frac{\sin x}{\cos x} \sin x = \tan x \sin x$

4- $\cos\theta = \frac{1-\tan^2\frac{\theta}{2}}{1+\tan^2\frac{\theta}{2}}$

$\frac{1-\tan^2\frac{\theta}{2}}{1+\tan^2\frac{\theta}{2}} = \frac{1-\frac{\sin^2\frac{\theta}{2}}{\cos^2\frac{\theta}{2}}}{1+\frac{\sin^2\frac{\theta}{2}}{\cos^2\frac{\theta}{2}}} = \frac{\frac{\cos^2\frac{\theta}{2}-\sin^2\frac{\theta}{2}}{\cos^2\frac{\theta}{2}}}{\frac{\cos^2\frac{\theta}{2}+\sin^2\frac{\theta}{2}}{\cos^2\frac{\theta}{2}}} = \left(\frac{\cos^2\frac{\theta}{2}-\sin^2\frac{\theta}{2}}{\cos^2\frac{\theta}{2}}\right) \times \left(\frac{\cos^2\frac{\theta}{2}}{\cos^2\frac{\theta}{2}+\sin^2\frac{\theta}{2}}\right) = \frac{\cos^2\frac{\theta}{2}-\sin^2\frac{\theta}{2}}{\cos^2\frac{\theta}{2}+\sin^2\frac{\theta}{2}} = \frac{\cos^2\frac{\theta}{2}-\sin^2\frac{\theta}{2}}{1}$, $\cos^2\frac{\theta}{2} + \sin^2\frac{\theta}{2} = 1$

$= \cos^2\frac{\theta}{2} - \sin^2\frac{\theta}{2} = \cos 2\frac{\theta}{2} = \cos\theta$, $\cos^2\theta - \sin^2\theta = \cos 2\theta$

السؤال الثالث:

1- $\sin x + 2\sin x \cos x = 0$, $[0,2\pi]$

$\sin x + 2\sin x \cos x = 0 \Rightarrow \sin x(1 + 2\cos x) = 0$

$\therefore \sin x = 0 \Rightarrow x = 0, \pi, 2\pi \in [0, 2\pi]$

Or $1 + 2\cos x = 0 \Rightarrow 2\cos x = -1 \Rightarrow \cos x = \frac{-1}{2}$

$\Rightarrow \therefore \cos x = \left|-\frac{1}{2}\right| = \frac{1}{2} \Rightarrow x = 60° = \frac{\pi}{3}$, سالب بالربع الثاني والثالث $(\cos x)$

$\Rightarrow x = \pi - \frac{\pi}{3} = \frac{3\pi-\pi}{3} = \frac{2\pi}{3}$ الربع الثاني,

$\Rightarrow x = \pi + \frac{\pi}{3} = \frac{3\pi+\pi}{3} = \frac{4\pi}{3}$ الربع الثالث, $\Rightarrow \therefore x = \frac{2\pi}{3}, x = \frac{4\pi}{3} \in [0, 2\pi]$

$\therefore x = 0, \frac{2\pi}{3}, \pi, \frac{4\pi}{3}, 2\pi \in [0, 2\pi]$

2- $\tan\frac{x}{2} - \sin x = 0$, $[0,2\pi]$

$\frac{\sin\frac{x}{2}}{\cos\frac{x}{2}} - \sin x = 0 \Rightarrow \frac{\sin\frac{x}{2}}{\cos\frac{x}{2}} - 2\sin\frac{x}{2}\cos\frac{x}{2} = 0$, $\sin 2x = 2\sin x\cos x \Rightarrow \sin x = 2\sin\frac{x}{2}\cos\frac{x}{2}$

$\Rightarrow \frac{\sin\frac{x}{2} - 2\sin\frac{x}{2}\cos^2\frac{x}{2}}{\cos\frac{x}{2}} = 0 \Rightarrow \frac{\sin\frac{x}{2}(1-2\cos^2\frac{x}{2})}{\cos\frac{x}{2}} = 0 \Rightarrow \sin\frac{x}{2}(1 - 2\cos^2\frac{x}{2}) = 0$

$\therefore \sin\frac{x}{2} = 0 \Rightarrow \therefore \frac{x}{2} = 0, \pi, 2\pi \Rightarrow x = 0 \times 2 = 0 \in [0, 2\pi], x = \pi \times 2 = 2\pi \in [0, 2\pi],$

$x = 2\pi \times 2 = 4\pi \notin [0, 2\pi] \Rightarrow \therefore x = 0, 2\pi \in [0, 2\pi]$

or $1 - 2\cos^2\frac{x}{2} = 0 \Rightarrow 1 = 2\cos^2\frac{x}{2} \Rightarrow \cos^2\frac{x}{2} = \frac{1}{2} \Rightarrow \cos\frac{x}{2} = \pm\frac{1}{\sqrt{2}}$, الزاوية $(\frac{x}{2})$ تقع في جميع الارباع

$\Rightarrow \therefore \cos\frac{x}{2} = \left|-\frac{1}{\sqrt{2}}\right| = \frac{1}{\sqrt{2}}$, $(\cos\frac{x}{2})$ موجب بالربع الاول والرابع

$\Rightarrow \frac{x}{2} = 45° = \frac{\pi}{4} \Rightarrow x = 2 \times \frac{\pi}{4} = \frac{\pi}{2} \in [0, 2\pi]$, $\frac{x}{2} = 2\pi - \frac{\pi}{4} = \frac{8\pi-\pi}{4} = \frac{7\pi}{4} \Rightarrow x = 2 \times \frac{7x}{4} = \frac{7\pi}{2} \notin [0, 2\pi]$

$\therefore x = \frac{\pi}{2} \in [0, 2\pi]$

$\Rightarrow \therefore \cos\frac{x}{2} = -\frac{1}{\sqrt{2}}$, $(\cos\frac{x}{2})$ سالب بالربع الثاني والثالث

$\Rightarrow \frac{x}{2} = 135° = \pi - \frac{\pi}{4} = \frac{4\pi-\pi}{4} = \frac{3\pi}{4} \Rightarrow x = 2 \times \frac{3\pi}{4} = \frac{3\pi}{2} \in [0, 2\pi]$, $\frac{x}{2} = \pi + \frac{\pi}{4} = \frac{4\pi+\pi}{4} = \frac{5\pi}{4}$

$\Rightarrow x = 2 \times \frac{5\pi}{4} = \frac{5\pi}{2} \notin [0, 2\pi]$

$\therefore x = \frac{3\pi}{2} \in [0, 2\pi] \Rightarrow \therefore x = 0, \frac{\pi}{2}, \frac{3\pi}{2}, 2\pi \in [0, 2\pi]$

3- $\sin(\cos x) = 0$, $[0,2\pi]$

$\sin(y) = 0$, $y = \cos x \Rightarrow \therefore y = 0, \pi, 2\pi \Rightarrow \therefore \cos x = 0 \Rightarrow \therefore x = \frac{\pi}{2}, \frac{3\pi}{2}$

$\therefore \cos x = \pi \Rightarrow \therefore \cos x = \phi$, $(\cos x)$ مدى في الفترة $[-1,1]$,

$\therefore \cos x = 2\pi \Rightarrow \therefore \cos x = \phi$, $(\cos x)$ مدى في الفترة $[-1,1]$,

4- $4\sin^2 x - 8\sin x + 3 = 0$, $[0,2\pi]$

$(2\sin x - 1)(2\sin x - 3) = 0$

$2\sin x - 1 = 0 \Rightarrow 2\sin x = 1 \Rightarrow \sin x = \frac{1}{2}$, $(\sin x)$ موجب بالربع الأول والثاني

$\therefore x = \frac{\pi}{6}$, $\therefore x = \pi - \frac{\pi}{6} = \frac{6\pi-\pi}{6} = \frac{5\pi}{6} \Rightarrow x = \frac{\pi}{6}, \frac{5\pi}{6} \in [0, 2\pi]$

or $2\sin x - 3 = 0 \Rightarrow 2\sin x = 3 \Rightarrow \sin x = \frac{3}{2} \Rightarrow \therefore \sin x = \phi$, $(\sin x)$ مدى في الفترة $[-1,1]$

5- $\cos^2 x + 3\cos x + 2 = 0$, $[0,2\pi]$

$(\cos x + 2)(\cos x + 1) = 0$

$\cos x + 2 = 0 \Rightarrow \cos x = -2 \Rightarrow \therefore \cos x = \phi$, $(\cos x)$ مدى في الفترة $[-1,1]$,

or $\cos x + 1 = 0 \Rightarrow \cos x = -1 \Rightarrow \therefore x = -\pi \in [0, 2\pi]$

6- $2\cos^2 x + 3\sin x = 0$, $[0,2\pi]$

$2(1 - \sin^2 x) + 3\sin x = 0$, $\cos^2 x = 1 - \sin^2 x$

$2 - 2\sin^2 x + 3\sin x = 0 \Rightarrow -2 + 2\sin^2 x - 3\sin x = 0$ بالضرب في $(-1$

$2\sin^2 x - 3\sin x - 2 = 0 \Rightarrow (2\sin x + 1)(\sin x - 2) = 0$

$2\sin x + 1 = 0 \Rightarrow 2\sin x = -1 \Rightarrow \sin x = -\frac{1}{2} \Rightarrow \sin x = \left|-\frac{1}{2}\right| = \frac{1}{2} \ \therefore x = \frac{\pi}{6},$

سالب بالربع الثالث والرابع $(\sin x)$ $, \therefore x = \pi + \frac{\pi}{6} = \frac{6\pi+\pi}{6} = \frac{7\pi}{6}, \therefore x = 2\pi - \frac{\pi}{6} = \frac{12\pi-\pi}{6} = \frac{11\pi}{6}$

$\Rightarrow x = \frac{7\pi}{6}, \frac{11\pi}{6} \in [0, 2\pi]$

$\sin x - 2 = 0 \Rightarrow \sin x = 2 \Rightarrow \therefore \sin x = \phi$,$[-1,1]$ في الفترة $(\sin x)$ مدى